An overview of Electroweak Precision Tests

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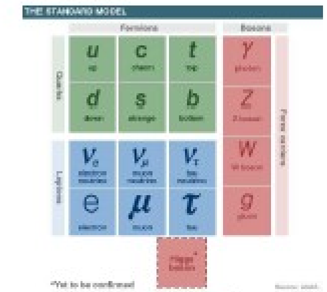
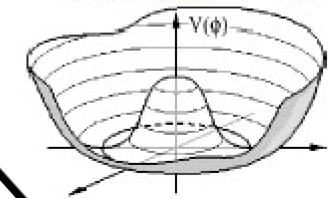
Standard Model



- Gauge group: $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}} \to \mathrm{U}(1)_{\mathrm{EM}}$

$$\mathcal{L}_{\mathrm{SM}} = \boxed{\overline{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}} + y \overline{\Psi} \Psi \Phi + \boxed{|D_\mu \Phi|^2} - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$



- Gauge interaction measured (1-10) per mille level at LEP.

$$\boxed{\Phi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}}$$

- Gauge boson masses are obtained as

$$|D_\mu \Phi|^2 \Longrightarrow \left| \left(-\frac{ig}{2} \tau^a W^a_\mu - \frac{ig'}{2} B_\mu \right) \Phi_0 \right|^2 = \frac{1}{8} \left| \begin{pmatrix} gW^3_\mu + g'B_\mu & \sqrt{2} g W^-_\mu \\ \sqrt{2} g W^+_\mu & -gW^3_\mu + g'B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \left(\frac{1}{2} gv \right)^2 W^+_\mu W^-_\mu + \frac{1}{8} v^2 (W^3_\mu \ B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}.$$

$$\frac{M_W}{M_Z} = \frac{\frac{1}{2} gv}{\frac{v}{2} \sqrt{g^2 + g'^2}} = \cos\theta_W$$

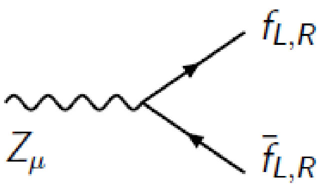
$$A_\mu = \frac{gB_\mu + g'W^3_\mu}{\sqrt{g^2 + g'^2}} \ : \ M_A = 0$$

$$Z_\mu = \frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2 + g'^2}} \ : \ M_Z = \frac{v}{2} \sqrt{g^2 + g'^2}$$

$$\boxed{\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}.}$$

Z boson properties

Zff coupling:



$$\frac{g}{\cos\theta_W}\gamma_\mu\left(a_L^f P_L + a_R^f P_R\right) \equiv \frac{g}{2\cos\theta_W}\gamma_\mu\left(v^f - a^f\gamma_5\right), \text{ where } v^f \equiv t_3^f - 2Q_f\sin^2\theta_W,\ a^f \equiv t_3^f$$

Decay width:

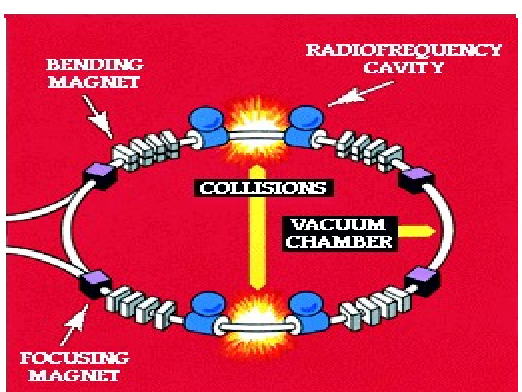
$$\Gamma_f = \frac{G_F}{6\pi\sqrt{2}} M_Z^3\left(v_f^2 + a_f^2\right) f\left(\frac{m_f}{M_Z}\right), \text{ where}$$

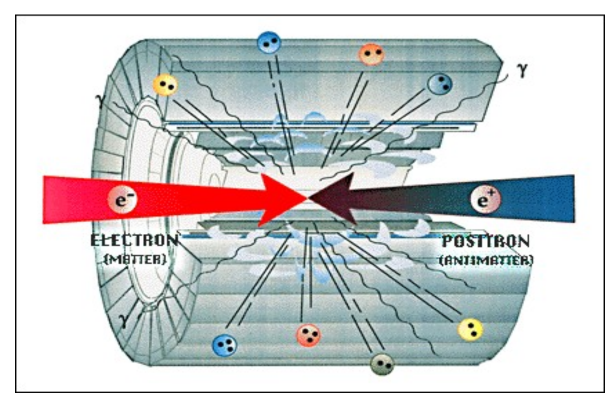
$$f(x) = \left(1-4x^2\right)^{1/2}\left(1 - x^2 + 3x^2\frac{v_f^2 - a_f^2}{v_f^2 + a_f^2}\right).$$

Total decay width 2.5 GeV, Hadronic decay width 1.75 GeV, Invisible decay width 499 MeV, Leptonic decay width per flavor 84 MeV.

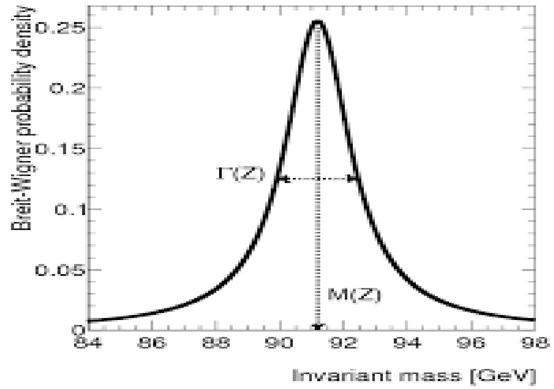
Remember:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2} = \frac{g^2}{8M_Z^2\cos^2\theta_W} = \frac{e^2}{8M_Z^2\sin^2\theta_W\cos^2\theta_W}$$





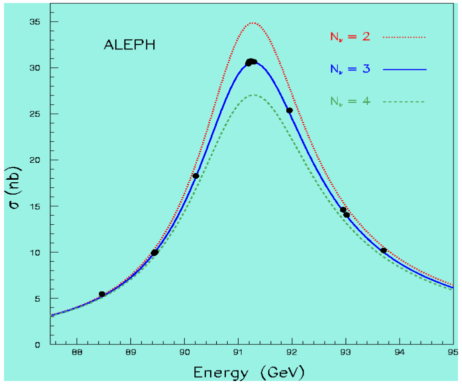
LEP observables



Cross section $\sigma(e^+e^- \to f\bar{f})$ vs. $\sqrt{s}$

Master formula $\sigma_f^0 = (12\pi\Gamma_e\Gamma_f/m_Z^2\Gamma_Z^2)$

1. From the peak position of the Breit-Wigner resonance, we can measure M_Z for any final state f.
2. The half-width at the maximum gives us the *total* width Γ_Z for any final state f.
3. By measuring Bhabha scattering cross section (σ^e) at the Z pole, we can calculate Γ_e.
4. By measuring the peak cross section for any other final state ($f = e, \mu, \tau$, hadron), we can calculate the corresponding Γ_f.
5. Since neutrinos are invisible, we cannot directly measure the neutrino decay width. But the total invisible decay width $\Gamma_{\text{inv}} = \Gamma_Z - \Gamma_{\text{visible}} = \Gamma_Z - \Gamma_e - \Gamma_\mu - \Gamma_\tau - \Gamma_{\text{had}}$.
6. The number of light neutrinos is $N_\nu = \Gamma_{\text{inv}}/\Gamma_\nu^{\text{SM}} = 2.984 \pm 0.008$, which for all practical purposes is 3.



Forward Backward Asymmetry

$$A_{\text{FB}}^l = \frac{\int_0^{\pi/2} d\theta \sin\theta \frac{d\sigma}{d\Omega} - \int_{\pi/2}^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}}{\int_0^{\pi/2} d\theta \sin\theta \frac{d\sigma}{d\Omega} + \int_{\pi/2}^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}} = \frac{3}{4} A_e A_f \qquad \text{where} \quad A_f = \frac{2 v_f a_f}{v_f^2 + a_f^2}$$

Top-less ruled out!

Even though top couldn't be produced at LEP, its presence was inferred from the partial Z decay width and FB asymmetry in bottom channel.

$$\Gamma_b^{\rm SM} = \frac{G_F M_Z^3}{3\pi\sqrt{2}}\left[\left(a_L^b\right)^2 + \left(a_R^b\right)^2\right] = \frac{G_F M_Z^3}{3\pi\sqrt{2}}\left[\left(t_3^b - Q_b \sin^2\theta_W\right)^2 + \left(-Q_b \sin^2\theta_W\right)^2\right]$$
$$= \frac{1.166\cdot 10^{-5}\,\mathrm{GeV}^{-2}\times(91.2\,\mathrm{GeV})^3}{3\pi\sqrt{2}}\left[\left(-\frac{1}{2}+\frac{1}{3}\times 0.23\right)^2 + \left(\frac{1}{3}\times 0.23\right)^2\right] \simeq 376\ \mathrm{MeV}$$

If bottom `were' isosinglet, the partial width would be 23.5 MeV. Expt number close to 376 MeV. The discrepancy is too much! Also, isospin of bottom has to be -1/2. Same conclusion from FB-asymmetry, which is sensitive to single power of isospin.

Thus even before top was discovered, not only its existence but also its gauge quantum numbers were comprehensively established by studying how the Z boson couples to the bottom quarks.

Measurements of electroweak radiative effects provided further hint to the top quark mass. Increasingly more precise measurements demanded more accurate theoretical predictions.

Early radiative corrections

To have a feeling, look back to summer 1992.

The measured $v_l^{\exp} = -0.0362^{+0.0035}_{-0.0032}$, when compared with its tree level SM prediction $v_l^{(\mathrm{SM,tree})} = -0.5 + 2\sin^2\theta_W = -0.076$ ($\sin^2\theta_W$ obtained from $G_\mu = \pi\alpha(0)/\sqrt{2}m_Z^2\sin^2\theta_W\cos^2\theta_W$), showed a 13σ discrepancy. Clearly, radiative corrections are necessary.

However, just the consideration of running $\alpha(0) \to \alpha(m_Z)$ and extracting $\sin^2\theta$ (to replace $\sin^2\theta_W$ in the expression of v_l) from $\cos^2\theta\sin^2\theta = \pi\alpha(m_Z)/\sqrt{2}G_\mu m_Z^2$, enabled one to obtain $v_l = -0.037$, *i.e.* within 1σ of its experimental value at that period.

> A significant consistency between data and predictions was established just by considering the running of α and it was only much later, with a significantly more data, that the weak loop effects were felt.

Rho parameter

An important parameter for weak scale physics

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W}.$$

If there are several representations of scalars whose electrically neutral members acquire vev's v_i, then

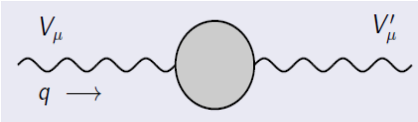
$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{\sum_{i=1}^{N} v_i^2 [T_i(T_i+1) - \frac{1}{4}Y_i^2]}{\sum_{i=1}^{N} \frac{1}{2} v_i^2 Y_i^2},$$

where T_i and Y_i are the weak isospin and hypercharge of the i-th multiplet. It is easy to check that only those scalars are allowed to acquire vevs which satisfy $(2T+1)^2 - 3Y^2 = 1$, as otherwise $\rho = 1$ will not be satisfied at the tree level. The simplest choice is to have a scalar with $T = \frac{1}{2}$ and $Y = 1$, which corresponds to the SM doublet Φ. More complicated scalar multiplets, e.g. one with $T = 3$ and $Y = 4$, also satisfy this relation.

Custodial symmetry

- The SM Higgs is a complex scalar doublet $\Rightarrow$ 4 real fields. Both kinetic and potential terms have $\mathrm{O}(4) = \mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry.
- One of them is $\mathrm{SU}(2)_\mathrm{L}$. Usually, the other is called $\mathrm{SU}(2)_\mathrm{R}$. Higgs carries a (2,2) rep.
- When Higgs gets a vev, $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \to \mathrm{SU}(2)_\mathrm{V}$. <u>This is called 'custodial SU(2)'</u>. So what?
- Consider the Lagrangian for gauge bosons after EWSB.
  $L = \Pi_\pm W^+ W^- + \Pi_{33} W^3 W^3 + \Pi_{3B} W^3 B + \Pi_{BB} BB$ where $\Pi_{ab} \sim < J_a J_b >$.
  Also $\Pi(p^2) = \Pi(0) + p^2 \Pi'(0)$.
- Each J transforms as (3,1) under $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ or as 3 under $\mathrm{SU}(2)_\mathrm{V}$. Recall $3 \times 3 = 1 + 3 + 5$.
  $A_i B_j : A_i B_i + (A_i B_j - A_j B_i) + \frac{1}{2}(A_i B_j + A_j B_i) - \frac{1}{3}(A.B)\delta_{ij}$
- $\Delta\rho = \alpha T \propto (\Pi_\pm(0) - \Pi_{33}(0))$, symmetric under indices and traceless, so transforms as 5 of $\mathrm{SU}(2)_\mathrm{V}$.
- Since $\mathrm{SU}(2)_\mathrm{V}$ is a symmetry of the vacuum, only singlets of $\mathrm{SU}(2)_\mathrm{V}$ can have non-vanishing expectation value. Hence $T = 0$ at leading order.

Vector boson self energies



$$X^{\mu\nu}(m_1,m_2,\lambda,\lambda') = (-)\int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}\left\{\gamma^\mu\frac{1-\lambda\gamma_5}{2}(\not{q}+\not{k}+m_1)\gamma^\nu\frac{1-\lambda'\gamma_5}{2}(\not{k}+m_2)\right\}}{\{(q+k)^2-m_1^2\}(k^2-m_2^2)}$$

$$= \frac{i}{16\pi^2}\int_0^1 dx\left[\Delta - \ln\left\{\frac{-q^2x(1-x)+m_1^2x+m_2^2(1-x)}{\mu^2}\right\}\right]\left[2(1+\lambda\lambda')x(1-x)(q_\mu q_\nu - q^2 g_{\mu\nu})\right.$$
$$\left. + (1+\lambda\lambda')(m_1^2x+m_2^2(1-x))g_{\mu\nu} - (1-\lambda\lambda')m_1m_2g_{\mu\nu}\right].$$

$$\Pi_{LL}(q^2,m_1^2,m_2^2) \;=\; -\frac{1}{4\pi^2}\int_0^1 dx\left[\Delta + \ln\frac{\mu^2}{-q^2x(1-x)+M^2(x)}\right]\left[q^2x(1-x)-\frac{1}{2}M^2(x)\right]$$

where $M^2(x) = m_1^2x + m_2^2(1-x)$.

$$\boxed{\begin{aligned}\Pi_{33}(q^2) &= \langle J_\mu^3, J_\mu^3\rangle = t_{3L}^2\,\Pi_{LL}(q^2,m^2,m^2)\,,\\ \Pi_{11}(q^2) &= \langle J_\mu^+, J_\mu^-\rangle = \frac{1}{2}\Pi_{LL}(q^2,m_1^2,m_2^2)\,.\end{aligned}}$$

Renormalization procedure

Follow the steps:

1. Write the bare Lagrangian and scale the fields and coupling constants by 'renormalization constants'.

$$\phi_i \to \sqrt{z_1^i}\,\phi_i \qquad g_j \to z_2^j\, g_j$$

2. Select renormalization inputs (the best measured experimental quantities).

- $\alpha^{-1}(0) = 137.0359895(61)$
- $G_\mu = 1.16639(2) \times 10^{-5}\ \mathrm{GeV}^{-2}$,
- $M_Z = 91.1867 \pm 0.0020\ \mathrm{GeV}$

3. Impose renormalization conditions, extract those effects that <u>cannot</u> be absorbed during renormalization.

Renormalization conditions

On-shell scheme

- The masses are defined as the pole positions of the corresponding propagators $\Rightarrow$ $\boxed{\hat{\Pi}_{VV}(m_V^2) = 0}$
- The residue of the photon propagator at $q^2 = 0$ is unity (QED demands it), *i.e.* $\boxed{\hat{\Pi}'_{\gamma\gamma} = 0}$ (prime means derivative w.r.t. q^2).
- There is no photon-Z mixing at $q^2 = 0$, *i.e.* $\boxed{\hat{\Pi}_{\gamma Z}(0) = 0}$ (QED is thus not contaminated by Z).
- The photon-electron-electron vertex at $q^2 = 0$ with electrons in their mass shell is $\boxed{ie\gamma_\mu}$.

How many parameters?

There are four Π functions: $\Pi_{\gamma\gamma}, \Pi_{\gamma Z}, \Pi_{WW}, \Pi_{ZZ}$. There are two energy scales $q^2 = 0, M_Z^2$. Use: $\boxed{\Pi(M_V^2) \simeq \Pi(0) + M_V^2 \Pi'(0)}$

Functions parametrizing New Physics

$$\Pi'_{\gamma\gamma}(0),\ \Pi'_{Z\gamma}(0),\ \Pi_{ZZ}(0),\ \Pi'_{ZZ}(0),\ \Pi_{WW}(0),\ \Pi'_{WW}(0)$$

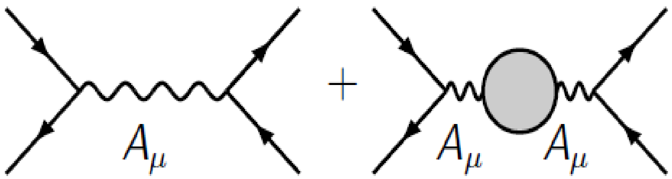
QED Ward identities $\Rightarrow \Pi_{\gamma\gamma}(0) = \Pi_{Z\gamma}(0) = 0$

$$\mathcal{L}_{new} = -\frac{\Pi'_{\gamma\gamma}(0)}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\Pi'_{WW}(0)}{2} W_{\mu\nu} W^{\mu\nu} - \frac{\Pi'_{ZZ}(0)}{4} Z_{\mu\nu} Z^{\mu\nu}$$

$$-\frac{\Pi'_{\gamma Z}(0)}{2} F_{\mu\nu} Z^{\mu\nu} - \Pi_{WW}(0) W_\mu^+ W^{\mu -} - \frac{\Pi_{ZZ}(0)}{2} Z_\mu Z^\mu$$

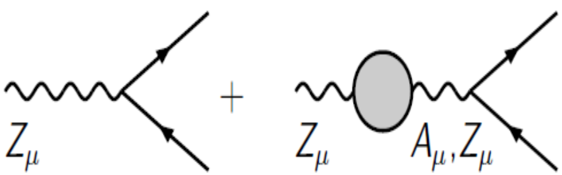
Three combinations will be absorbed in a redefinition of α, G_μ and M_Z. The remaining three will show up as radiative corrections. These are the S,T,U parameters.

Renormalization effects



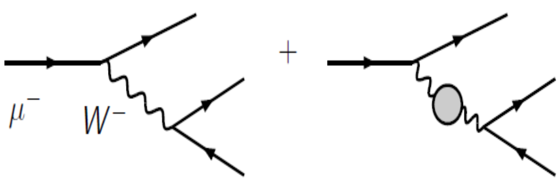
$$-i\left.\frac{4\pi\hat{\alpha}}{q^2}\right|_{q^2\to 0} = \frac{-ie^2}{q^2}\left[1+\frac{\Pi_{\gamma\gamma}(q^2)}{q^2}\right]_{q^2\to 0}$$

$\alpha(q^2) = \alpha(0)/(1+\mathrm{Re}\hat{\Pi}'_{\gamma\gamma}(q^2)) \Rightarrow \boxed{\alpha(0)\simeq(137.0)^{-1}\to\alpha(M_Z^2)\simeq(128.9)^{-1}}$.

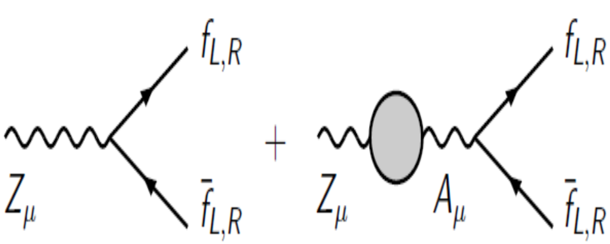


Residue of the Z propagator at the Z-pole is *not* unity $(\hat{\Pi}'_{ZZ}(q^2 = M_Z^2) \neq 0)$
$\Rightarrow$ non-trivial wave function renormalization on on-shell Z (decaying to $f\bar{f}$)
$\Rightarrow$ **celebrated ρ-parameter:**

$$\boxed{\rho = (1-\Delta r)/(1+\hat{\Pi}'_{ZZ}(M_Z^2))}$$



The muon decay radiative correction Δr (which is indeed a charged-current radiative correction) enters when we use G_μ in Z decay width formula.



Non-zero photon-Z mixing at $q^2 = M_Z^2$ (*i.e.* $\hat{\Pi}_{\gamma Z}(M_Z^2)\neq 0$). This modifies $\sin^2\theta_W \to \sin^2\theta_{\mathrm{eff}}$.

$$(\hat{s}^2_{\mathrm{eff}})^2 = s^2 - sc\frac{\Pi_{\gamma Z}(q^2 = m_Z^2)}{m_Z^2}$$

STU parameters

Peskin, Takeuchi; Marciano, Rosner; Kennedy, Langacker (1990); Altarelli, Barbieri (1991)

RAPID COMMUNICATIONS

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Oblique electroweak corrections and new physics

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Oblique electroweak parameters $\tilde{S}$, $\tilde{T}$, and $\tilde{U}$, defined so as to be nonvanishing only for physics beyond the standard model, are determined by direct use of high-statistics data from the CERN e^+e^- collider LEP at different energy points around the Z peak. Additional information from related electroweak measurements are used as constraints. The results are $\tilde{S}=-0.76\pm0.71$, $\tilde{T}=-0.70\pm0.49$, and $\tilde{U}=-0.11\pm1.07$. The consequent restrictions on extra fermion generations and an extra neutral gauge boson are discussed.

$$S \equiv \frac{16\pi}{M_Z^2}[\Pi_{33}(M_Z^2)-\Pi_{33}(0)-\Pi_{3Q}(M_Z^2)]$$

$$= \frac{16\pi}{M_Z^2}[\Pi_{3Y}(0)-\Pi_{3Y}(M_Z^2)]\,, \qquad (2a)$$

$$T \equiv \frac{4\pi}{s^2c^2M_Z^2}[\Pi_{11}(0)-\Pi_{33}(0)]\,, \qquad (2b)$$

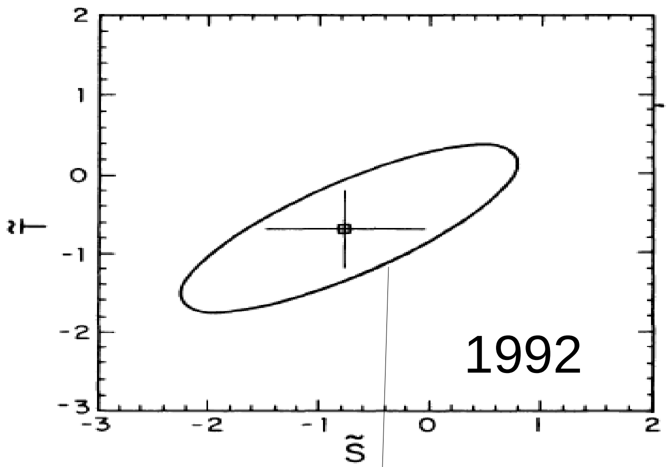
$$U \equiv \frac{16\pi}{M_W^2}[\Pi_{11}(M_W^2)-\Pi_{11}(0)]-\frac{16\pi}{M_Z^2}[\Pi_{33}(M_Z^2)-\Pi_{33}(0)]\,. \qquad (2c)$$

How to measure S and T ?

The vector and axial-vector couplings of Z are modified due to wave-function renormalization as

$$v_f = \sqrt{\rho}\left(t_3^f - 2Q_f \sin^2\bar{\theta}_W\right), \quad a_f = \sqrt{\rho}\, t_3^f.$$

$$\sin^2\bar{\theta}_W = (\sin^2\bar{\theta}_W)^{SM} + \frac{\alpha}{4(c^2-s^2)}(\tilde{S} - 4c^2s^2\tilde{T})$$

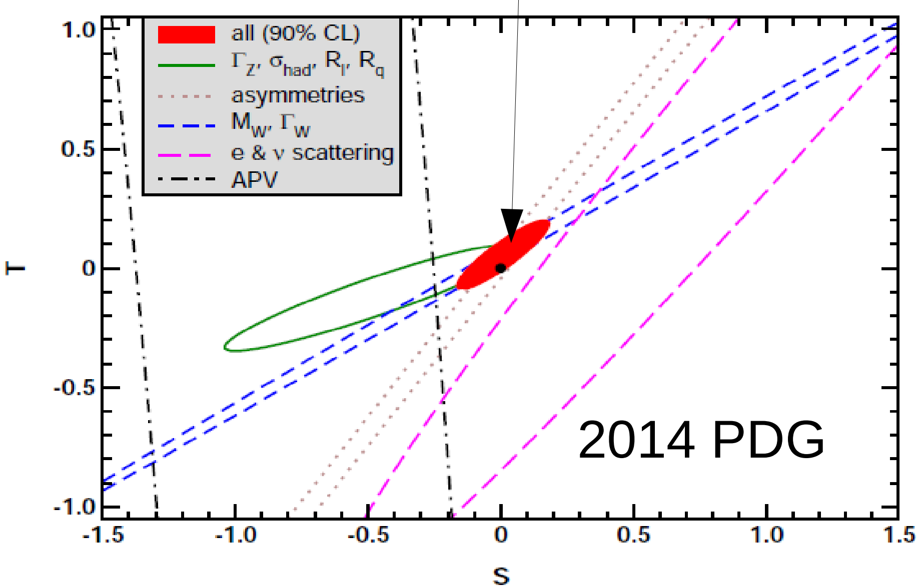


$$\Gamma_f = N_c^f \frac{G_\mu M_Z^3}{6\pi\sqrt{2}}(v_f^2 + a_f^2) ,$$

where

$$N_c^f = 1 + \frac{3\alpha}{4\pi}Q_f^2 \quad (f = \text{lepton})$$

$$= 3\left[1 + \frac{3\alpha}{4\pi}Q_f^2\right]\left[1 + \frac{\alpha_s(M_Z)}{\pi} + 1.405\frac{\alpha_s^2(M_Z)}{\pi^2}\right] \quad (f = \text{quark})$$



Top and Higgs from T and S

T is quadratically sensitivity to top mass

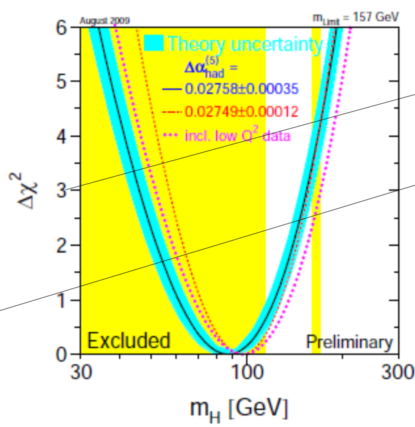
$$\Delta\rho^{t-b} = \alpha\frac{4\pi}{\sin^2\theta_W\cos^2\theta_W M_Z^2}\frac{N_c}{32\pi^2}\left[\frac{m_t^2+m_b^2}{2} - \frac{m_t^2 m_b^2}{m_t^2-m_b^2}\ln\frac{m_t^2}{m_b^2}\right] \simeq \frac{\alpha}{\pi}\frac{m_t^2}{M_Z^2}.$$

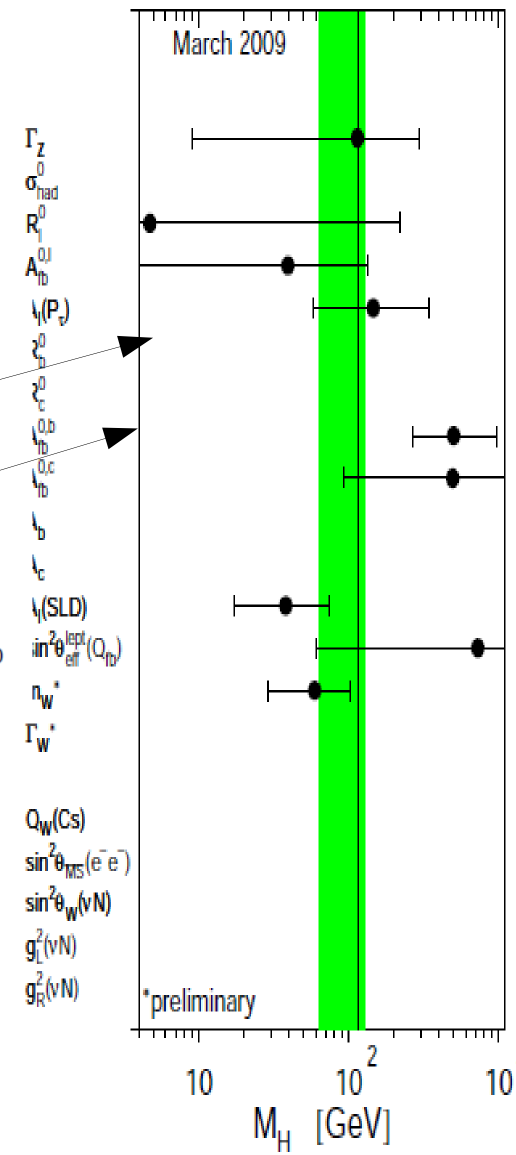
Log sensitivity to Higgs mass (Veltman screening)

$$\Delta\rho^h = -\frac{3G_F}{8\pi^2\sqrt{2}}(M_Z^2 - M_W^2)\ln\left(\frac{m_h^2}{M_Z^2}\right) \simeq -\frac{\alpha}{2\pi}\ln\frac{m_h}{M_Z}$$

$$S \equiv \frac{16\pi}{M_Z^2}\left[\Pi_{3Y}(0) - \Pi_{3Y}(M_Z^2)\right] \xrightarrow{\text{Higgs}} \frac{1}{6\pi}\ln\left(\frac{m_h}{M_Z}\right)$$

$$\boxed{\Delta\rho^{\rm SM} \simeq \frac{\alpha}{\pi}\frac{m_t^2}{M_Z^2} - \frac{\alpha}{2\pi}\ln\left(\frac{m_h}{M_Z}\right)}$$

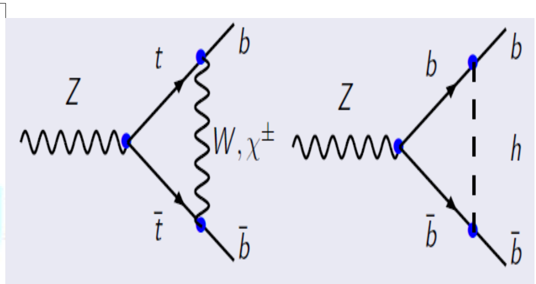




Zbb vertex

$$v_b(a_b) = v_d(a_d) - 19\Delta V_b^t/60, \quad \text{where}$$

$$\Delta V_b^t \simeq -(\alpha/\pi)\left[m_t^2/m_Z^2 + (13/6)\ln(m_t^2/m_Z^2)\right]$$



Constraints on extra chiral family

- A multiplet of heavy chiral family contributes to S irrespective of its mass. The contribution for a degenerate chral family is $2/3\pi$

$$S = \frac{C}{3\pi}\sum_i \Big(t_{3L}(i) - t_{3R}(i)\Big)^2$$

- A fourth chiral family is hugely disfavored by S and T (combining the Higgs data, a fourth chiral family with a single Higgs doublet is ruled out)

- Examples of T-violating (but S-preserving) operator $\left|H^\dagger D_\mu H\right|^2$

  and S violating (but T-preserving) operator $H^\dagger W_{\mu\nu} B^{\mu\nu} H$

$$S = -0.03 \pm 0.10$$
$$T = 0.01 \pm 0.12$$
$$U = 0.05 \pm 0.10$$

With U = 0

$$S = 0.00 \pm 0.08$$
$$T = 0.05 \pm 0.07$$

Outlook

- Standard `Model' should now be called Standard `Theory'.
- Measuring gauge interaction to a per mille precision, fixing the number of light neutrinos to 3, constraining the heavy chiral families irrespective of the mass are all outstanding achievements.
- Precision measurements have guided us to the experimental discovery of the top quark and Higgs boson.
- Higgs precision measuments would now take over. New Physics Model building is now more difficult than ever!
- Let's keep our fingers crossed.