Road map of Left-Right Symmetric Model

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References

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Important features of the model that can be specified:

- **Small masses of light neutrinos :**

  Three heavy (presumably Majorana) neutrinos: seesaw mechanism.

- **Physical interpretation of the $U(1)_{B-L}$ and Charge Quantization:**

  The SM: anomaly free global B-L symmetry .

  The right-handed neutrinos: B-L symmetry is local and anomaly free.

  The arbitrary weak hypercharge Y can be expressed in terms of B(aryon) and L(epton) numbers and so the the electric charges.

- **Smallness of CP violation in the quark sector:**

  The smallness of the CP effect is related directly to the suppression of the V+A current: the bigger right-gauge boson, the smaller CP violation.

- **Solution of the strong CP problem**:

  The strong CP parameter Θ changes under parity to $-\Theta$, thus in the L-R models $\Theta = 0$ naturally.

Let me introduce the L-R model...

- In the left-right symmetric models both left-handed and right-handed fields are treated in the same way.

- Violation of the space inversion symmetry is not an ad hoc assumption but follows from vacuum structure of the theory.

- At high energy parity is conserved, but at some energy scale v_R (connected with the masses of heavy gauge bosons) the space inversion symmetry is broken spontaneously.

- Observed near maximal parity violation at low energies is explained by the large difference between masses of heavy and light gauge bosons.

Dictionary on L-R symmetry:

In L-R symmetric models every L field has an R counterpart.

This allows the definition of the space inversion symmetry

$$(Px = (t, -\vec{x}),\ \varepsilon(\mu) = 1\ for\ \mu = 0\ and\ -1\ for\ \mu = 1,2,3)$$

$$\begin{aligned}
\psi_{L,R}(x) &\rightarrow \psi_{R,L}(Px), \\
\vec{W}^{\mu}_{L,R}(x) &\rightarrow \varepsilon(\mu)\vec{W}^{\mu}_{R,L}(Px), \\
B^{\mu}(x) &\rightarrow \varepsilon(\mu)B^{\mu}(Px), \\
\Delta_{L,R}(x) &\rightarrow \Delta_{R,L}(Px), \\
\phi(x) &\rightarrow \phi^{\dagger}(Px).
\end{aligned}$$

P symmetry implies $g_L = g_R \equiv g$, $h_{l,q} = h^{\dagger}_{l,q}$, $\tilde{h}_{l,q} = \tilde{h}^{\dagger}_{l,q}$.

$$\underline{SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}}$$

$$SU(2)_R \otimes U(1)_{B-L} \quad \rightarrow \quad U(1)_Y$$

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2}.$$

$$SU(2)_L \otimes U(1)_Y \quad \rightarrow \quad U(1)_Q$$

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}.$$

Fermion Representation

Leptons(L) and Quarks(Q)under $\underline{SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}}$:

$$L_{iL} = (\nu_i \;\; l_i)_L \;\; : \;(2,1,-1), \quad L_{iR} = (\nu_i \;\; l_i)_R : \quad (1,2,-1),$$

$$Q_{iL} = (u_i \;\; d_i)_L \;\; : \;(2,1,1/3), \quad Q_{iR} = (u_i \;\; d_i)_R : \quad (1,2,1/3).$$

$i = 1,2,3$ runs over number of generations.

$$\begin{aligned} \Psi'_L &= \left[e^{-ig'\frac{Y}{2}\Theta(x)} e^{-ig_L\frac{\vec{\tau}}{2}\vec{\Theta}(x)}\right]\Psi_L, \\ \Psi'_R &= \left[e^{-ig'\frac{Y}{2}\Theta(x)} e^{-ig_R\frac{\vec{\tau}}{2}\vec{\Theta}(x)}\right]\Psi_R. \end{aligned}$$

$$\begin{aligned} D_\mu \Psi_L &= \left(\partial_\mu - ig_L\frac{\vec{\tau}}{2}\vec{W}_{L\mu} - ig'\frac{Y}{2}B_\mu\right)\Psi_L, \\ D_\mu \Psi_R &= \left(\partial_\mu - ig_R\frac{\vec{\tau}}{2}\vec{W}_{R\mu} - ig'\frac{Y}{2}B_\mu\right)\Psi_R, \end{aligned}$$

$$L_f^{kin} = \sum_{\Psi=(Q),(L)} \bar{\Psi}_L \gamma^\mu \left(i\partial_\mu + g_L\frac{\vec{\tau}}{2}\vec{W}_{L\mu} + g'\frac{Y}{2}B_\mu\right)\Psi_L + (L \rightarrow R)\,.$$

$$
\begin{aligned}
D_\mu \Psi_L &= \left(\partial_\mu - i g_L \frac{\vec{\tau}}{2}\vec{W}_{L\mu} - i g' \frac{Y}{2} B_\mu\right)\Psi_L, \\
D_\mu \Psi_R &= \left(\partial_\mu - i g_R \frac{\vec{\tau}}{2}\vec{W}_{R\mu} - i g' \frac{Y}{2} B_\mu\right)\Psi_R,
\end{aligned}
$$

$$
L_f = \sum_{\Psi=(Q),(L)} \bar{\Psi}_L \gamma^\mu \left(i\partial_\mu + g_L \frac{\vec{\tau}}{2}\vec{W}_{L\mu} + g' \frac{Y}{2} B_\mu\right)\Psi_L + (L \to R)\,.
$$

The second gives the gauge-gauge one

$$
L_g = -\frac{1}{4} W_{Li}^{\mu\nu} W_{Li\mu\nu} - \frac{1}{4} W_{Ri}^{\mu\nu} W_{Ri\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu},
$$

where

$$
\begin{aligned}
W_{iL,R}^{\mu\nu} &= \partial^\mu W_{iL,R}^\nu - \partial^\nu W_{iL,R}^\mu + g_{L,R}\epsilon_{ijk} W_{jL,R}^\mu W_{kL,R}^\nu, \\
B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu.
\end{aligned}
$$

Scalar Representation

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \quad \rightarrow \quad SU(2)_L \otimes U(1)_Y .$$

$$\Delta_{L/R} = \begin{pmatrix} \delta^+_{L/R} & \delta^{++}_{L/R} \\ \delta^0_{L/R} & -\delta^+_{L/R} \end{pmatrix} \quad : (1,3,2)\,, \qquad \phi = \begin{pmatrix} \phi^0_1 & \phi^+_1 \\ \phi^-_2 & \phi^0_2 \end{pmatrix} \quad : (2,2,0)\,,$$

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2}.$$

$$SU(2)_L \otimes U(1)_Y \quad \rightarrow \quad U(1)_Q .$$

$$H = (h^+ \;\; h^0)^T \equiv (2,1).$$

$$Q = T_{3L} + \frac{Y}{2}.$$

Gauge transformation of bi-doublet

$$\phi' = e^{-ig_L \frac{\vec{\tau}}{2}\vec{\Theta}(x)}\ \phi\ e^{ig_R \frac{\vec{\tau}}{2}\vec{\Theta}(x)}.$$

The gauge transformations for the triplets fields

$$\Delta'_{L,R} = e^{-ig' \frac{Y}{2}\Theta(x)} e^{-ig_{L,R} \frac{\vec{\tau}}{2}\vec{\Theta}(x)}\ \Delta_{L,R}\ e^{ig_{L,R} \frac{\vec{\tau}}{2}\vec{\Theta}(x)}.$$

$$L^{kin}_{Higgs} = Tr\left[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)\right] + Tr\left[(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R)\right] + Tr\left[(D_\mu \phi)^\dagger (D^\mu \phi)\right],$$

$$D_\mu \phi = \partial_\mu \phi - ig_L \vec{W}_{L\mu}\frac{\vec{\tau}}{2}\phi + ig_R \phi \frac{\vec{\tau}}{2}\vec{W}_{R\mu},$$

$$D_\mu \Delta_{L,R} = \partial_\mu \Delta_{L,R} - ig_{L,R}\left[\frac{\vec{\tau}}{2}\vec{W}_{L,R\mu}, \Delta_{L,R}\right] - ig' B_\mu \Delta_{L,R}.$$

$$
\begin{aligned}
V(\phi,\Delta_L,\Delta_R) = & - \mu_1^2\left(Tr\left[\phi^\dagger\phi\right]\right) - \mu_2^2\left(Tr\left[\tilde{\phi}\phi^\dagger\right] + Tr\left[\tilde{\phi}^\dagger\phi\right]\right) - \mu_3^2\left(Tr\left[\Delta_L\Delta_L^\dagger\right] + Tr\left[\Delta_R\Delta_R^\dagger\right]\right) \\
& + \lambda_1\left(\left(Tr\left[\phi\phi^\dagger\right]\right)^2\right) + \lambda_2\left(\left(Tr\left[\tilde{\phi}\phi^\dagger\right]\right)^2 + \left(Tr\left[\tilde{\phi}^\dagger\phi\right]\right)^2\right) + \lambda_3\left(Tr\left[\tilde{\phi}\phi^\dagger\right]Tr\left[\tilde{\phi}^\dagger\phi\right]\right) \\
& + \lambda_4\left(Tr\left[\phi\phi^\dagger\right]\left(Tr\left[\tilde{\phi}\phi^\dagger\right] + Tr\left[\tilde{\phi}^\dagger\phi\right]\right)\right) + \rho_1\left(\left(Tr\left[\Delta_L\Delta_L^\dagger\right]\right)^2 + \left(Tr\left[\Delta_R\Delta_R^\dagger\right]\right)^2\right) \\
& + \rho_2\left(Tr\left[\Delta_L\Delta_L\right]Tr\left[\Delta_L^\dagger\Delta_L^\dagger\right] + Tr\left[\Delta_R\Delta_R\right]Tr\left[\Delta_R^\dagger\Delta_R^\dagger\right]\right) + \rho_3\left(Tr\left[\Delta_L\Delta_L^\dagger\right]Tr\left[\Delta_R\Delta_R^\dagger\right]\right) \\
& + \rho_4\left(Tr\left[\Delta_L\Delta_L\right]Tr\left[\Delta_R^\dagger\Delta_R^\dagger\right] + Tr\left[\Delta_L^\dagger\Delta_L^\dagger\right]Tr\left[\Delta_R\Delta_R\right]\right) \\
& + \alpha_1\left(Tr\left[\phi\phi^\dagger\right]\left(Tr\left[\Delta_L\Delta_L^\dagger\right] + Tr\left[\Delta_R\Delta_R^\dagger\right]\right)\right) + \alpha_2\left(Tr\left[\phi\tilde{\phi}^\dagger\right]Tr\left[\Delta_R\Delta_R^\dagger\right] + Tr\left[\phi^\dagger\tilde{\phi}\right]Tr\left[\Delta_L\Delta_L^\dagger\right]\right) \\
& + \alpha_2^*\left(Tr\left[\phi^\dagger\tilde{\phi}\right]Tr\left[\Delta_R\Delta_R^\dagger\right] + Tr\left[\tilde{\phi}^\dagger\phi\right]Tr\left[\Delta_L\Delta_L^\dagger\right]\right) + \alpha_3\left(Tr\left[\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right] + Tr\left[\phi^\dagger\phi\Delta_R\Delta_R^\dagger\right]\right) \\
& + \beta_1\left(Tr\left[\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right] + Tr\left[\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) + \beta_2\left(Tr\left[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger\right] + Tr\left[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger\right]\right) \\
& + \beta_3\left(Tr\left[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger\right] + Tr\left[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger\right]\right).
\end{aligned}
$$

Spontaneous Symmetry Breaking..

$$< \phi >= \begin{pmatrix} \kappa_1/\sqrt{2} & 0 \\ 0 & \kappa_2/\sqrt{2} \end{pmatrix} \quad , \quad < \Delta_{L,R} >= \begin{pmatrix} 0 & 0 \\ v_{L,R}/\sqrt{2} & 0 \end{pmatrix}.$$

$< \phi^0_{1,2} >$, $< \delta^0_{L,R} >$ can be complex, but the freedom of gauge symmetry transformation gives a chance to make two of them real. Usually v_R and κ_1 are taken to be real and v_L and κ_2 remain complex

$$v_L = |v_L| e^{i\Theta_L}, \qquad \kappa_2 = |\kappa_2| e^{i\Theta_2}.$$

Following six minimization conditions connect the VEVs with the Higgs potential parameters:

$$\frac{\partial V}{\partial v_R} = \frac{\partial V}{\partial \kappa_1} = \frac{\partial V}{\partial |v_L|} = \frac{\partial V}{\partial \Theta_L} = \frac{\partial V}{\partial |\kappa_2|} = \frac{\partial V}{\partial \Theta_2} = 0.$$

We can perform a consistent choice:

$$\Theta_2 = v_L = \beta_1 = \beta_2 = \beta_3 = 0.$$

Gauge Boson Mass

$$L_{Higgs}^{kin} = Tr\left[\left(D_\mu \Delta_L\right)^\dagger \left(D^\mu \Delta_L\right)\right] + Tr\left[\left(D_\mu \Delta_R\right)^\dagger \left(D^\mu \Delta_R\right)\right] + Tr\left[\left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right)\right],$$

$$L_M = \left(W_L^{+\mu},\ W_R^{+\mu}\right) \tilde{M}_W^2 \begin{pmatrix} W_{L\mu}^- \\ W_{R\mu}^- \end{pmatrix} + \frac{1}{2} \left(W_{3L}^\mu,\ W_{3R}^\mu,\ B^\mu\right) \tilde{M}_0^2 \begin{pmatrix} W_{3L\mu} \\ W_{3R\mu} \\ B_\mu \end{pmatrix},$$

$$\tilde{M}_W^2 = \frac{g^2}{4} \begin{pmatrix} \kappa_+^2 & -2\kappa_1\kappa_2 \\ -2\kappa_1\kappa_2 & \kappa_+^2 + 2v_R^2 \end{pmatrix},$$

$$\tilde{M}_0^2 = \frac{1}{2} \begin{pmatrix} \frac{g^2}{2}\kappa_+^2 & -\frac{g^2}{2}\kappa_+^2 & 0 \\ -\frac{g^2}{2}\kappa_+^2 & \frac{g^2}{2}\left(\kappa_+^2 + 4v_R^2\right) & -2gg'v_R^2 \\ 0 & -2gg'v_R^2 & 2g'^2 v_R^2 \end{pmatrix},$$

where $\kappa_+ = \sqrt{\kappa_1^2 + \kappa_2^2}$.

The symmetric mass matrices are diagonalized by the orthogonal transformations

$$\begin{pmatrix} W_L^{\pm} \\ W_R^{\pm} \end{pmatrix} = \begin{pmatrix} cos\xi & sin\xi \\ -sin\xi & cos\xi \end{pmatrix} \begin{pmatrix} W_1^{\pm} \\ W_2^{\pm} \end{pmatrix},$$

and

$$\begin{pmatrix} W_{3L} \\ W_{3R} \\ B \end{pmatrix} = \begin{pmatrix} c_W c & c_W s & s_W \\ -s_W s_M c - c_M s & -s_W s_M s + c_M c & c_W s_M \\ -s_W c_M c + s_M s & -s_W c_M s - s_M c & c_W c_M \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \\ A \end{pmatrix}$$

$$\begin{aligned} g &= \frac{e}{sin\Theta_W}, \;\; g' = \frac{e}{\sqrt{\cos 2\Theta_W}}, \;\; c_W = \cos\Theta_W, \;\; s_W = \sin\Theta_W, \\ c_M &= \frac{\sqrt{\cos 2\Theta_W}}{\cos\Theta_W}, \;\; s_M = tg\Theta_W, \;\; s = \sin\phi, \;\; c = \cos\phi. \end{aligned}$$

Masses of the physical gauge bosons are the following

$$M^2_{W_{1,2}} = \frac{g^2}{4}\left[\kappa_+^2 + v_R^2 \mp \sqrt{v_R^4 + 4\kappa_1^2\kappa_2^2}\right],$$

$$M^2_{Z_{1,2}} = \frac{1}{4}\left\{\left[g^2\kappa_+^2 + 2v_R^2\left(g^2+g'^2\right)\right] \mp \sqrt{\left[g^2\kappa_+^2 + 2v_R^2\left(g^2+g'^2\right)\right]^2 - 4g^2\left(g^2+2g'^2\right)\kappa_+^2 v_R^2}\right\}.$$

The mixing angles are given by

$$\tan 2\xi = -\frac{2\kappa_1\kappa_2}{v_R^2}, \quad \sin 2\phi = -\frac{g^2\kappa_+^2\sqrt{\cos 2\Theta_W}}{2\cos^2\Theta_W\left(M^2_{Z_2} - M^2_{Z_1}\right)}.$$

In the approximation, $v_R >> \kappa_+$, the gauge boson masses are:

$$M^2_{W_1} \simeq \frac{g^2}{4}\kappa_+^2\left(1 - \frac{(\kappa_1\kappa_2)^2}{\kappa_+^2 v_R^2}\right), \quad M^2_{W_2} \simeq \frac{g^2 v_R^2}{2},$$

$$M^2_{Z_1} \simeq \frac{g^2\kappa_+^2}{4\cos^2\Theta_W}\left(1 - \frac{\cos^2 2\Theta_W \kappa_+^2}{2\cos^4\Theta_W v_R^2}\right), \quad M^2_{Z_2} \simeq \frac{v_R^2 g^2\cos^2\Theta_W}{\cos 2\Theta_W}, \quad \sin 2\phi \simeq -\frac{\kappa_+^2\left(\cos 2\Theta_W\right)^{\frac{3}{2}}}{2v_R^2\cos^4\Theta_W}.$$

Yukawa Sector

$$L^{lepton}_{Yukawa} = -\bar{L}_L \left[h_l \phi + \tilde{h}_l \tilde{\phi} \right] L_R - \bar{L}^c_R \Sigma_L h_M L_L - \bar{L}^c_L \Sigma_R h_M L_R \ + h.c. \ ,$$

where

$$\Sigma_{L,R} = i\tau_2 \Delta_{L,R} = \left(\begin{array}{cc} \delta^0_{L,R} & -\delta^+_{L,R}\sqrt{2} \\ -\delta^+_{L,R}\sqrt{2} & -\delta^{++}_{L,R} \end{array} \right),$$

$$\tilde{\phi} = \tau_2 \phi^* \tau_2 = \left(\begin{array}{cc} \phi^{0*}_2 & -\phi^+_2 \\ -\phi^-_1 & \phi^{0*}_1 \end{array} \right)$$

and the 3×3 complex matrix h_M is symmetric ($h_M = h_M^T$). After SSB, the neutrino and charged lepton mass Lagrangians are obtained

$$L^\nu_{mass} = -\frac{1}{2} \left(\bar{n}'^c_L M_\nu n'_R + \bar{n}'^c_R M^*_\nu n'_L \right), \ \ n'_R = \left(\begin{array}{c} \nu'^c_R \\ \nu'_R \end{array} \right), \ \ n'_L = \left(\begin{array}{c} \nu'_L \\ \nu'^c_L \end{array} \right), \ \ \nu'^c_{L,R} = C \bar{\nu}'^T_{R,L},$$

$$L^l_{mass} = -\bar{l}'_L M_l l'_R - \bar{l}'_R M^\dagger_l l'_L,$$

where $\nu'_{L,R}$ ($l'_{L,R}$) are three dimensional vectors constructed from neutrino (charged lepton) fields and

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} = M_\nu^T, \;\; M_D = \frac{1}{\sqrt{2}} \left(h_l \kappa_1 + \tilde{h}_l \kappa_2 \right) = M_D^\dagger, \qquad (1)$$
$$M_R = \sqrt{2} h_M v_R = M_R^T, \;\; M_l = \frac{1}{\sqrt{2}} \left(h_l \kappa_2 + \tilde{h}_l \kappa_1 \right) = M_l^\dagger.$$

Quark sector

$$L^q_{mass} = -\bar{U}'_L M_u U'_R - \bar{D}'_L M_d D'_R + h.c. \; ,$$

where

$$M_u = \frac{1}{\sqrt{2}} \left(h_q \kappa_1 + \tilde{h}_q \kappa_2 \right), \;\; M_d = \frac{1}{\sqrt{2}} \left(\tilde{h}_q \kappa_1 + h_q \kappa_2 \right),$$

and $U'_{L,R}$, $D'_{L,R}$ are three dimensional vectors built of the weak quark fields (e.g. $\bar{U}'_L = (\bar{u}'_L \bar{c}'_L \bar{t}'_L)$).

$$M_u = M_u^\dagger, \quad M_d = M_d^\dagger.$$

These matrices are diagonalized by a bi-unitary transformation

$$U'_{L,R} = V^u_{L,R} U_{L,R}, \quad D'_{L,R} = V^d_{L,R} D_{L,R},$$

$$V_L^{u\dagger} M_u V_R^u = diag(m_u, \; m_c, \; m_t),$$
$$V_L^{d\dagger} M_d V_R^d = diag(m_d, \; m_s, \; m_b).$$

Scalar Sector after SSB

- The Higgs sector is described by 20 degrees of freedom:

  8 real fields for bidoublet and (6×2) degrees of freedom for two triplets.

- The initial 20 degrees of freedom give two charged $G^{\pm}_{L,R}$ and two neutral $\tilde{G}^0_{1,2}$ Goldstone bosons and 14 physical particles. These physical degrees of freedom produce:

  (i) four neutral scalars with $J^{PC} = 0^{++}$ $\left(H_i^0 \;\; i = 0, 1, 2, 3\right)$,

  (ii) two neutral pseudoscalars with $J^{PC} = 0^{+-}$ $\left(A_i^0 \;\; i = 1, 2\right)$,

  (iii) two singly charged bosons $\left(H_i^{\pm} \;\; i = 1, 2\right)$,

  (iv) two doubly charged Higgs particles $\left(\delta_L^{\pm\pm},\, \delta_R^{\pm\pm}\right)$.

$$
\begin{aligned}
\phi_1^0 &= \frac{1}{\sqrt{2}\kappa_+}\left[H_0^0\left(\kappa_1 a_0-\kappa_2 b_0\right)+H_1^0\left(\kappa_1 a_1-\kappa_2 b_1\right)+H_2^0\left(\kappa_1 a_2-\kappa_2 b_2\right)+i\kappa_1\tilde{G_1^0}-i\kappa_2 A_1^0\right], \\
\phi_2^0 &= \frac{1}{\sqrt{2}\kappa_+}\left[H_0^0\left(\kappa_2 a_0+\kappa_1 b_0\right)+H_1^0\left(\kappa_2 a_1+\kappa_1 b_1\right)+H_2^0\left(\kappa_2 a_2+\kappa_1 b_2\right)-i\kappa_2\tilde{G_1^0}-i\kappa_1 A_1^0\right], \\
\delta_L^0 &= \frac{1}{\sqrt{2}}\left(H_3^0+iA_2^0\right), \\
\delta_R^0 &= \frac{1}{\sqrt{2}}\left(c_0 H_0^0+c_1 H_1^0+c_2 H_2^0+i\tilde{G_2^0}\right), \\
\phi_1^+ &= \frac{\kappa_1}{\kappa_+\sqrt{1+\left(\frac{\kappa_-^2}{\sqrt{2}\kappa_+ v_R}\right)^2}}H_2^+-\frac{\kappa_1}{\kappa_+\sqrt{1+\left(\frac{\sqrt{2}\kappa_+ v_R}{\kappa_-^2}\right)^2}}G_R^+-\frac{\kappa_2}{\kappa_+}G_L^+\equiv a_{12}H_2^++a_{1R}G_R^++a_{1L}G_L^+, \\
\phi_2^+ &= \frac{\kappa_2}{\kappa_+\sqrt{1+\left(\frac{\kappa_-^2}{\sqrt{2}\kappa_+ v_R}\right)^2}}H_2^+-\frac{\kappa_2}{\kappa_+\sqrt{1+\left(\frac{\sqrt{2}\kappa_+ v_R}{\kappa_-^2}\right)^2}}G_R^++\frac{\kappa_1}{\kappa_+}G_L^+\equiv a_{22}H_2^++a_{2R}G_R^++a_{2L}G_L^+, \\
\delta_L^+ &= H_1^+, \\
\delta_R^+ &= \frac{1}{\sqrt{1+\left(\frac{\kappa_-^2}{\sqrt{2}\kappa_+ v_R}\right)^2}}G_R^++\frac{1}{\sqrt{1+\left(\frac{\sqrt{2}\kappa_+ v_R}{\kappa_-^2}\right)^2}}H_2^+\equiv a_{RR}G_R^++a_{R2}H_2^+.
\end{aligned}
$$

For $v_R >> \kappa_+$

$$
\begin{aligned}
\phi_1^0 &\simeq \frac{1}{\kappa_+\sqrt{2}}\left[\kappa_1 H_0^0 - \kappa_2 H_1^0 + i\kappa_1 \tilde{G_1^0} - i\kappa_2 A_1^0\right], \\
\phi_2^0 &\simeq \frac{1}{\kappa_+\sqrt{2}}\left[\kappa_2 H_0^0 + \kappa_1 H_1^0 - i\kappa_2 \tilde{G_1^0} - i\kappa_1 A_1^0\right], \\
\delta_R^0 &= \frac{1}{\sqrt{2}}\left(H_2^0 + iG_2^0\right), \\
\phi_1^+ &\simeq \frac{\kappa_1}{\kappa_+} H_2^+ - \frac{\kappa_2}{\kappa_+} G_L^+, \\
\phi_2^+ &\simeq \frac{\kappa_2}{\kappa_+} H_2^+ + \frac{\kappa_1}{\kappa_+} G_L^+, \\
\delta_R^+ &\simeq G_R^+.
\end{aligned}
$$

Scalar Spectrum

$$M^2_{H^0_0} \simeq 2\kappa_+^2\left[\lambda_1+\varepsilon^2\left(2\lambda_1+\lambda_3\right)+2\lambda_4\varepsilon\right],$$
$$M^2_{H^0_1} \simeq \frac{1}{2}\alpha_3 v_R^2\frac{1}{\sqrt{1-\varepsilon^2}},$$
$$M^2_{H^0_2} \simeq 2\rho_1 v_R^2.$$
$$M^2_{H^0_3} = \frac{1}{2}v_R^2\left(\rho_3-2\rho_1\right).$$

$$M^2_{A^0_1} = \frac{1}{2}\alpha_3 v_R^2\frac{1}{\sqrt{1-\varepsilon^2}}-2\kappa_+^2\left(2\lambda_2-\lambda_3\right),$$
$$M^2_{A^0_2} = \frac{1}{2}v_R^2\left(\rho_3-2\rho_1\right).$$

$$M^2_{H^\pm_1} = \frac{1}{2}v_R^2\left(\rho_3-2\rho_1\right)+\frac{1}{4}\alpha_3\kappa_+^2\sqrt{1-\varepsilon^2},$$
$$M^2_{H^\pm_2} = \frac{1}{2}\alpha_3\left[v_R^2\frac{1}{\sqrt{1-\varepsilon^2}}+\frac{1}{2}\kappa_+^2\sqrt{1-\varepsilon^2}\right].$$

$$M^2_{\delta_L^{\pm\pm}} = \frac{1}{2}\left[v_R^2\left(\rho_3-2\rho_1\right)+\alpha_3\kappa_+^2\sqrt{1-\varepsilon^2}\right],$$
$$M^2_{\delta_R^{\pm\pm}} = 2\rho_2 v_R^2+\frac{1}{2}\alpha_3\kappa_+^2\sqrt{1-\varepsilon^2}.$$

Theoretical Constraints

Boundedness of Scalar Potential

- What'll happen if the potential is not bounded from below?
- How to ensure the boundedness? Is ensuring a problem, if not then what?
- We need to first guarantee that the boundedness criteria ensures the exploration of largest parameter space – otherwise it can not be used as a theoretical constraint !
- Any better proposal than the existing ones- Copositivity of Quadratic form !
- Problem with Copositivity – Basis dependency !
- Further improvement – Open issue.

1. Copositivity of order two matrix

Let us consider a symmetric matrix of order two:

$$\mathcal{S}_2 = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ & \lambda_{22} \end{pmatrix}. \tag{A1}$$

This matrix is copositive if and only if

$$\lambda_{11} \geq 0, \quad \lambda_{22} \geq 0, \quad \text{and} \quad \lambda_{12} + \sqrt{\lambda_{11}\lambda_{22}} \geq 0. \tag{A2}$$

2. Copositivity of order three matrix

Let us consider a symmetric matrix of order three:

$$\mathcal{S}_3 = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ & \lambda_{22} & \lambda_{23} \\ & & \lambda_{33} \end{pmatrix}. \tag{A3}$$

This matrix copositive if and only if,

$$\lambda_{ii} \geq 0, \qquad \lambda_{ij} + \sqrt{\lambda_{ii}\lambda_{jj}} \geq 0,$$
$$\sqrt{\prod_{i=1,2,3} \lambda_{ii}} + \sum_{i,j,k} \lambda_{ij}\sqrt{\lambda_{kk}} + \sqrt{2\prod_{i,j,k}(\lambda_{ij} + \sqrt{\lambda_{ii}\lambda_{jj}})} \geq 0, \tag{A4}$$

where $\{i, j, k\}$ = Permutation of $\{1, 2, 3\}$ with $i < j$.

3. Copositivity of order four matrix

Let us consider a symmetric matrix of order four:

$$\mathcal{S}_4 = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} \\ & \lambda_{22} & \lambda_{23} & \lambda_{24} \\ & & \lambda_{33} & \lambda_{34} \\ & & & \lambda_{44} \end{pmatrix}. \tag{A5}$$

Case I: If all the off-diagonal elements of $\mathcal{S}_4$ are positive then this is copositive if and only if $\lambda_{ii} \geq 0$.

Case II: If $\lambda_{ij} \leq 0$ and other off-diagonal elements are positive then $\mathcal{S}_4$ is copositive if and only if $(\lambda_{ii}\lambda_{jj} - \lambda_{ij}^2) \geq 0$.

Case III: If $\lambda_{ij}, \lambda_{lk} \leq 0$ and other off-diagonal elements are positive then the matrix is copositive if and only if $(\lambda_{ii}\lambda_{jj} - \lambda_{ij}^2) \geq 0$, $(\lambda_{ll}\lambda_{kk} - \lambda_{lk}^2) \geq 0$.

Case IV: If $\lambda_{ij}, \lambda_{ik} \leq 0$ then we must have $(\lambda_{ii}\lambda_{jk} - \lambda_{ij}\lambda_{ik} + \sqrt{(\lambda_{ii}\lambda_{jj} - \lambda_{ij}^2)(\lambda_{ii}\lambda_{kk} - \lambda_{ik}^2)}) \geq 0$ to make this matrix copositive.

Case V: If $\lambda_{ij}, \lambda_{jk}, \lambda_{ik} \leq 0$ while the other off-diagonal elements are positive then $\mathcal{S}_4$ is copositive if and only if the following order three matrix is copositive:

$$\begin{pmatrix} \lambda_{ii} & \lambda_{ij} & \lambda_{ik} \\ & \lambda_{jj} & \lambda_{jk} \\ & & \lambda_{kk} \end{pmatrix}.$$

Case VI: If $\lambda_{ij}, \lambda_{ik}, \lambda_{il} \leq 0$ and other off-diagonal elements are positive then the following matrix:

$$\begin{pmatrix} \lambda_{ii}\lambda_{jj} - \lambda_{ij}^2 & \lambda_{ii}\lambda_{jk} - \lambda_{ij}\lambda_{ik} & \lambda_{ii}\lambda_{jl} - \lambda_{ij}\lambda_{il} \\ & \lambda_{ii}\lambda_{kk} - \lambda_{ik}^2 & \lambda_{ii}\lambda_{kl} - \lambda_{ik}\lambda_{il} \\ & & \lambda_{ii}\lambda_{ll} - \lambda_{il}^2 \end{pmatrix}$$

must be copositive in order to make $\mathcal{S}_4$ to be copositive.

$$ {}^{3F}V_{10}(\phi^0,\phi^+,\delta^+) = \left(\lambda_2+\frac{\lambda_3}{2}\right)\delta^{+4}+\frac{\lambda}{4}\phi^{04}+\frac{\lambda}{4}\phi^{+4} + \frac{\lambda}{2}\phi^{+2}\phi^{02}+\left(\lambda_1+\frac{\lambda_4}{2}\right)\delta^{+2}\phi^{+2} + \left(\lambda_1+\frac{\lambda_4}{2}\right)\delta^{+2}\phi^{02}. \quad \text{(D20)} $$

In matrix form it can be represented in basis $(\phi^{02},\phi^{+2},\delta^{+2})$:

$$ \begin{pmatrix} \frac{\lambda}{4} & \frac{\lambda}{4} & \frac{\lambda_1+\frac{\lambda_4}{2}}{2} \\ & \frac{\lambda}{4} & \frac{\lambda_1+\frac{\lambda_4}{2}}{2} \\ & & \lambda_2+\frac{\lambda_3}{2} \end{pmatrix}. $$

Copositivity conditions:

$$ \lambda \geq 0, \qquad \lambda_2+\frac{\lambda_3}{2}\geq 0, $$
$$ \kappa_1=\kappa_3=\frac{\lambda_1+\frac{\lambda_4}{2}}{2}+\sqrt{\frac{\lambda}{4}\left(\lambda_2+\frac{\lambda_3}{2}\right)}\geq 0, $$
$$ \kappa_2=\frac{\lambda}{4}+\sqrt{\frac{\lambda}{4}\cdot\frac{\lambda}{4}}=\frac{\lambda}{2}\geq 0, \quad \text{(D21)} $$

$$ \sqrt{\left(\frac{\lambda}{4}\right)\left(\frac{\lambda}{4}\right)\left(\lambda_2+\frac{\lambda_3}{2}\right)}+\frac{\lambda}{4}\sqrt{\lambda_2+\frac{\lambda_3}{2}}+\frac{\lambda_1+\frac{\lambda_4}{2}}{2}\sqrt{\frac{\lambda}{4}} + \frac{\lambda_1+\frac{\lambda_4}{2}}{2}\sqrt{\frac{\lambda}{4}}+\sqrt{2(\kappa_1)(\kappa_2)(\kappa_3)}\geq 0. $$

$$ {}^{4F}V_1(\phi_1^0,\phi_1^+,\delta^0,\delta^+) = \lambda_5(\delta^{02}+\delta^{+2})^2+\lambda_6\delta^{+4}+\lambda_1(\phi_1^{02}+\phi_1^{+2})^2 + \frac{1}{2}\lambda_{12}(2\delta^{02}\phi_1^{+2}+2\sqrt{2}\phi_1^0\phi_1^+\delta^0\delta^+ +\delta^{+2}(\phi_1^{02}+\phi_1^{+2}))+\lambda_9\delta^{+2}(\phi_1^{02}+\phi_1^{+2}). $$

In matrix form it can be represented in basis $(\phi_1^{02},\phi_1^{+2},\delta^{+2},\delta^{++2},\phi_1^0\phi_1^+,\delta^+\delta^{++})$:

$$ \begin{pmatrix} \lambda_1 & (1-C)\lambda_1 & \frac{1}{4}(\lambda_{12}+2\lambda_9) & \frac{1}{4}(\lambda_{12}+2\lambda_9) & 0 & 0 \\ & \lambda_1 & 0 & \frac{\lambda_{12}}{2} & 0 & 0 \\ & & \lambda_5+\lambda_6 & (1-K)\lambda_5 & 0 & 0 \\ & & & \lambda_5 & 0 & 0 \\ & & & & 2C\lambda_1 & \frac{\lambda_{12}}{\sqrt{2}} \\ & & & & & 2K\lambda_5 \end{pmatrix}. $$

Tree Unitarity– Contact terms

The quartic part of the scalar potential can be written in terms of the physical fields as follows:

$$V(H^0_{0,1,2,3}; A^0_{1,2}; H^{\pm}_{1,2}; H^{\pm\pm}_{1,2}) = \sum_{m=1,..,72} \Lambda_m H_i H_j H_k H_L,$$

where $H_i, H_j, H_k, H_l \in (H^0_{0,1,2,3}; A^0_{1,2}; H^{\pm}_{1,2}; H^{\pm\pm}_{1,2})$.

$$H_i + H_j \to H_p + H_q,$$

where $H_{i,j,p,q}$ are the physical Higgs fields.

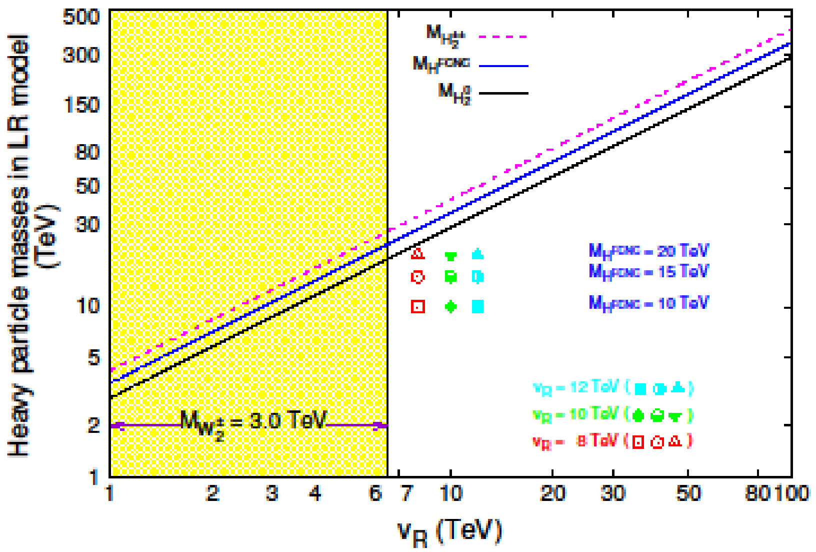
Focus is on the Quartic terms of the scalar potential

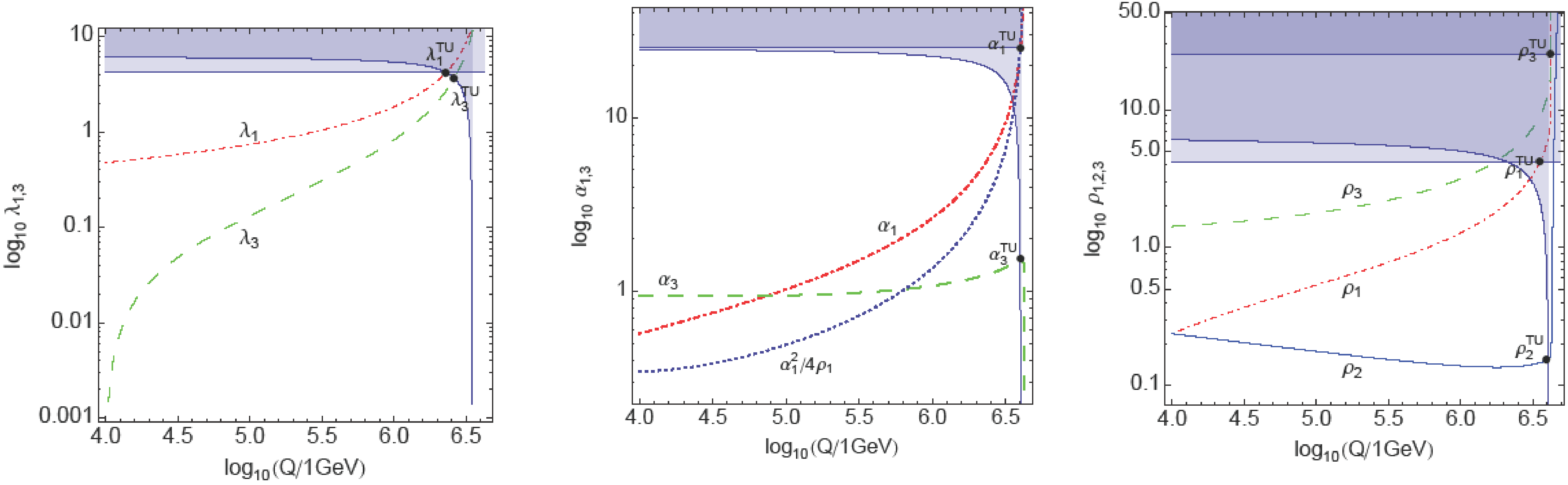
As all terms with quartic couplings in four-scalar scatterings must be smaller than 8π, the following constraints on the quartic couplings follow:

$$\lambda_1 < 4\pi/3, \ (\lambda_1 + 4\lambda_2 + 2\lambda_3) < 4\pi,$$
$$(\lambda_1 - 4\lambda_2 + 2\lambda_3) < 4\pi,$$
$$\lambda_4 < 4\pi/3,$$
$$\alpha_1 < 8\pi, \ \alpha_2 < 4\pi, \ (\alpha_1 + \alpha_3) < 8\pi,$$
$$\rho_1 < 4\pi/3, \ (\rho_1 + \rho_2) < 2\pi, \ \rho_2 < 2\sqrt{2}\pi,$$
$$\rho_3 < 8\pi, \ \rho_4 < 2\sqrt{2}\pi.$$

$$
\begin{aligned}
M^2_{H^0_0} &= 2\left(\lambda_1 - \frac{\alpha_1^2}{4\rho_1}\right)\kappa_+^2, \\
M^2_{H^0_1} &= \frac{1}{2}\alpha_3 v_R^2 < 4\pi v_R^2, \\
M^2_{H^0_2} &= 2\rho_1 v_R^2 < (8\pi/3) v_R^2, \\
M^2_{H^0_3} &= \frac{1}{2}(\rho_3 - 2\rho_1) v_R^2 < (4\pi v_R^2 - M^2_{H^0_2}/2), \\
M^2_{A^0_1} &= \frac{1}{2}\alpha_3 v_R^2 - 2\kappa_+^2(2\lambda_2 - \lambda_3) < 4\pi v_R^2, \\
M^2_{A^0_2} &= \frac{1}{2}(\rho_3 - 2\rho_1) v_R^2 < (4\pi v_R^2 - M^2_{H^0_2}/2),
\end{aligned}
$$

$$
\begin{aligned}
M^2_{H^\pm_1} &= \frac{1}{2}(\rho_3 - 2\rho_1) v_R^2 + \frac{1}{4}\alpha_3 k_+^2 \\
&< (4\pi v_R^2 - M^2_{H^0_2}/2), \\
M^2_{H^\pm_2} &= \frac{1}{2}\alpha_3 v_R^2 + \frac{1}{4}\alpha_3 k_+^2 < 4\pi v_R^2, \\
M^2_{H^{\pm\pm}_1} &= \frac{1}{2}(\rho_3 - 2\rho_1) v_R^2 + \frac{1}{2}\alpha_3 k_+^2 \\
&< (4\pi v_R^2 - M^2_{H^0_2}/2), \\
M^2_{H^{\pm\pm}_2} &= 2\rho_2 v_R^2 + \frac{1}{2}\alpha_3 k_+^2 < 4\sqrt{2}\pi v_R^2.
\end{aligned}
$$


Heavy particle masses in LR model (TeV)
v_R (TeV)
M_W2± = 3.0 TeV
M_H^FCNC = 20 TeV
M_H^FCNC = 15 TeV
M_H^FCNC = 10 TeV
v_R = 12 TeV
v_R = 10 TeV
v_R = 8 TeV

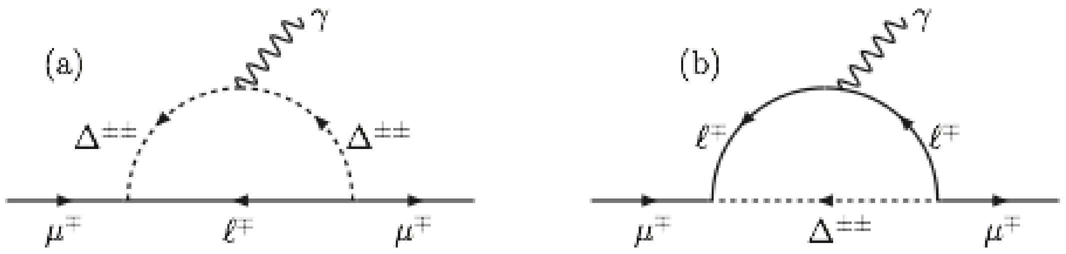

log10 λ1,3
log10(Q/1GeV)
λ1^TU
λ3^TU
λ1
λ3
log10 α1,3
α1^TU
α3^TU
α1
α3
α1^2/4ρ1
log10 ρ1,2,3
ρ3^TU
ρ1^TU
ρ2^TU
ρ3
ρ1
ρ2

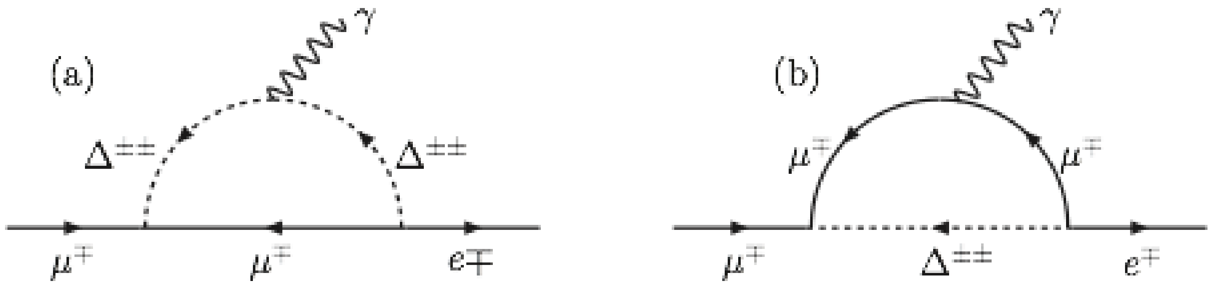
Interesting phenomenological phenomena connected with:

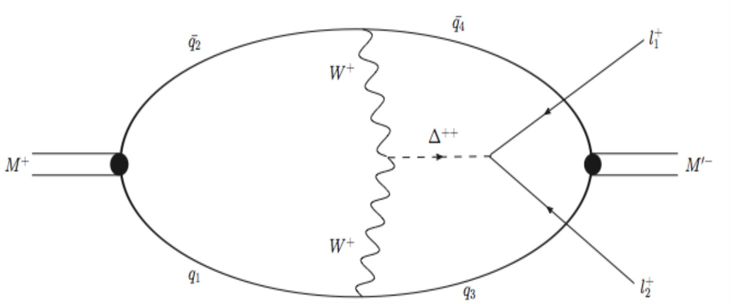
- extra gauge bosons at lepton and hadron colliders;
- heavy neutrinos at e^+e^-, $e^-\gamma$ and hadron colliders;
- doubly, singly charged and neutral Higgs particles;
- lepton number violating processes at both high and low energies;
- CP effects in Higgs, lepton and quark sectors;
- electromagnetic properties and nature of neutrinos;
- flavour changing neutral processes (FCNC);
- Kaon and B physics.

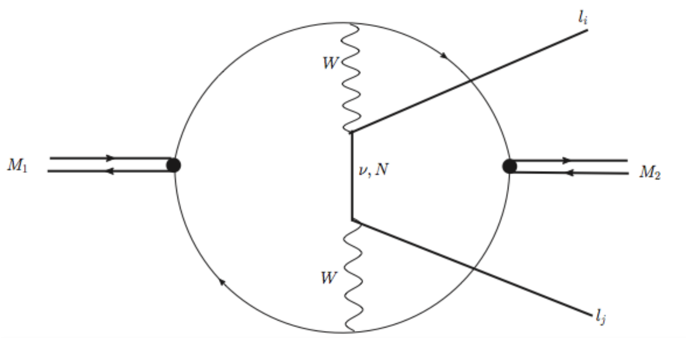
Contribution to Low energy processes

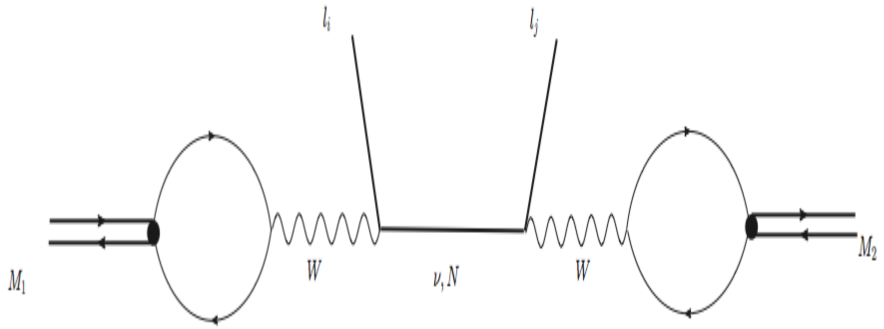
1. Anomalous magnetic moment (Δa_μ)
2. Lepton Flavour Violation ($l_i \to l_j \gamma, l_i \to l_j l_k l_m$)
3. Rare Meson decay
4. Neutrinoless double beta decay
5. Muon decay (affecting G_F)
6. Rare Tau decay ($\tau \to M_i^+ M_j^- \ell$)
7. FCNC via heavy scalars

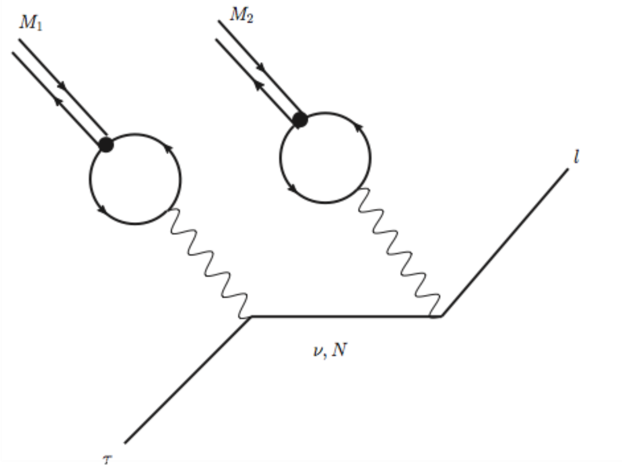

(a)
γ
$\Delta^{\pm\pm}$
$\Delta^{\pm\pm}$
$\mu^{\mp}$
$\ell^{\mp}$
$\mu^{\mp}$
(b)
γ
$\ell^{\mp}$
$\ell^{\mp}$
$\mu^{\mp}$
$\Delta^{\pm\pm}$
$\mu^{\mp}$

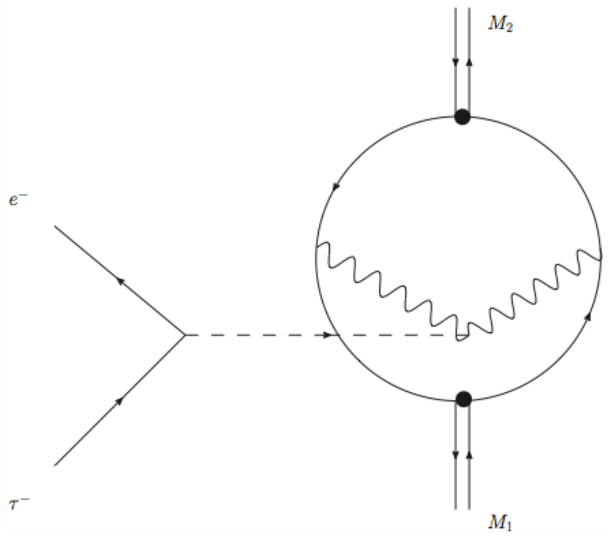

(a)
γ
$\Delta^{\pm\pm}$
$\Delta^{\pm\pm}$
$\mu^{\mp}$
$\mu^{\mp}$
$e^{\mp}$
(b)
γ
$\mu^{\mp}$
$\mu^{\mp}$
$\mu^{\mp}$
$\Delta^{\pm\pm}$
$e^{\mp}$

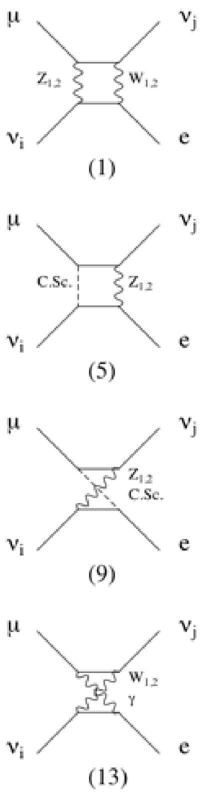

$\bar{q}_2$
$\bar{q}_4$
l_1^+
W^+
Δ^{++}
M^+
M'^-
W^+
q_1
q_3
l_2^+

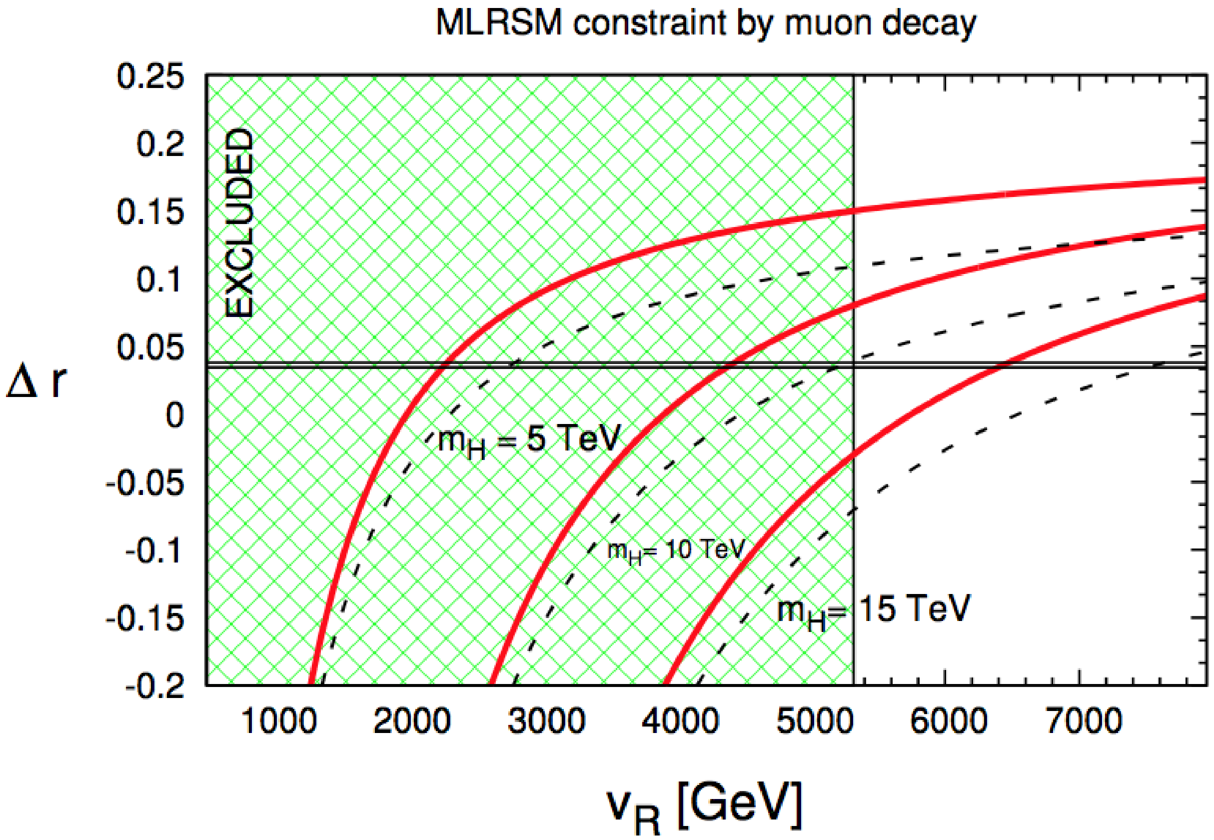

l_i
W
M_1
ν, N
M_2
W
l_j


l_i
l_j
M_1
W
ν, N
W
M_2


M_1
M_2
l
ν, N
τ

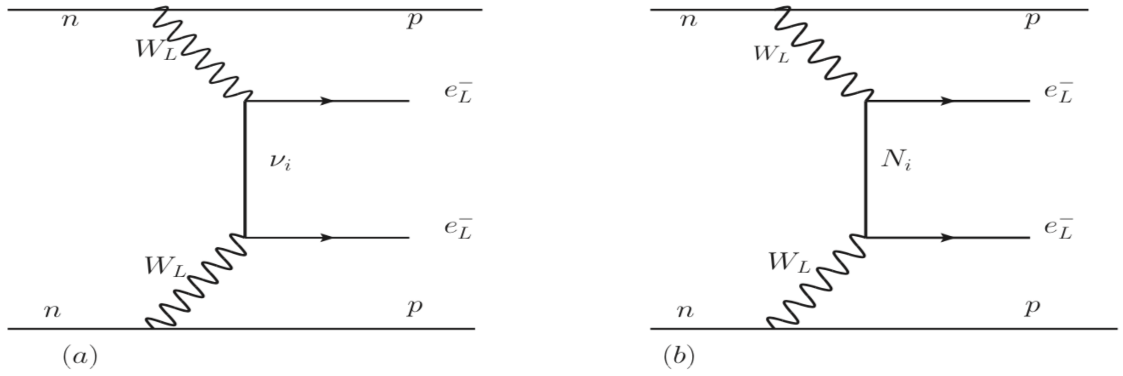

M_2
e^-
τ^-
M_1

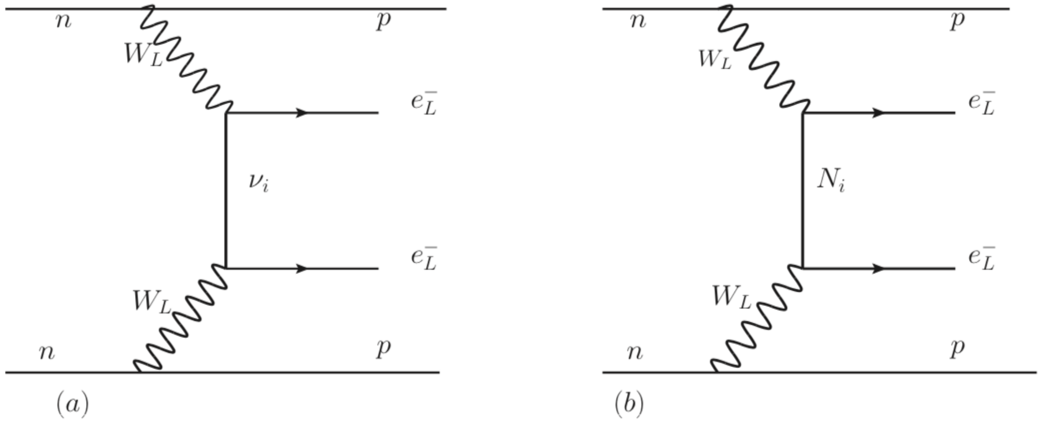


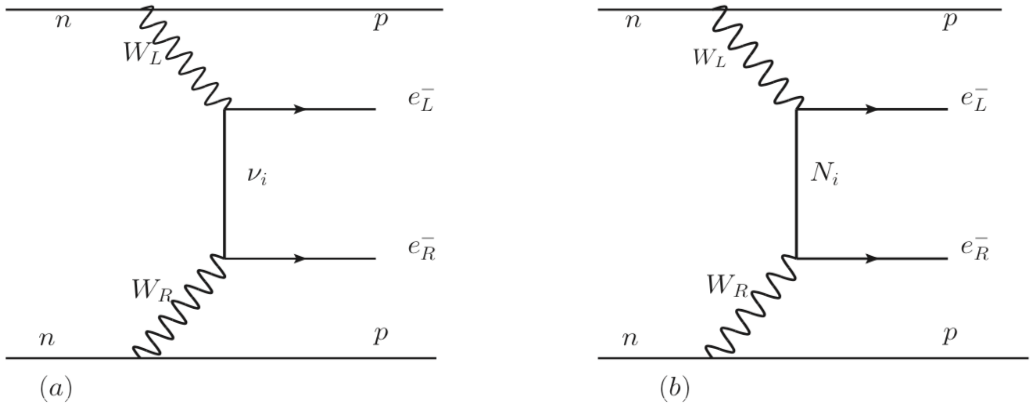


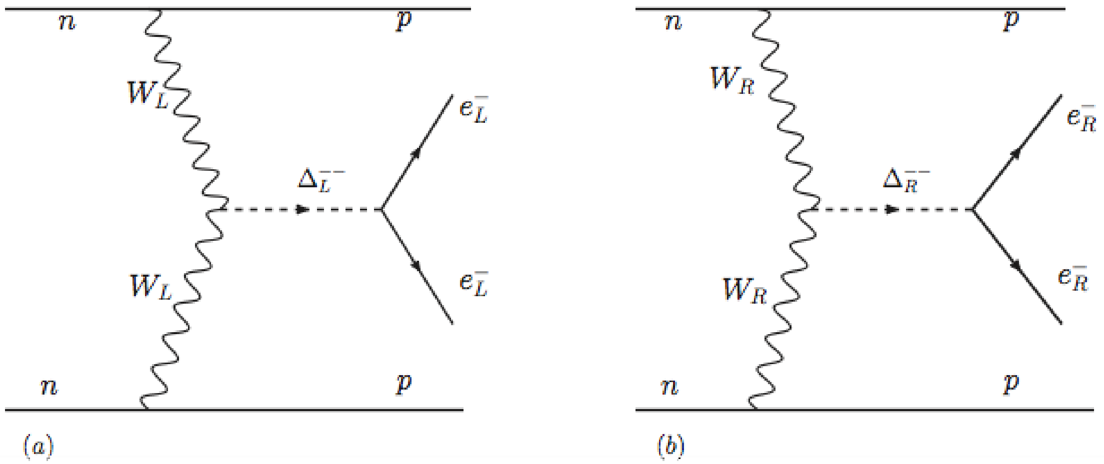
$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8(1 - M_W^2/M_Z^2)M_W^2}(1 + \Delta r).$$

$$2n \rightarrow 2p + 2e^-$$





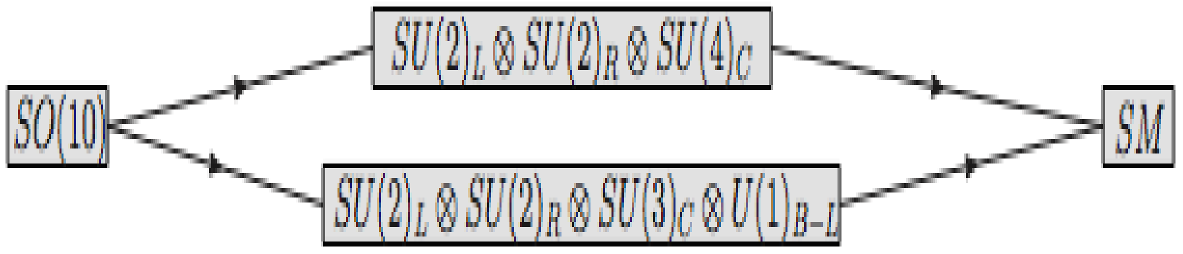


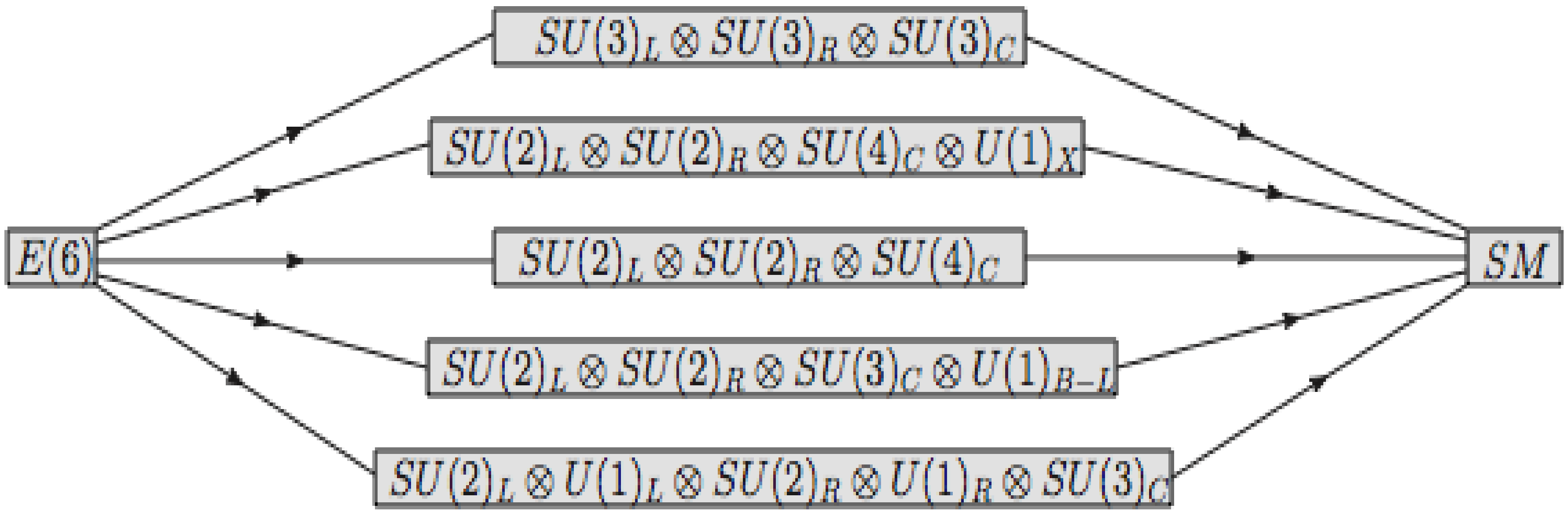


Adjudging LR model at the colider experiments

Primary production	Secondary production	Signal
I. $H_1^+H_1^-$	$\ell^+\ell^-\nu_L\nu_L$	$\ell^+\ell^- \oplus MET$
–	$\ell^+\ell^-N_RN_R$	depends on N_R decay modes
–	$\ell^+\ell^-\nu_LN_R$	depends on N_R decay modes
II. $H_2^+H_2^-$	$\ell^+\ell^-\nu_L\nu_L$	$\ell^+\ell^- \oplus MET$
–	$\ell^+\ell^-N_RN_R$	depends on N_R decay modes
–	$\ell^+\ell^-\nu_LN_R$	depends on N_R decay modes
III. $H_1^{++}H_1^{--}$	–	$\ell^+\ell^+\ell^-\ell^-$
–	$H_1^+H_1^+H_1^-H_1^-$	See I
–	$H_1^\pm H_1^\pm H_2^\mp H_2^\mp$	See I & II
–	$H_2^+H_2^+H_2^-H_2^-$	See II
–	$W_i^+W_i^+W_j^-W_j^-$	depends on W's decay modes
IV. $H_2^{++}H_2^{--}$	–	$\ell^+\ell^+\ell^-\ell^-$
–	$H_2^+H_2^+H_2^-H_2^-$	See II
–	$H_1^\pm H_1^\pm H_2^\mp H_2^\mp$	See I & II
–	$H_1^+H_1^+H_1^-H_1^-$	See I
–	$W_i^+W_i^+W_j^-W_j^-$	depends on W's decay modes
V. $H_1^{\pm\pm}H_1^\mp$	–	$\ell^\pm\ell^\pm\ell^\mp\nu_L$
VI. $H_2^{\pm\pm}H_2^\mp$	–	$\ell^\pm\ell^\pm\ell^\mp\nu_L$
VII. $H_1^\pm Z_i, H_1^\pm W_i$	–	See I & Z_i, W_i decay modes
VIII. $H_2^\pm Z_i, H_2^\pm W_i$	–	See II & Z_i, W_i decay modes
IX. $H_1^\pm\gamma$	–	See I
X. $H_2^\pm\gamma$	–	See II

Other possible Left-Right$(SU(N)_L \otimes SU(N)_R \otimes ..)$ models





Our group @ IIT Kanpur

- **Statistics, Neural Network, Machine and Deep Learning** (in collaboration with **Statistics** and **EE** departments.)


Sunando Patra


Anisha

- **Effective Field Theory** **“PANEKA”** loading....


Supratim Das Bakshi

- **Dark Matter and Cosmology**


Himadri Roy

- **Grand Unified Theory**
- **Low scale phenomenology**


Rinku Maji


Tripurari Srivastava