DARK MATTER

1. The DM problem: the fact that ~ 85% of the U's mass seen by its gravitational effects is unaccounted for.

2. The solution to that problem

At present, all evidence points to a new, massive particle (Matter) that is non-relativistic (cold), weakly-interacting (Dark)

↓

CDM

PLAN

I: Evidence for DM
- astronomy
- cosmology

II DM candidates
- The WIMP
- Alternatives

III Finding DM

I EVIDENCE FOR DARK MATTER

1. The 1930's Zwicky and the Coma cluster

By counting galaxies and estimating their kinetic energies. virial theorem
$2\langle T \rangle = -\langle V \rangle$ requires $\frac{M}{L} = 500 \frac{M_\odot}{L_\odot}$

Local stars $\langle \frac{M}{L} \rangle \sim 2-3$

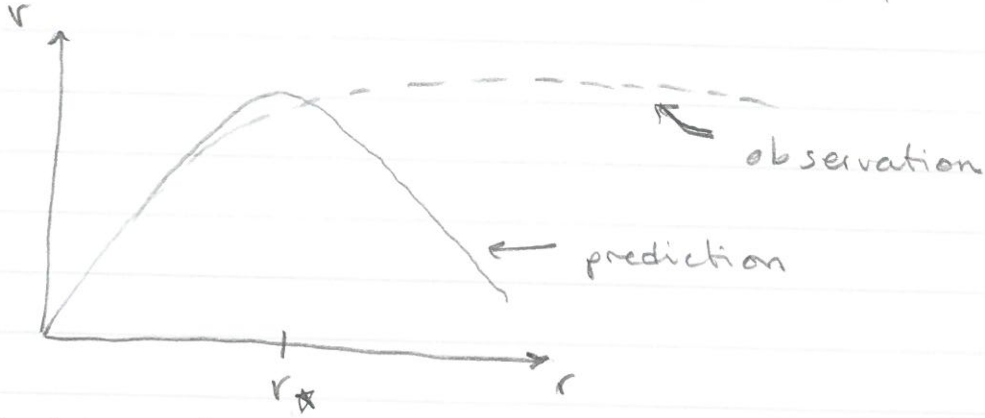
Zwicky, Ap. J 86, #3 (1937)

2. Vera Rubin (1970s): Developments in spectroscopy allowed rotational speeds at the edge of galaxies to be measured for the first time, far beyond stellar radii

If a galaxy ~ cylinder

$$\frac{v^2}{r} \sim \frac{GM(<r)}{r^2}$$

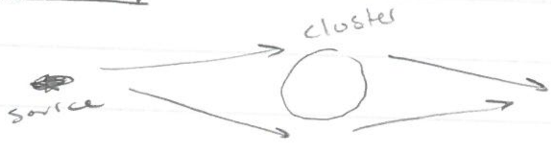
$\Rightarrow$ $v^2 \propto r$ inside the disk
$v^2 \propto 1/r$ beyond the disk



Rubin observed ~20 galaxies and found this to be the case in every one!

⇒ There is a dark (invisible) component that extends far beyond the extent of stars and gas.

Lensing



Can infer mass of cluster from lensed image

again: inferred mass ≫ luminous component

Colliding clusters

- Bullet cluster
- "Train wreck"
- many others

Most of the lensing comes from <u>collisionless</u> components of the two clusters that passed through each other.

So something is affecting the dynamics of objects in space, from scales of ~ kpc (local stars, dwarf galaxies) to tens of Mpc.

If this is a sign of a new particle, it must be
- Invisible: no electric charge
- Weakly interacting (otherwise it would brem off E_K, form a disk like ordinary matter)
- Non-relativistic

COLD DARK MATTER

BUT could be something else

- MACHOs : massive compact halo objects
  i.e space rocks
- Gravity is wrong

For a definitive answer we turn to cosmo

Interlude: COSMOLOGY 101

Einstein's equations $\underbrace{G_{\mu\nu}}_{\text{dynamics}} = 8\pi G_N \underbrace{T_{\mu\nu}}_{\text{matter}}$ (1)

On the largest scales, the universe is
- homogeneous
- Isotropic

⇒ Can write, generally: $ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$

(Friedman-Lemaître-Robertson-Walker universe)

(2) $$d\vec{x}^2 = \frac{dr^2}{1-kr^2} + r^2 d\Omega$$

$kr^2 \cong 0$

Plugging (2) into (1), one obtains the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho(t) + (\text{curvature})$$

$$H^2 = \frac{8\pi G}{3}\rho \quad \left(= \frac{1}{M_p^2}\rho \right)$$

By convention $a(t_0 \equiv \text{today}) = 1$

Useful : redshift $1+z = \frac{1}{a} \Rightarrow \frac{dz}{dt} = \frac{d}{dt}\frac{1}{a} = \frac{\dot{a}}{a^2}$ $\equiv (1+z)H(z)$

Can split $\rho = \rho_m + \rho_r + \rho_\Lambda$ $\Rightarrow dt = \frac{-dz}{(1+z)H(z)}$

$$\rho_m \propto \frac{1}{a^3}$$

$$\rho_r \propto \frac{1}{a^4}$$

$$\Rightarrow H^2 = \frac{8\pi G}{3}\rho_c\left(\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda\right)$$

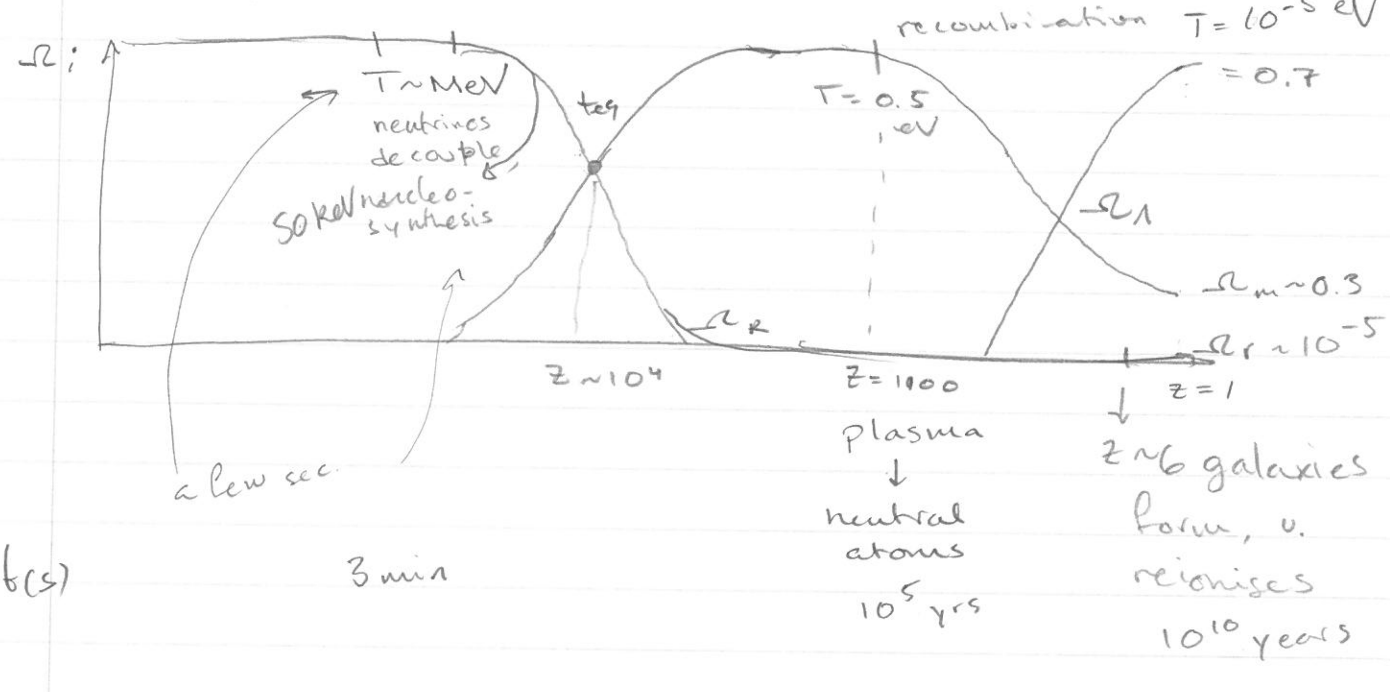
ρ_c $\downarrow$ "critical density" <u>today</u>

$$H(t) \equiv H_0\left(\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda\right)^{1/2}$$

$H(t)$ ↳ Hubble parameter

H_0 ↳ Hubble "constant" $= h \times 100\ \frac{\text{km}}{\text{s Mpc}}$

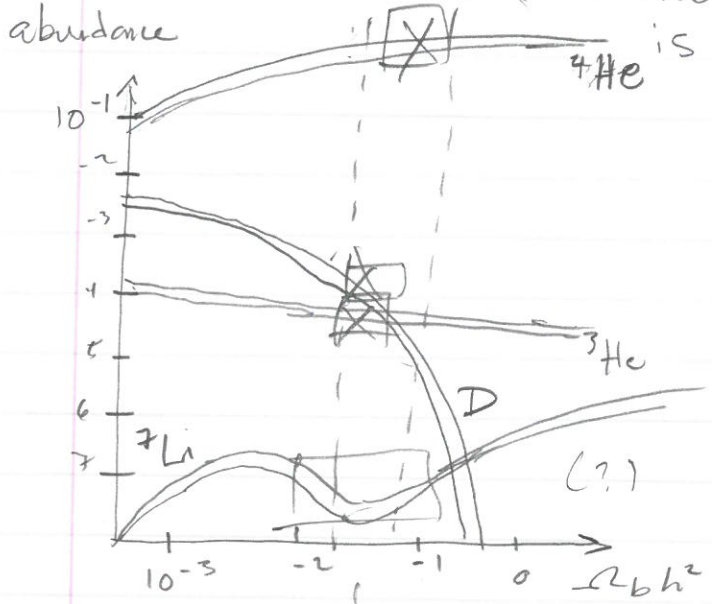
History of the universe



We separate Ω_m into $\Omega_b + \Omega_c$

① BBN ($T \sim$ MeV ... 50 keV)

- expansion speed → how many neutrons decay
- reation rates → how much D, He, Li is produced



By measuring $H(z)$
$\Omega_m \sim 0.3$
$\Omega_\Lambda \sim 0.7$

So → matter that is not baryonic. Otherwise we get the wrong abundances.

BBN tells us $\Omega_b h^2 \sim 0.022$
(i.e) $\Omega_b \sim 5\%$

What is the remaining 85% of matter? Can't be space rocks.

② Recombination ($T \sim 0.5$ eV, $z = 1100$)

★ Recommended ref: Ma & Bertschinger astro-ph/9506072

As hydrogen recombines, the mean free path of photons goes from $\lambda \sim 0$ (plasma) to $\lambda \sim \infty$

This results in a uniform* background radiation with a blackbody temperature

$T_{CMB} = 2.71$ K (redshifted from $T \sim 3000$)

* Not completely uniform : $\frac{\Delta T}{T} = \frac{\Delta \rho}{\rho} = 10^{-5}$

Density perturbations

- Patch moving away: redshift (Dop)
- Dense region: redshift (grav)
  ∴ ΔT negative
- " $\leftrightarrow$ moving toward us underdense

↓ matter continues to oscillate/collapse, but photons free-stream

Can find the power spectrum $P(k)$ of the Fourier-Transformed perturbations.

In GR: $T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p g^{\mu\nu}$

four-velocity

Perturb the metric:

$$ds^2 = a^2(\tau)\left(-(1+2\psi)d\tau^2 + (1-2\phi)\,dx^i dx_i\right)$$

comoving time

Matter

$\delta T^0{}_0 = -\delta$ Density perturbation ($\sim 10^{-5}$)

$\delta T^0{}_i = (\rho + P) v_i \rightarrow$ velocity perturbation

Use $\theta \equiv i k_i v^i \equiv \nabla \cdot \vec{v}$

Euler equations:

$$\dot{\delta}_b = -\Theta_b + 3\dot{\phi}$$

$$\dot{\Theta}_b = \underbrace{-\mathcal{H}\Theta_b}_{\text{redshift } ①} + \underbrace{c_s^2 k^2 \delta_b}_{\text{restorative force } ②} + \underbrace{k^2\Psi}_{\text{gravity } ③} - \underbrace{\frac{4\rho_\gamma}{3\rho_b} a n_e (\Theta_b - \Theta_\gamma)}_{\text{damping term } ④}$$

Adding γ, ν, can solve coupled equations

Terms ② + ④ ensure perturbations don't grow as quickly under gravity + are damped

Solve eqs IC to recomb (e.g. CLASS, CAMB)

Can then plot the power spectrum projected onto the sky (sph. harmonics). Isotropy: $\sum_m a_{\ell m}$

not what is seen!

If I add collisionless matter

$$\boxed{\dot{\Theta}_c = -\mathcal{H}\Theta_c + k^2\Psi}$$

I have added the Euler eq for a <u>cold</u> (non-rel) colisionless (on CMB scales) fluid, i.e. a new, weakly-interacting particle.

☆ No modification of gravity is able to avoid Silk damping and obtain even the 3rd peak.

Can also measure $P(k)$ of matter (after recomb)
↳ structure formation
simulations $\Rightarrow \Omega_{CDM} \sim 0.25$.
↳ galaxy formation → full circle.

II WHAT COULD DM BE?

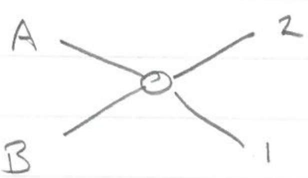
We know it must be:

	n	ν	γ	dust
- Electrically neutral				
- present in the early universe	✓	✓	✓	✗
- Long-lived	✗	✓	✓	
- Massive		✓	✗	
- N-R at recombination		✗		

⇒ Need <u>new</u> particle.

In the Hot Big Bang model, the abundance of every particle is set by its thermal history.

Early universe: hot, dense plasma → chemical equilibrium

(✡) 

$A, B \rightleftarrows 1, 2$

⇒ equipartition.

If $m_{A,B} \gg m_{1,2}$, sector A, B will begin to deplete as $T < m_{A,B}$. ✡ becomes

(✡✡) $A, B \rightarrow 1, 2$

As long as $\Gamma \gg H$, ($\star$) is efficient (or $\star\star$ if $T < m_i$)

But the interaction rate $\Gamma \propto \sigma v n$
$\downarrow$
which is decreasing quickly!

At some point the reaction stops, and the two sectors decouple. This is called Freeze-out.

Cosmology 202: Thermodynamics

Kolb + Turner

Radiation domination: $H = 1.66\, g_\star^{1/2} \dfrac{T^2}{M_P}$

$M_P = 1.22 \times 10^{19}$ GeV

Phase space distribution $f(p) = \dfrac{1}{e^{(E-\mu)/T} \pm 1}$

$E^2 = p^2 + m^2$

Can define
$$n = \frac{g}{2\pi^3} \int f(\bar{p})\, d^3p \,,$$
$$\rho = \frac{g}{2\pi^3} \int E\, f(p)\, d^3p \,,$$
$$P = \frac{g}{2\pi^3} \int \frac{p^2}{3E} f(p)\, d^3p .$$

Assuming no μ, equilibrium

Relativistic		NR
$n = \dfrac{\zeta(3)}{\pi^2} g T^3$	$\times \frac{3}{4}$	$n = \left(\dfrac{g m T}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}$
$\rho = \dfrac{\pi^2}{30} g T^4$	$\times \frac{7}{8}$	$\rho = m n$
$P = \rho/3$	Fermi	$P = nT \ll \rho$

You can show that the entropy density is
$s = (p+\rho)/T$.

$$s = \frac{2\pi^2}{45} g_{*s} T^3$$

$$g_{*s} = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g \left(\frac{T_i}{T}\right)^3$$

$$g_* = \quad " \quad \left(\frac{T_i}{T}\right)^4 \qquad\qquad \left(\frac{T_i}{T}\right)^4$$

Define a quantity called the <u>Yield</u>

$$Y \equiv \frac{n}{s}$$

For a relativistic species T drops out:

$$g_{eff} \equiv \begin{cases} g & \text{Bose} \\ \frac{3}{4} g & \text{Fermi} \end{cases}$$

(a) $$Y_{eq} = \frac{45}{2\pi^4} \zeta(3) \frac{g_{eff}}{g_{*s}} \sim 0.278 \frac{g_{eff}}{g_{*s}}$$

NR

(b) $$Y_{eq} = \frac{45}{2\pi^4} \left(\frac{\pi}{8}\right)^{1/2} \frac{g_{eff}}{g_{*s}} \left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$

Ex: What yield do we need to reproduce the correct DM density $\Omega h^2 \simeq 0.1$? After freeze-out, no #-changing processes $\Rightarrow$ Y const.

$$\Omega h^2 = \frac{\rho_x}{\rho_c} h^2 = \frac{m_x Y_\infty s_0 h^2}{\rho_c} \qquad Y_\infty = Y_{FO}$$

Measured entropy: $s_0 = 2970 \text{ cm}^{-3}$

$\rho_c = 1.054 \times 10^{-5} h^2 \text{ GeV cm}^{-3}$

$$\Rightarrow Y_{FO} \cong 3.55 \times 10^{-10} \left(\frac{m_x}{\text{GeV}}\right)^{-1}$$

Can plug this back into (b) and solve numerically

$$x_{FO} \equiv \left(\frac{m}{T}\right)_{FO} \simeq 20$$

Could neutrinos be DM? They froze out while relativistic $\Rightarrow Y_{FO} = 0.278 \frac{g_{eff}}{g_{*s}}$

at ~ MeV, $g_{*s} = 10.75$ $(e^{\pm}, \gamma, \nu)$, $g_{eff}/\text{flavor} = 3/2$

$$\Rightarrow \Omega h^2 = \frac{\sum m_{\nu i} Y_\infty s_0 h^2}{\rho_c} \simeq \frac{\sum m_i}{91 \text{eV}}$$

$\Rightarrow$ for $\Omega h^2 \sim 0.1$, need $\sum m_i = 9$ eV. $\gg$ current bound.

Time evolution of the number density

The Boltzmann equation

$$\hat{L}[f] = C[f]$$

$\hookrightarrow$ Liouville operator

1) $\hat{L} = \frac{d}{d\tau} = p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^\mu_{\sigma\rho} p^\sigma$

In an FLRW universe

$$\hat{L} = E\frac{\partial}{\partial t} - Hp^2 \frac{\partial}{\partial E}$$

Integrating over phase space: $\overbrace{\frac{g}{2\pi^3}\int \hat{L}[f] d^3\vec{p}}^{A} = \frac{g}{2\pi^3}\int C[f] d^3 p$

Can show $\frac{g}{2\pi^3}\int \frac{\hat{L}[f]}{E} d^3\vec{p} = \frac{dn}{dt} + 3Hn$

2) Collision operator. Write the phase space of each particle in $\underbrace{A, B}_{SM} \leftrightarrow \underbrace{1, 2}_{DM}$ as

$$d\Pi_i = \frac{g_i}{2\pi^3} \frac{d^3\vec{p}_i}{2E_i}$$

$$\Rightarrow \frac{g}{2\pi^3}\int \frac{C[f]}{E} d^3p = \frac{g}{2\pi^3}\int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta(p_A + p_B - p_1 - p_2) \times \left[|\mathcal{M}_{12\to AB}|^2 f_1 f_2 - |\mathcal{M}_{AB\to 12}|^2 f_A f_B\right]$$

Now, assuming time-reversal (i.e. no ~~CP~~)

$$|\mathcal{M}_{AB\to 12}|^2 = |\mathcal{M}_{12\to AB}|^2$$

We're interested in the case where the SM (A,B) is in equilibrium, but DM (1,2) is not necessarily.

$$\Rightarrow \quad f_A f_B = f_A^{eq} f_B^{eq} = f_1^{eq} f_2^{eq} \neq f_1 f_2$$

Define the thermally-averaged cross section

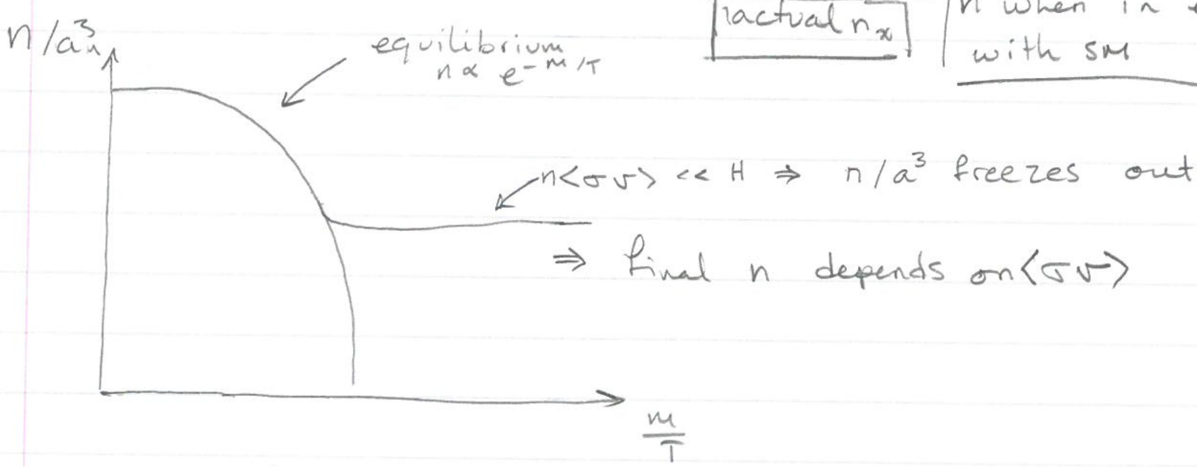
$$\langle \sigma v \rangle \equiv \frac{1}{n_{eq}^2} \int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta(\Sigma p_i) |\mathcal{M}|^2 f_1^{eq} f_2^{eq}$$

then, when the dust settles

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{eq}^2)$$

actual n_x

n when in eq with SM



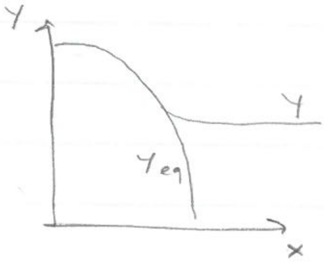
Can define $x = \frac{m}{T}$, and reintroduce Y

this simplifies to $$\boxed{\frac{dY}{dx} = \frac{\lambda \langle \sigma v \rangle}{x^2}(Y^2 - Y_{eq}^2)}$$

(Ricatti eq)

$$\lambda \equiv \frac{2\pi^2}{45} \frac{M_P\, g_{*s}}{1.66\, g_*^{1/2}}\, m \simeq 3.22 \times 10^{19} \frac{m}{\text{GeV}}$$

Without solving



$$\frac{dY}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(Y^2 - Y_{eq}^2\right)$$

Introduce $\Delta \equiv Y - Y_{eq}$

1) Early times ; $x << x_{today}$, $Y \simeq Y_{eq}$

$$\cancel{\frac{d\Delta}{dx}} + \frac{dY_{eq}}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(\Delta^2 + 2\Delta Y_{eq}\right)$$

$$\simeq -\frac{\lambda\langle\sigma v\rangle 2\Delta Y_{eq}}{x^2}$$

$$\Rightarrow \quad \Delta_{FO} \simeq -\frac{x_{FO}^2}{2\lambda\langle\sigma v\rangle}\frac{Y_{eq}'}{Y_{eq}}$$

for $x_{FO} \sim 20$, $Y_{eq}' \simeq -Y_{eq}$

$$\Rightarrow \quad \Delta_{FO} \simeq \frac{x^2_{FO}}{2\lambda\langle\sigma v\rangle}$$

2) Late times $Y >> Y_{eq}$

$$\Rightarrow \quad \frac{d\Delta}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\Delta^2 \quad \Rightarrow \text{integrate from } x_{FO} \text{ to } \infty$$

$$\int_{\Delta_{FO}}^{\Delta_\infty}\frac{d\Delta}{\Delta^2} = -\lambda\langle\sigma v\rangle\int_{x_{FO}}^{\infty}\frac{dx}{x^2}$$

$$\frac{1}{\Delta_\infty} = \frac{1}{\Delta_{FO}} + \frac{\lambda\langle\sigma v\rangle}{x_{FO}} = \underbrace{\cancel{\frac{2\lambda\langle\sigma v\rangle}{x^2_{FO}}}}_{O(1/10)} + \frac{\lambda\langle\sigma v\rangle}{x_{FO}}$$

$$\Rightarrow \quad \Delta_\infty \sim Y_\infty \simeq \frac{x_{FO}}{\lambda\langle\sigma v\rangle}$$

Recall $Y_\infty \sim 3.55\times10^{-10}\left(\frac{GeV}{m}\right)$ → to get $\Omega_c h^2 \sim 0.1$

$x_{FO} \sim 20$

λ $3.22\times10^{19}\left(\frac{m}{GeV}\right)GeV^2$

$\langle\sigma v\rangle \simeq 1.74\times10^{-9}\ GeV^{-2}$

$\hbar c = 1.97\times10^{-14}$ GeV cm

$\langle\sigma v\rangle \simeq 2\times10^{-26}\ \frac{cm^3}{s}$

Now, say we have a new particle that interacts at the weak scale

$$\langle \sigma v \rangle \simeq \frac{G_F^2 m^2}{2\pi} \cong 10^{-10} \frac{m^2}{GeV^2} \; GeV^{-2}$$

So

$$\langle \sigma v \rangle_{FO} \simeq \langle \sigma v \rangle_{weak}$$

WOW!

The WIMP "miracle"

☆ Real model ☆ - Solve numerically
- $\langle \sigma v \rangle \neq$ const.

eg micromegas

<u>Other considerations</u>

- Hut-Lee-Weinberg 1977 : $\Omega h^2 \cong \dfrac{10^{-10}}{\langle \sigma v \rangle}$

$$\simeq \frac{2\pi \times \cancel{10^{-10}}}{\cancel{G_F^2}\, m_\chi^2} \; GeV^2$$

So for $\Omega h^2 \lesssim 1$ $\quad m_\chi >$ a few GeV

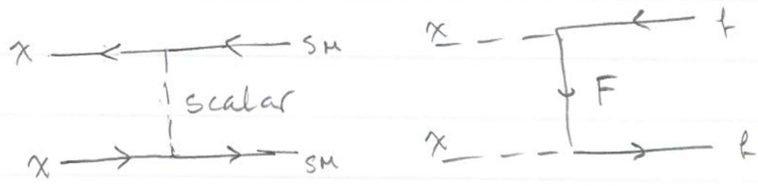
- Unitarity bound (Griest & Kamionkowski 1990

$$m_\chi \lesssim 300 \text{ TeV}$$

Can we get around these? Sure. New mediator
$G_{new} \sim \dfrac{\alpha^2}{M_{new}^2}$

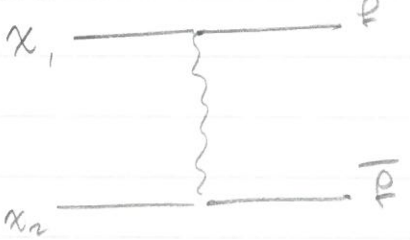
But there is still <u>a</u> LW bound.

t-channel



+ chiral couplings → can remove the m_χ dependence

Co-annihilations $\longrightarrow$ solve coupled boltzmann eq



~~Other mechanisms for thermal production~~

So what is a WIMP

very restrictive

1 - Weakly Interacting Massive particle G_F

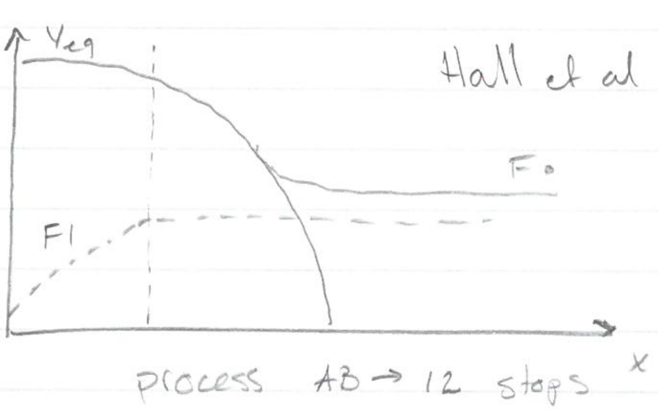
★ 2 - Weakly " " " : produced in thermal FO (not G_F)

3 - Any DM particle

wrong

Other production methods for DM?

• Freeze-in



Hall et al 2009

process $AB \rightarrow 12$ stops

"FIMP"
↳ feebly

Sterile neutrino DM can be produced like this.

Asymmetric DM

Production related to baryogenesis. Only χ (no $\bar{\chi}$) survives. If $n_\chi \sim n_b$, $\Omega_\chi \sim 5\,\Omega_b$

$$\Rightarrow m_\chi \sim 5\, m_b \sim 5\,\text{GeV}$$

Non-thermal: e.g. misalignment (axion)

Symmetry-breaking in early U. $\Rightarrow$ "condensate" of ground-state particles

...