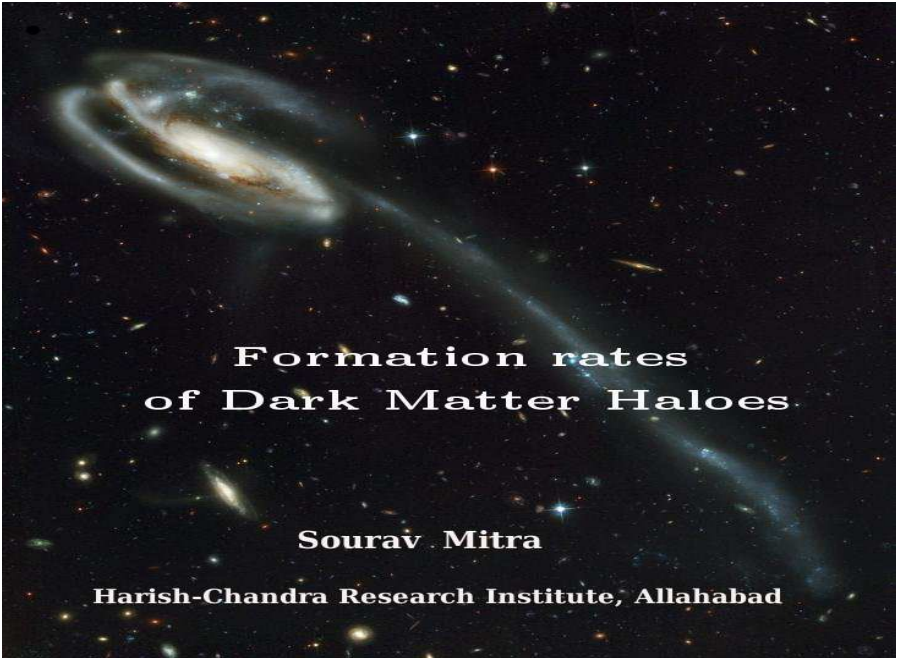

Formation rates of Dark Matter Haloes

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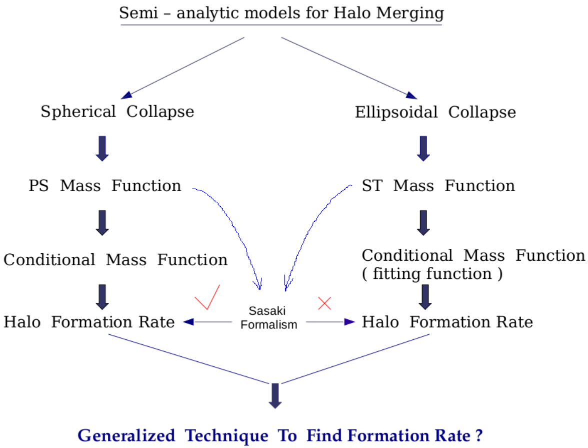
Motivation

<u>Numerical Simulations</u>

Most powerful methods to study the formation of dark matter halos



Spherical Collapse vs. Ellipsoidal Collapse

- **Press-Schechter Mass Function** : The number of virialized objects with mass between M and $M + dM$

$$\frac{dn}{dM}dM = \sqrt{\frac{2}{\pi}} \frac{\rho_M}{M^2} \frac{\delta_c}{\sigma} \left| \frac{d \ln \sigma}{d \ln M} \right| \exp\left[-\frac{\delta_c^2}{2\sigma^2} \right] dM$$

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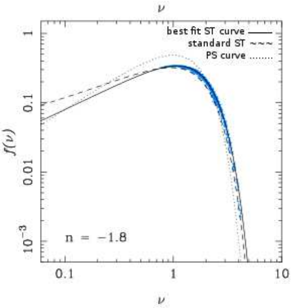


- fewer high-mass and more low-mass objects than are seen in N-body simulations

- **Sheth -Tormen (1999)** : discrepancy reduced substantially if bound structures are assumed to form an **ellipsoidal collapse**.

Nontrivial Barrier Shapes and Ellipsoidal Collapse

- Ellipsoidal barrier is higher for higher S (lower mass)

$$B(\sigma, z) = \delta_{ec}(\sigma, z) = \sqrt{a}\delta_{sc}(z)\left[1 + \beta(a\nu^2)^{-\gamma}\right]$$

where $\nu = \delta_{sc}(z)/\sigma(m)$ and $a = 0.75$, $\beta = 0.485$, $\gamma = 0.615$

- Simulations obeyed a mass function where the fraction of mass in collapsed objects is modified to -

$$f_{ST}(\nu) = A\sqrt{\frac{2a}{\pi}}\left(1 + \frac{1}{(\sqrt{a}\nu)^{2p}}\right)\nu \exp\left[-\frac{a\nu^2}{2}\right]$$

where $p = 0.3$ and $A = 0.322$

We use here best fit parameters for ST mass function[1]

- **Sheth and Tormen(2001)** $\Rightarrow$ for various barrier shapes $B(S)$, the first-crossing distribution (approximated)

$$f(S)dS = \frac{|T(S)|}{\sqrt{2\pi S^{3/2}}} \exp\left[-\frac{B(S)^2}{2S}\right] dS$$

where

$$T(S) = \sum_{n=0}^{5} \frac{(-S)^n}{n!} \frac{\partial^n B(S)}{\partial S^n}$$

[1] arXiv:0908.2702, Bagla, Khandai & Kulkarni (2009)

Halo Formation rate - for PS formalism

- **Shin Sasaki** (1994)
- In the PS formalism, the co-moving number density of bound objects

$$N_{PS}(M,t) = \sqrt{\frac{2}{\pi}}\frac{\rho_0}{M}\frac{\delta_c}{D(t)}\left(-\frac{1}{\sigma^2}\frac{d\sigma}{dM}\right)\exp\left[-\frac{\delta_c^2}{2\sigma^2(M)D^2(t)}\right]$$

- The total change in number density of objects in unit time, $\dot{N}_{PS}$

$$\dot{N}_{PS}(M,t) = \dot{N}_{form}(M,t) - \dot{N}_{dest}(M,t)$$

In general,

$$\dot{N}_{dest}(M,t) = \int_M^\infty N_{PS}(M,t)\tilde{Q}(M,M';t)dM' \text{ (here } M' > M)$$
$$\equiv \phi(M,t)N_{PS}(M,t)$$
$$\dot{N}_{form}(M,t) = \int_{M_{min}}^M N_{PS}(M',t)Q(M',M;t)dM' \text{ (here } M > M')$$

$\tilde{Q}(M,M';t)$ the probability that an object of mass M grows into a part of an object of mass M' per unit time.
$Q(M',M;t)$ the distribution that an object of mass M' is one of the progenitors when an object of mass M forms.

Halo Formation rate - for PS formalism

- We can evaluate the formation rate by obtaining only $\phi(M,t)$ even if we do not know the functions Q and $\tilde{Q}$

$$\dot{N}_{form}(M,t) = \dot{N}_{PS}(M,t) + \phi(M,t)N_{PS}(M,t)$$

- **Crucial Assumption**

  $\Rightarrow$ the efficiency of the destruction rate has no characteristic mass scale

$$\phi(M,t) = M^{\alpha}\tilde{\phi}(t)$$

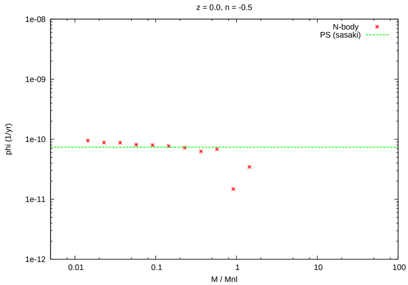
- As, ϕ must be finite to describe the destruction rate, we must set $\alpha = 0$

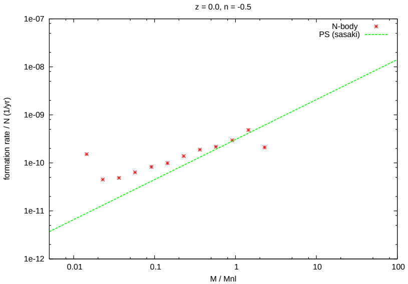
- We can get

$$\phi(M,t) = \frac{1}{D}\frac{dD}{dt}$$

$$\dot{N}_{form}(M,t) = \frac{1}{D}\frac{dD}{dt}N_{PS}(M,t)\frac{\delta_c^2}{\sigma^2(M)D^2(t)}$$

comparison of $\phi(M, t)$ and $\dot{N}_{form}(M, t)$ as obtained from Sasaki formalism and that calculated by N-body Simulation





Sasaki Formalism for ST Mass Function - **Negative Formation rate !!**

- Follow the same steps using *Sasaki's* prescription.
- efficiency of the destruction rate

$$\phi(M,t) = \frac{1}{D}\frac{dD}{dt}\left[1-2p\right]$$

- Formation rate

$$\dot{N}^{ST}_{form}(M,t) = -\frac{1}{D}\frac{dD}{dt}\left[\frac{2p}{1+\left(\frac{a\delta_c^2}{\sigma^2(M)D^2(t)}\right)^{-p}} - \frac{a\delta_c^2}{\sigma^2(M)D^2(t)}\right]N_{ST}(M,t)$$

- It is not guaranteed that the formation rate will always be positive. [1]
- Negative formation rate is **unphysical**

[1] arXiv:0902.3141, Samui, Subramanian & Srianand (2009)

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Generalization of *Sasaki* formalism for ST mass function is **incorrect**.

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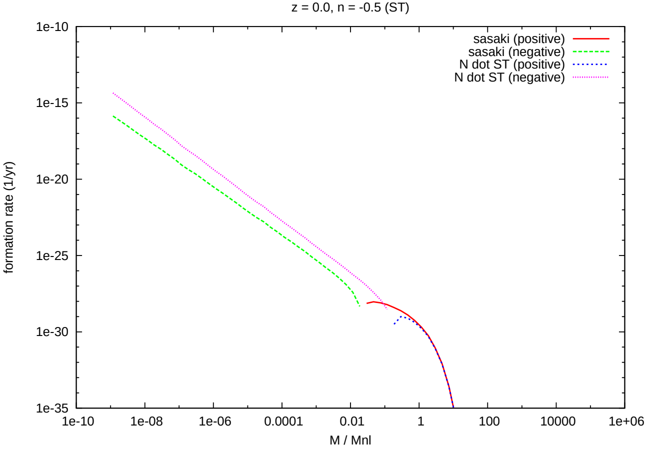
$\dot{N}_{form}(M,t)$ **for ST obtained from Sasaki formalism and that calculated by taking the derivative of mass function**



A Generalized Technique To Find Formation Rate - Working Formulae

- We check the validity of *Sasaki's* **crucial ansatz** using the formula

$$\phi(M_1, t) = \int_{M_1}^{\infty} \tilde{Q}(M_1, M_2; t) dM_2 \text{ (here } M_2 > M_1)$$

where $\tilde{Q}(M_1, M_2; t)$represents the probability that an object of mass M_1 grows into a part of an object of mass M_2 per unit time

- Conditional probability that a halo of mass M_1 present at time t_1 will have merged to form a halo of mass between M_2 and $M_2 + dM_2$ at time $t_2 > t_1$ for PS mass function[2]

$$f(M_2|M_1) d \ln M_2 = \sqrt{\frac{2}{\pi}} \frac{\delta_2(\delta_1 - \delta_2)}{\delta_1} \sigma_2^2 \left[\frac{\sigma_1^2}{\sigma_2^2(\sigma_1^2 - \sigma_2^2)} \right]^{\frac{3}{2}} \times \exp\left[-\frac{(\delta_2\sigma_1^2 - \delta_1\sigma_2^2)^2}{2\sigma_1^2\sigma_2^2(\sigma_1^2 - \sigma_2^2)} \right] \left| \frac{d \ln \sigma_2}{d \ln M_2} \right| d \ln M_2$$

- Taking the limit as z_2 tends to z_1 i.e. δ_2 tends to δ_1, we can determine the mean transition rate

[2] Lacey and Cole, Mon. Not. Roy. Astron. Soc., 1993

A Generalized Technique To Find Formation Rate - Working Formulae

- The probability that, a halo of mass M_1 will accrete or merge to form another halo of mass M_2 per unit redshift (**same as** $\tilde{Q}(M_1, M_2; z_2)$)

$$\frac{df}{dz_2} d\ln M_2 \overset{z_2 \to z_1}{=} \sqrt{\frac{2}{\pi}} \sigma_2^2 \left[\frac{\sigma_1^2}{\sigma_2^2(\sigma_1^2 - \sigma_2^2)} \right]^{\frac{3}{2}} \left| \frac{d\delta_2}{dz_2} \right| \times \exp\left[-\frac{\delta_2^2(\sigma_1^2 - \sigma_2^2)}{2\sigma_1^2\sigma_2^2} \right] \left| \frac{d\ln\sigma_2}{d\ln M_2} \right| d\ln M_2$$

- Note that, the integrand diverges at $M_2 = M_1$.Our modified formula for efficiency of the destruction rate

$$\phi(M_1, z) = \int_{M_1(1+\epsilon)}^{\infty} \tilde{Q}(M_1, M_2; z) dM_2$$

- We calculated the formation rate for **PS mass function**

$$\dot{N}_{form}^{PS}(M_1, z) = \dot{N}_{PS}(M_1, z) + \phi(M_1, z) N_{PS}(M_1, z)$$

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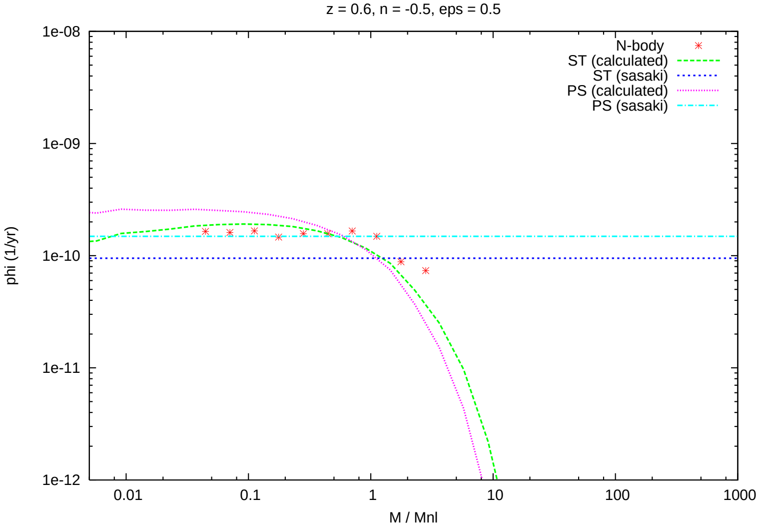
- Follow the same steps for the **ST mass function**

Results and Comparison - comparison of $\phi(M, t)$ as obtained from Sasaki formalism and that calculated by our method with N-body Simulation



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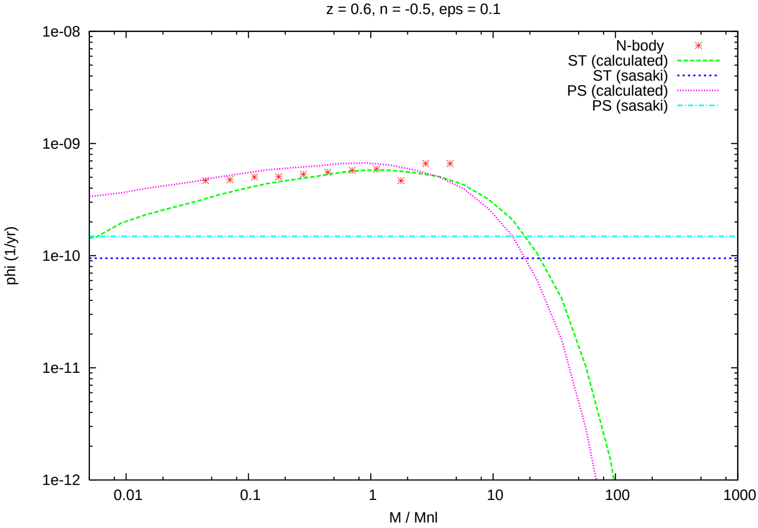

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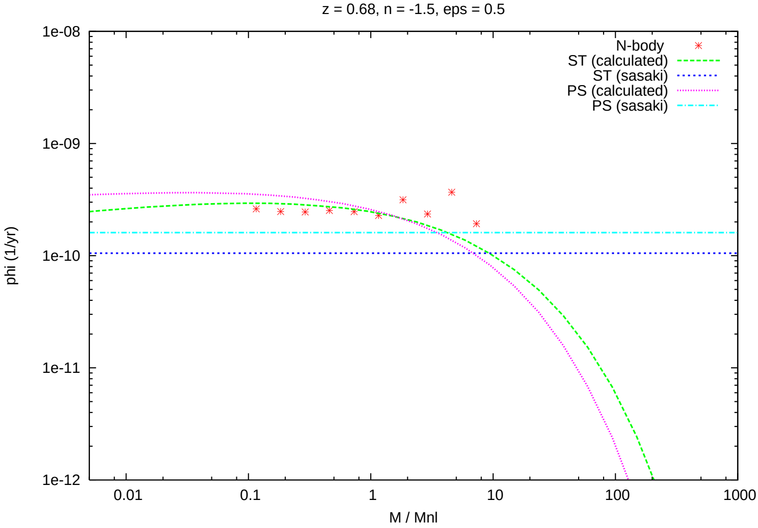

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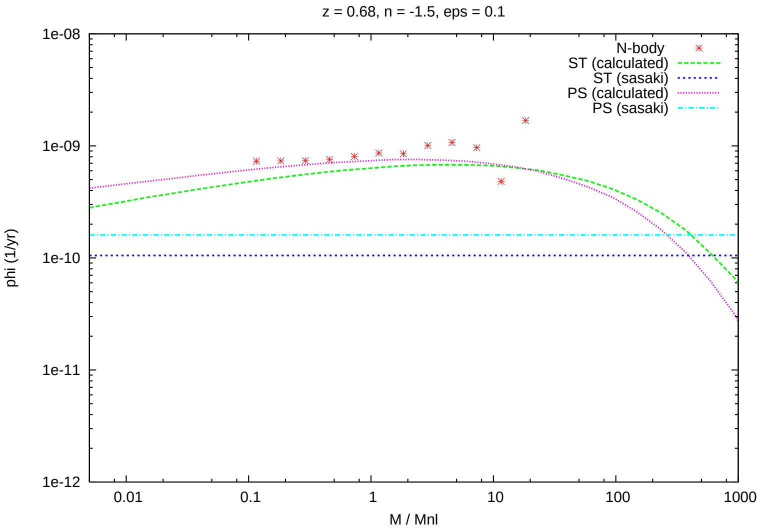
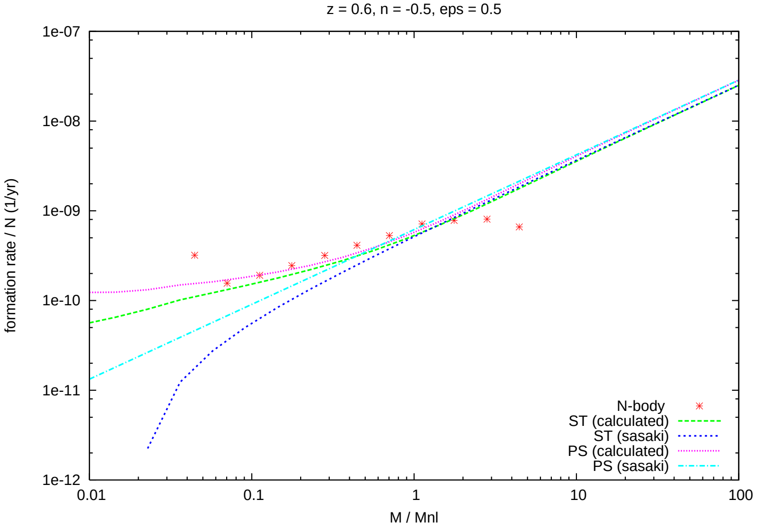


Results and Comparison - comparison of $\dot{N}_{form}(M,t)$ as obtained from Sasaki formalism and that calculated by our method with N-body Simulation



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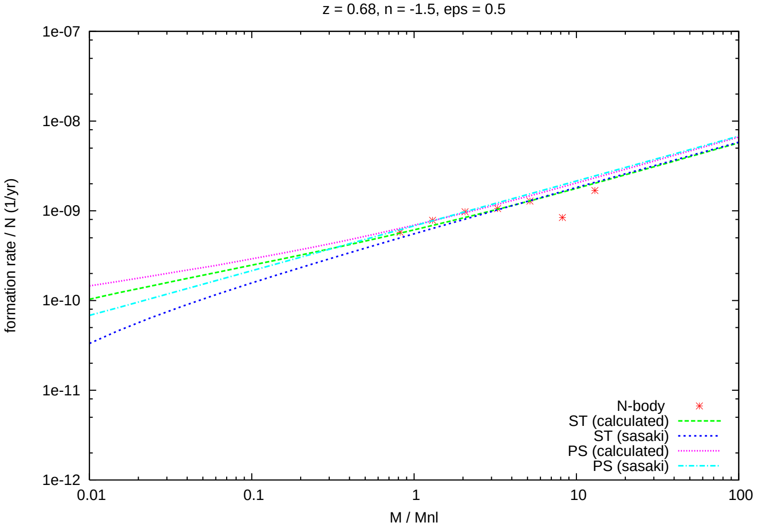
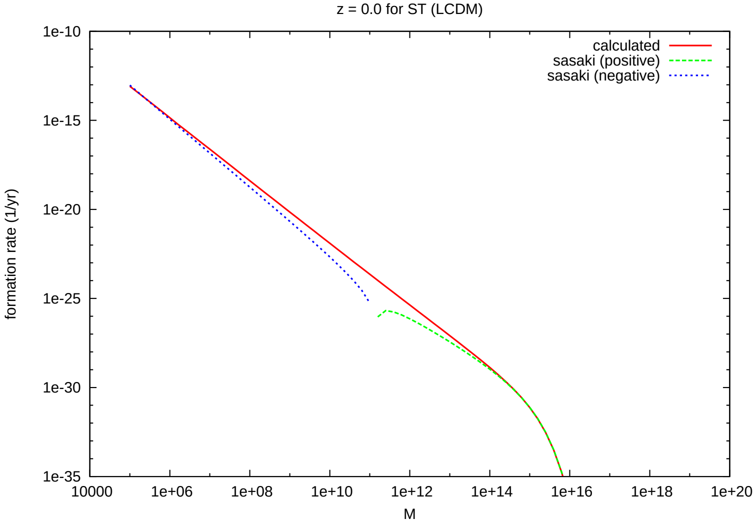
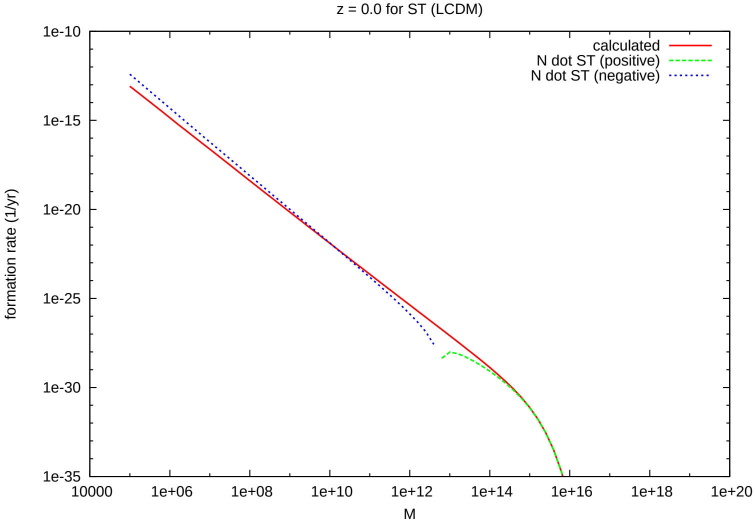


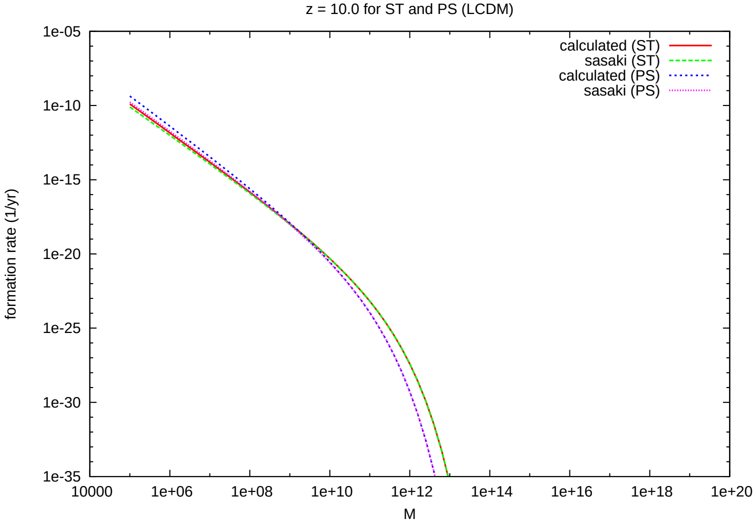
Results and Comparison - comparison of $\dot{N}_{form}(M, t)$ as obtained from Sasaki formalism and that calculated by our method (for ST)



Results and Comparison - comparison of $\dot{N}_{form}(M, t)$ as obtained by taking the derivative of mass function and that calculated by our method (for ST)



Results and Comparison - comparison of $\dot{N}_{form}(M, t)$ as obtained from Sasaki formalism and that calculated by our method (for ST and PS)



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5. We obtained the formation rate $\dot{N}_{form}$ which is **always positive** for any mass of halos at any redshift.

Ongoing Work with -

- J. S. Bagla [1]
- Girish Kulkarni [2]
- Jaswant K. Yadav [3]

Thank You !

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