SANGAM@ Harish Chandra Institute, 5-10 March 2018

Leptogenesis and Baryogenesis

Pasquale Di Bari

(University of Southampton)

http://www.hep.phys.soton.ac.uk/~pdb1d08

Plan

- Lecture I: Cosmological background;
  Matter-antimatter asymmetry of the universe
  and models of Baryogenesis,

- Lecture II: Neutrino physics and Leptogenesis

- Lecture III: Leptogenesis and BSM physics

Lecture I

Cosmological background; Matter-antimatter asymmetry and models of Baryogenesis

References

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  Electroweak Baryogenesis, arXiv:1206.2942
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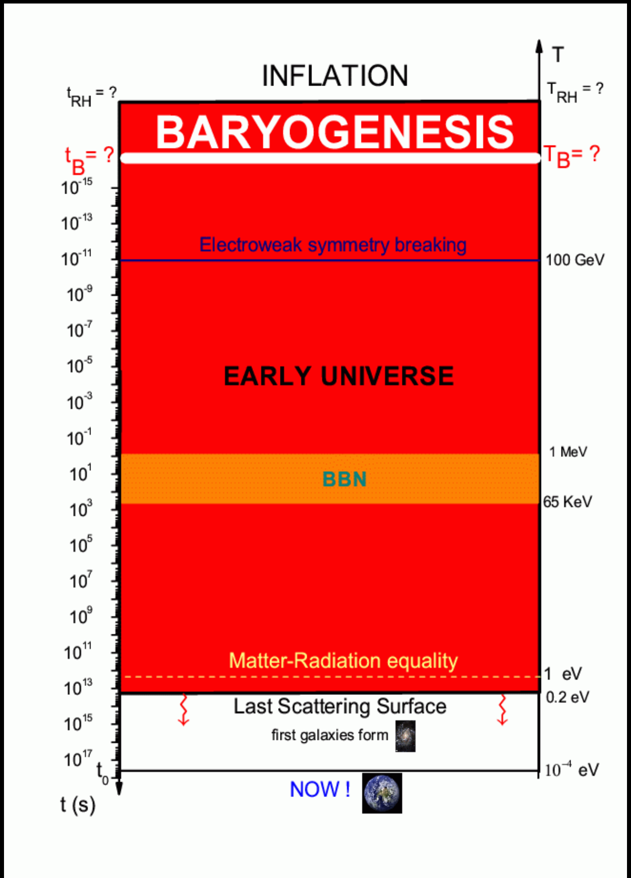
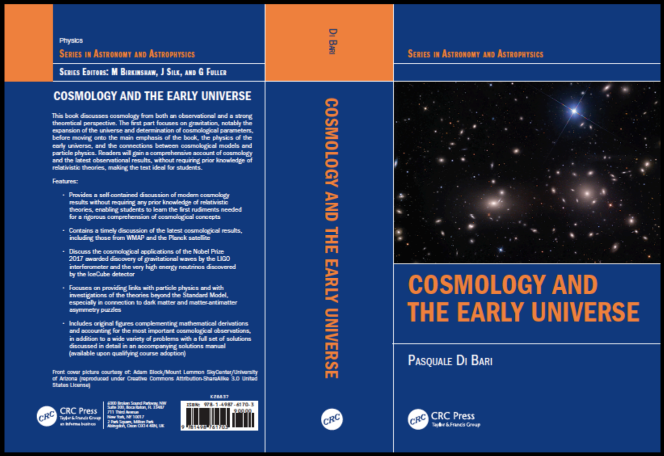




- PDB, *Cosmology and the early universe*, CRC Press, 2018.

Geometry of the Universe

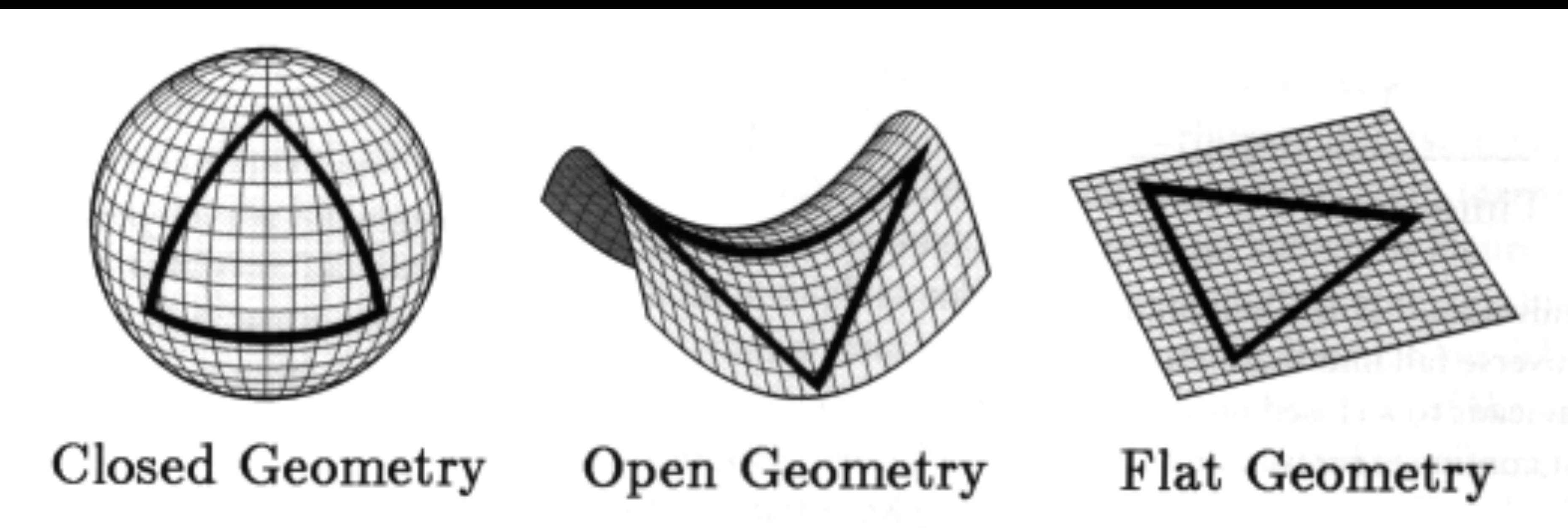
Assuming homogeneity and isotropy of space (cosmological principle)

⇒ Friedmann-Robertson-Walker metric (comoving system):

$$\boxed{ds^2 = c^2\,dt^2 - a^2(t)\,R_0^2\left(\frac{dr^2}{1-k\,r^2} + r^2\,d\Omega^2\right)}$$

$a(t) \equiv R(t)/R_0$ is the *scale factor*

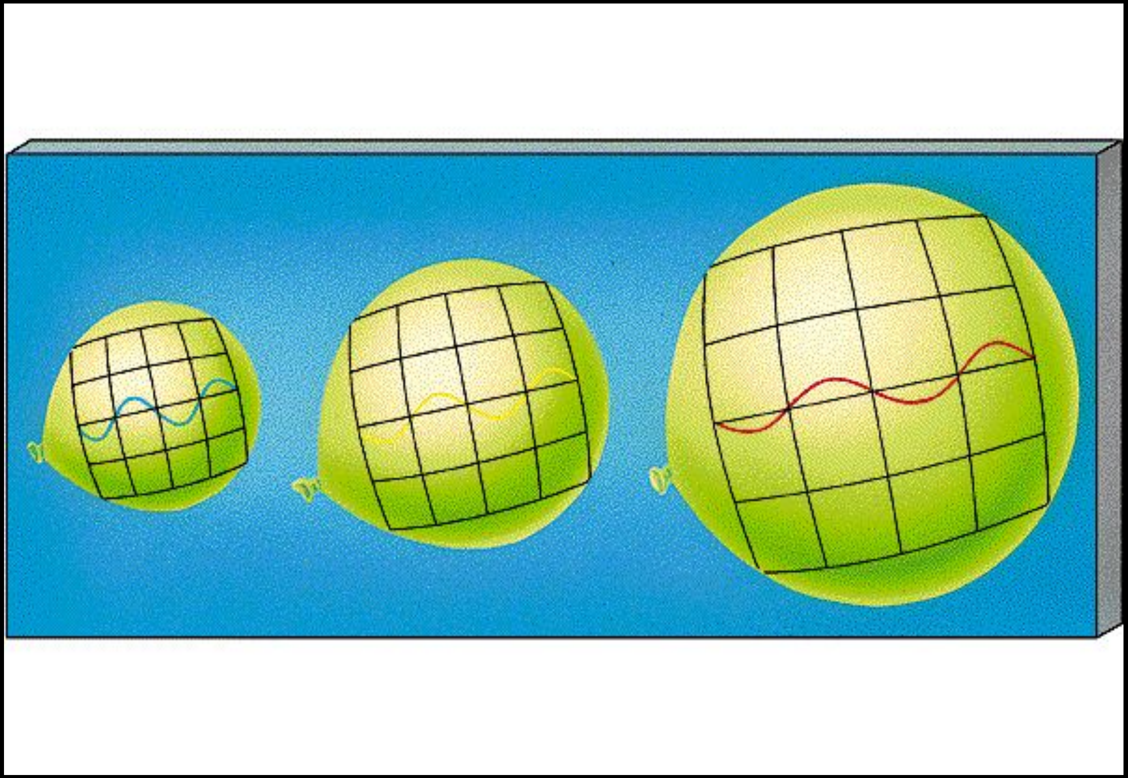
κ=+1 κ=-1 κ=0



Cosmological redshift

For photons: $|\vec{p}| = \hbar/\lambda$ $\Rightarrow$ $\lambda(t) = \lambda_0 \frac{R(t)}{R_0} = a(t)\,\lambda_0\,.$

The wavelength of photons is “stretched by the expansion” !



$$z \equiv \frac{\lambda_0}{\lambda_{\rm em}} - 1 \equiv \frac{R_0}{R_{\rm em}} - 1 = a_{\rm em}^{-1} - 1$$

Hubble's law from theory

$d_{pr,0}$

Proper distance

$$d_{\rm pr}(t) = a(t)\, R_0 \int_0^r \frac{dr'}{\sqrt{1 - k\, r'^2}}$$

$\Rightarrow$

$$\dot{d}_{\rm pr}(t) = \dot{a}(t)\, d_{\rm pr}(t_0)$$

Expansion rate

$$H(t) \equiv \frac{\dot{a}}{a}$$

Proper velocity

$$v_{\rm pr}(t) \equiv \dot{d}_{\rm pr}(t)$$

Lemaitre's equation

$$\dot{d}_{\rm pr}(t) = \dot{a}(t)\, d_{\rm pr,0}\,.$$

At the present time one can relate the proper distance to the luminosity distance and redshift:

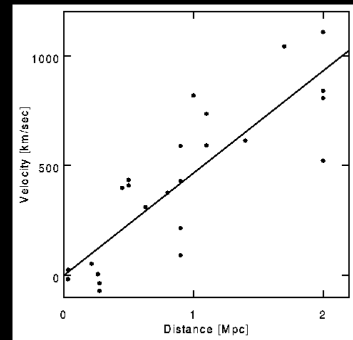
$$d_L(z) = (1+z)\, d_{\rm pr,0}(z) = c\, H_0^{-1} \left[z + z^2 \left(\frac{1 - q_0}{2} \right) \right] + \mathcal{O}(z^3)\,.$$

Hubble's law

$$d_L(z) = c\, H_0^{-1}\, z + \mathcal{O}(z^2)$$

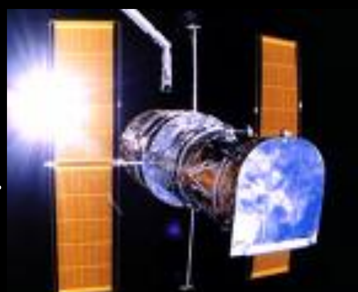
Hubble constant measurements

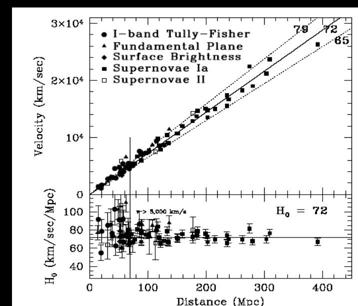
Edwin Hubble (1929)



$$H_0 \sim 500\, km\ s^{-1} Mpc^{-1}$$

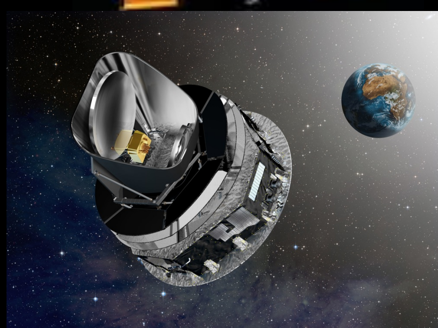
Hubble Space Telescope Key Project (2001)

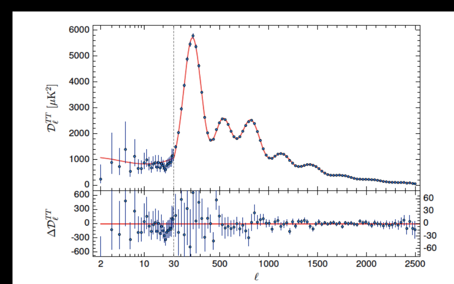




$$H_0 = (72 \pm 8)\, km\ s^{-1} Mpc^{-1}$$

Planck 2015 +ΛCDM



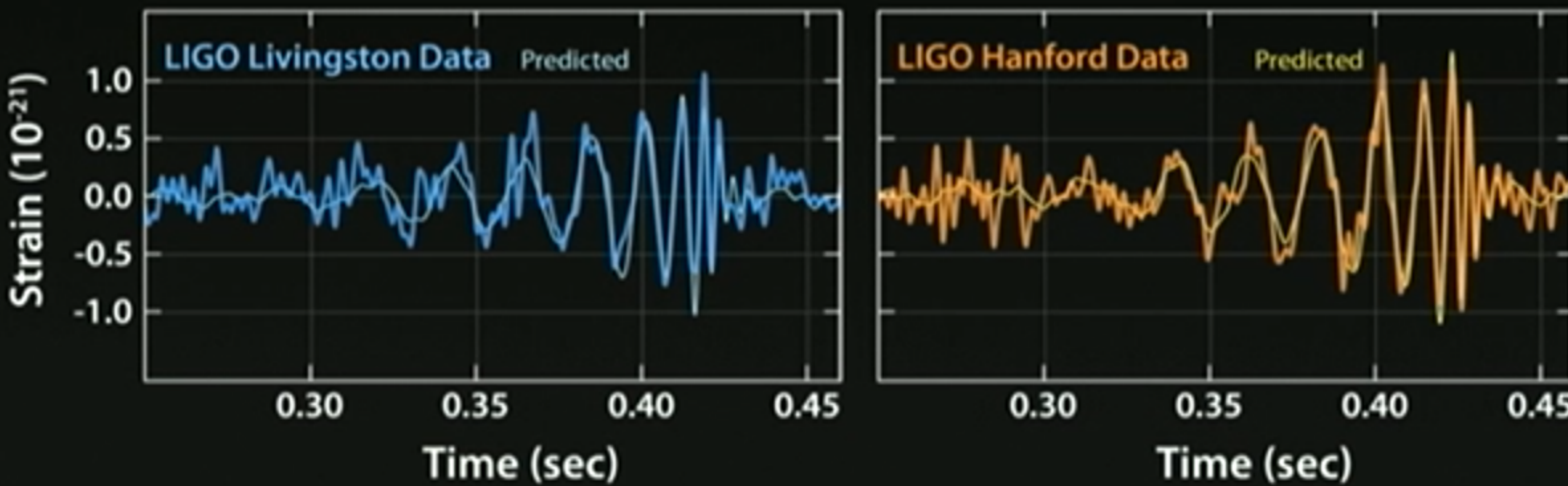


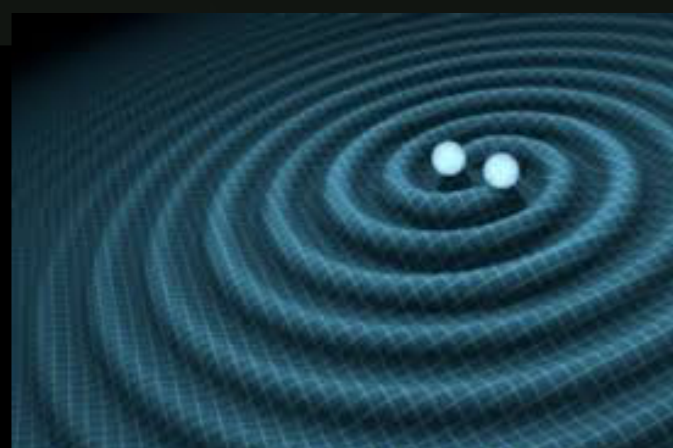
$$H_0 = (67.3 \pm 1.2)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

3.7σ tension

Hubble Space Telescope , Riess et al. (2018)

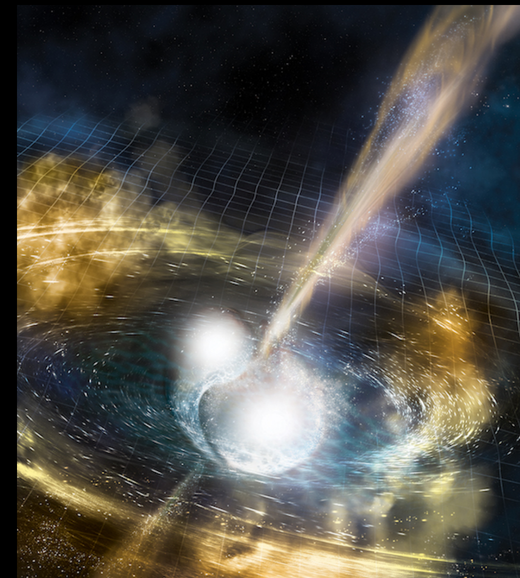
$$H_0 = (73.48 \pm 1.66)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$


LIGO Livingston Data
Predicted
LIGO Hanford Data
Predicted
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (sec)
Time (sec)



GW170817: The first observation of gravitational waves from from a binary neutron star inspiral

(almost) coincident detection of GW's and light: one can measure distance from GW's "sound" and redshift from light: STANDARD SIREN!



A GRAVITATIONAL-WAVE STANDARD SIREN MEASUREMENT OF THE HUBBLE CONSTANT

THE LIGO SCIENTIFIC COLLABORATION AND THE VIRGO COLLABORATION, THE 1M2H COLLABORATION, THE DARK ENERGY CAMERA GW-EM COLLABORATION AND THE DES COLLABORATION, THE DLT40 COLLABORATION, THE LAS CUMBRES OBSERVATORY COLLABORATION, THE VINROUGE COLLABORATION, THE MASTER COLLABORATION, et al.

arXiv:1710.05835

$$H_0 = 70^{+12}_{-8}\ km\ s^{-1}\ Mpc^{-1}$$

~50 more detections of standard sirens should reduce the error below and solve the current tension between Planck and HST measurements

Fundamental equations of Friedmann cosmology

Einstein equations $G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \Rightarrow \left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\varepsilon - \frac{k}{a^2 R_0^2}$ Friedmann equation

Energy-momentum tensor conservation $T^{\mu\nu}{}_{;\nu} = 0 \Rightarrow \frac{d(\varepsilon a^3)}{dt} = -p\frac{da^3}{dt}$ Fluid equation

Friedmann equation + Fluid equation $\Rightarrow \ddot{a} = -4\pi G(\varepsilon + 3p)a$ acceleration equation

Critical energy density $\varepsilon_c \equiv \frac{3H^2}{8\pi G}$ energy density parameter $\Omega \equiv \frac{\varepsilon}{\varepsilon_c} = \sum_i \Omega_{X_i}$

$k \equiv H_0^2 R_0^2 (\Omega_0 - 1) \Rightarrow$

- $\Omega_0 < 1 \Leftrightarrow k = -1 \Leftrightarrow$ open Universe
- $\Omega_0 = 1 \Leftrightarrow k = 0 \Leftrightarrow$ flat Universe ;
- $\Omega_0 > 1 \Leftrightarrow k = +1 \Leftrightarrow$ closed Universe

Building a cosmological model: general strategy

- Assume an equation of state: $p=p(\varepsilon)$
- Plug the equation of state into the fluid equation

$$\frac{d(\varepsilon a^3)}{dt} = -p\frac{da^3}{dt} \Rightarrow \varepsilon = \varepsilon(a)$$

- Finally plug $\varepsilon(a)$ into the Friedmann equation

$$\boxed{\dot{a}^2(t) = H_0^2\,\Omega_0\,a^2(t)\,\frac{\varepsilon(t)}{\varepsilon_0} + H_0^2\,(1-\Omega_0)} \Rightarrow a=a(t) \Rightarrow \varepsilon = \varepsilon(t)$$

- Example: Matter universe

$p_M=0 \Rightarrow \varepsilon_M = \varepsilon_{M0}/a^3 \Rightarrow$ (flat universe) $a(t)= (t/t_0)^{2/3}$, $t_0=2H_0^{-1}/3$

Flat Universe with 1 fluid : summary of the results

	equation of state	fluid equation	Friedmann equation (Flat Universe)	Age of the Universe (Flat Universe)	energy density vs. time (Flat Universe)
Matter Universe	$p_M = 0$	$\varepsilon_M(a) = \frac{\varepsilon_{M,0}}{a^3} \propto a^{-3}$	$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$	$t_0 = \frac{2}{3} H_0^{-1} = (9.40 \pm 0.15)\ \mathrm{Gyr}$	$\varepsilon_M(t) = \varepsilon_0 \left(\frac{t_0}{t}\right)^2$
Radiation Universe	$p_R = \frac{1}{3} \varepsilon_R$.	$\varepsilon_R = \frac{\varepsilon_{R,0}}{a^4} \propto a^{-4}$	$a(t) = \left(\frac{t}{t_0}\right)^{1/2}$	$t_0 = \frac{1}{2} H_0^{-1} = (7.05 \pm 0.12)\ \mathrm{Gyr}$	$\varepsilon_R(t) = \varepsilon_0 \left(\frac{t_0}{t}\right)^2$
	if $\varepsilon_R \propto T_R^4$,	$\Rightarrow$ $T_R \propto a^{-1}$			
Fluids with constant p/ε	$p = \omega\, \varepsilon$ $(\omega \neq -1)$	$\varepsilon(a) = \frac{\varepsilon_0}{a^{3(1+\omega)}}$	$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+\omega)}}$ $\omega > -1$	$t_0 = \frac{2}{3(1+\omega)} H_0^{-1}$	$\varepsilon(t) = \varepsilon_0 \left(\frac{t_0}{t}\right)^2$
de Sitter model	p=-ε ω=-1	$\varepsilon=\varepsilon_0$=const	$a(t) = e^{H_0(t-t_0)}$	INFINITE !	$\varepsilon=\varepsilon_0$=const

Matter-radiation equality

Consider and admixture of 2 fluids: matter (M) and radiation (R) :

$$p = p_M + p_R, \quad \varepsilon = \varepsilon_M + \varepsilon_R$$

with equations of state:

$$p_M = 0, \; p_R = \frac{1}{3}\varepsilon_R \; ,$$

That, from the fluid equation, lead to :

$$\varepsilon_M = \frac{\varepsilon_{M,0}}{a^3}, \quad \varepsilon_R = \frac{\varepsilon_{R,0}}{a^4}$$

The equality matter-radiation time is defined as:

$$\frac{\varepsilon_{M0}}{a_{eq}^3} = \frac{\varepsilon_{R0}}{a_{eq}^4} \Rightarrow a_{eq} = \frac{\varepsilon_{R0}}{\varepsilon_{M0}} = \frac{\Omega_{R0}}{\Omega_{M0}}$$

Friedmann cosmology as a conservative system

In terms of H_0 and Ω_0 the Friedmann equation can be recast as:

$$\frac{\dot{a}^2}{H_0^2} = \Omega_0 \frac{\varepsilon a^2}{\varepsilon_0} + (1 - \Omega_0)$$

If $\varepsilon = \varepsilon(a)$ then we can define:

$$V(a) = -\Omega_0 \frac{\varepsilon a^2}{\varepsilon_0}, \quad E_0 \equiv 1 - \Omega_0 \;\Rightarrow\; \frac{\dot{a}^2}{H_0^2} + V(a) \equiv E(a) = E_0$$

Showing that the Friedmann equation has an integral of motion, E(a) , and is, therefore, a conservative system: this will be useful to find the set of solutions for specific models

Lemaitre models

Admixture of 3 fluids: matter (M) + radiation (R) + Λ-like fluid (Λ) :

$$p = p_M + p_R + p_\Lambda , \quad \varepsilon = \varepsilon_M + \varepsilon_R + \varepsilon_\Lambda$$

with equations of state:

$$p_M = 0, \; p_R = \frac{1}{3}\varepsilon_R \, , \; p_\Lambda = -\varepsilon_\Lambda$$

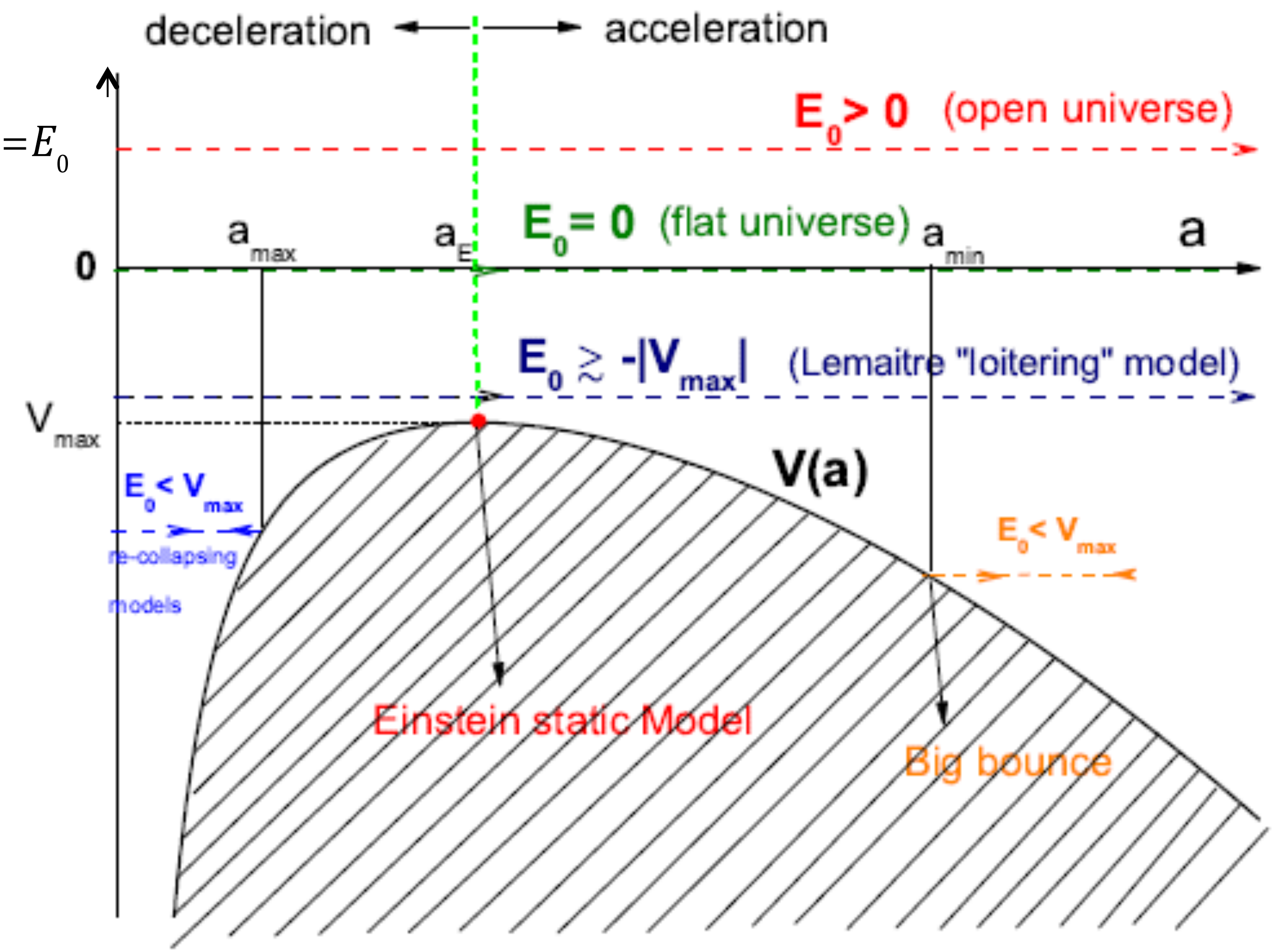
That, from the fluid equation, lead to :

$$\varepsilon_M = \frac{\varepsilon_{M,0}}{a^3}, \;\; \varepsilon_R = \frac{\varepsilon_{R,0}}{a^4} \, , \quad \varepsilon_\Lambda = \varepsilon_{\Lambda,0}$$

$$\Rightarrow V(a) = -a^2 \left(\frac{\Omega_{R,0}}{a^4} + \frac{\Omega_{M,0}}{a^3} + \Omega_{\Lambda,0} \right)$$

Lemaitre models

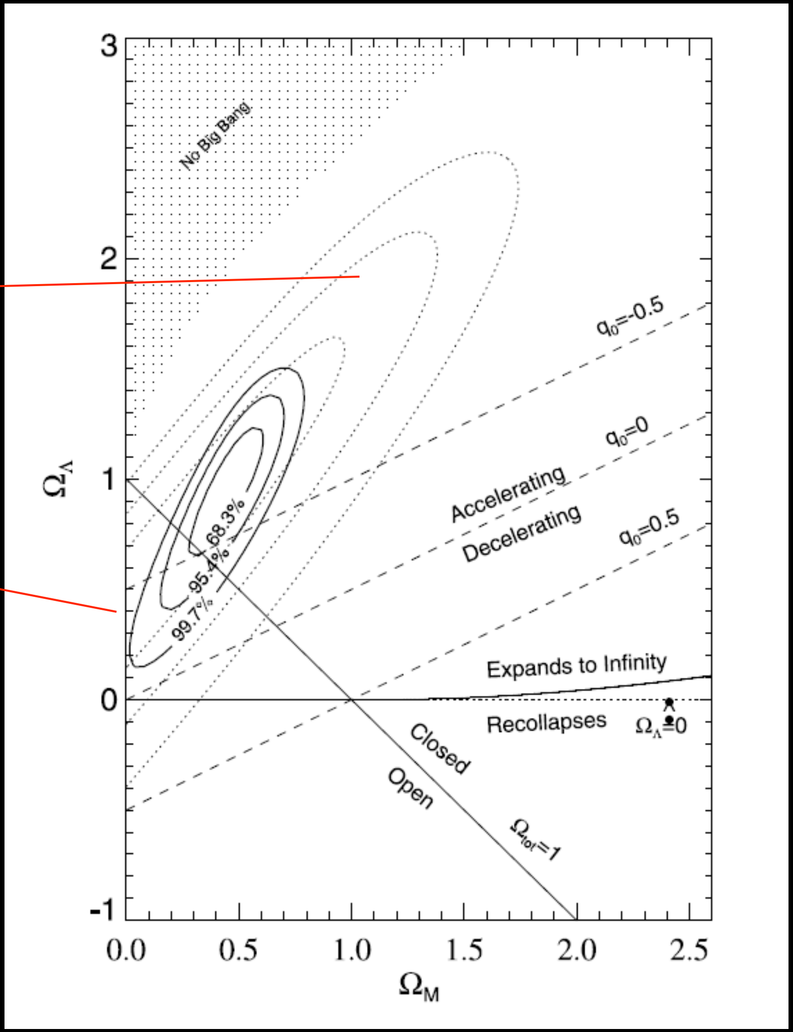
$$E(a) \equiv \frac{\dot{a}^2}{H_0^2} + V(a) = E_0$$



Supernovae type Ia

Old results

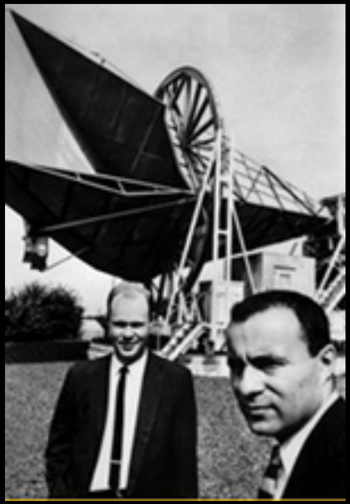
New results



A. G. Riess *et al.* [Supernova Search Team Collaboration], *Type Ia Supernova Discoveries at z¿1 From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution*, Astrophys. J. **607** (2004) 665.

The discovery of the cosmic microwave background radiation

COBE satellite

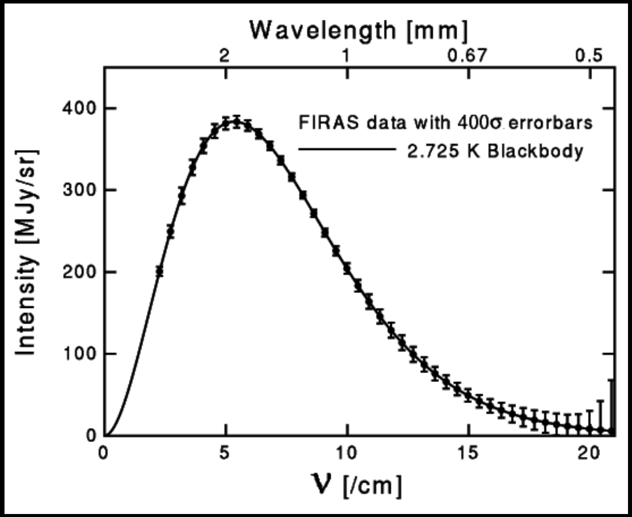


Penzias and Wilson (1965)

$T_{\gamma 0}$= (3.5 ± 1) ^{0}K

FIRAS instrument of COBE (1990)

$T_{\gamma 0}$= (2.725 ± 0.002) ^{0}K ⇒ $n_{\gamma 0} \simeq 411$ cm^{-3}



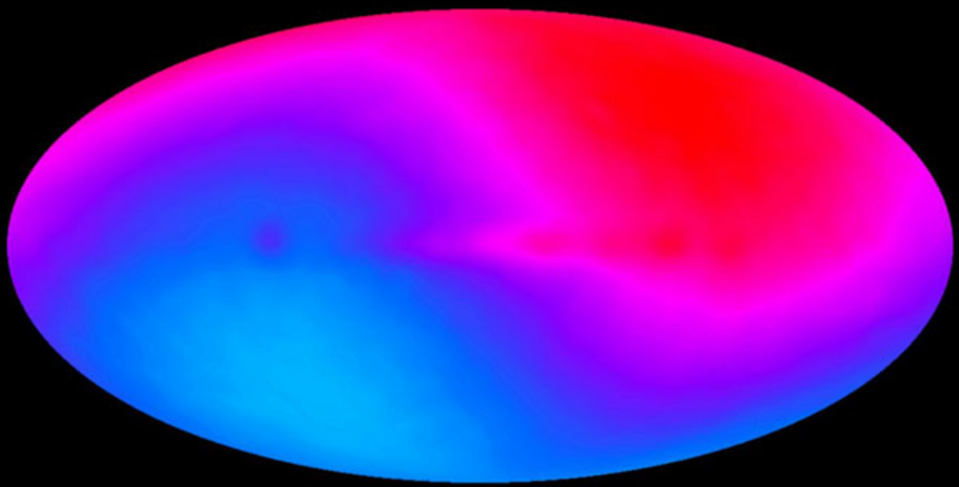
⇒ $\Omega_{\gamma 0} \simeq 0.54 \times 10^{-4}$

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{m=\ell} a_{\ell m} Y_{\ell m}(\theta,\phi) \,.$$

$$\Delta\theta = \frac{180^\circ}{\ell}\,.$$

Example: the dipole anisotropy ($\Delta\Theta$=180°) corresponds to l =1

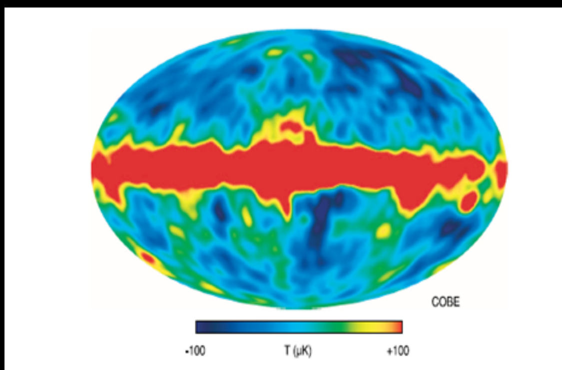
COBE DMR microwave map of the sky in Galactic coordinates: temperature variation with respect to the mean value <T> =2.725 K. The color change indicates a fluctuation of $\Delta T \sim 3.5$ mK $\Rightarrow$ $\Delta T/T \sim 10^{-3}$



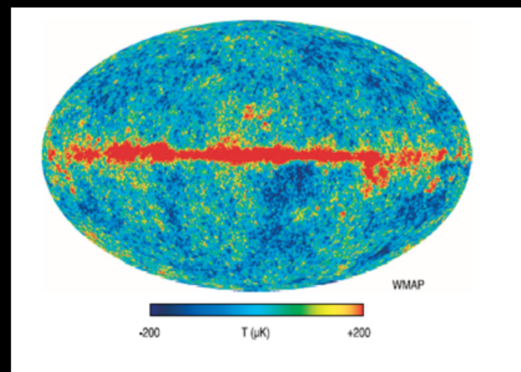
CMB temperature anisotropies

After subtraction of the dipole anisotropy, higher multipole anisotropies are measured with a much lower amplitude than the dipole anisotropy $\Rightarrow T/T \sim 10^{-5}$

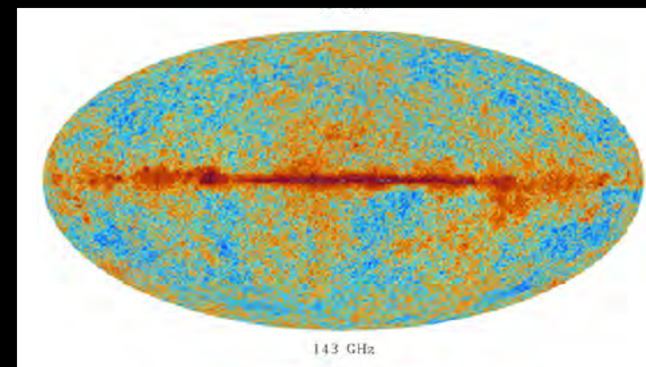
COBE (1992)



WMAP (2003)

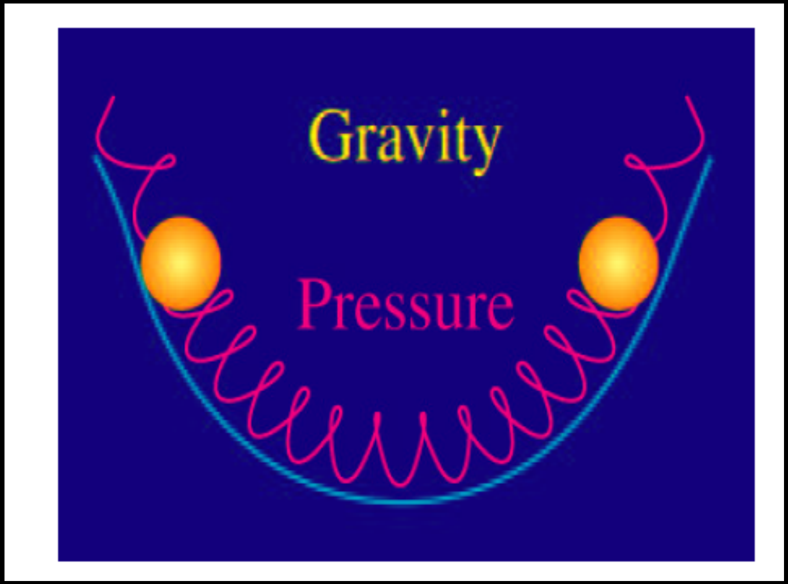


Planck (2013)



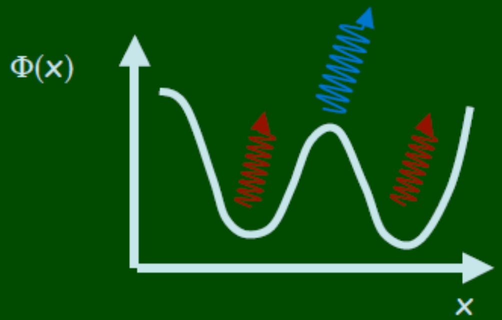
The angular resolution of COBE was about $\delta\Theta^{COBE} \simeq 7°$, that one of WMAP is $\delta\Theta^{WMAP} \simeq 10'$, while that one of Planck is $\delta\Theta^{Planck} \simeq 3'$

Acoustic oscillations

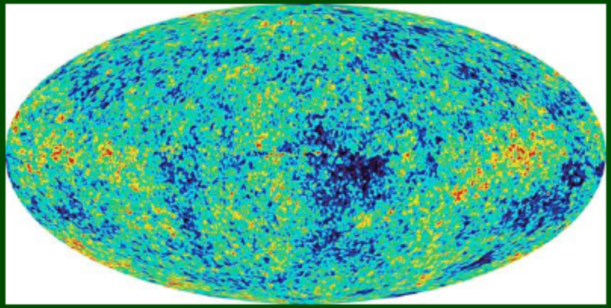


- Photons escape from gravitational potential

Cold spots = high density
Hot spots = low density

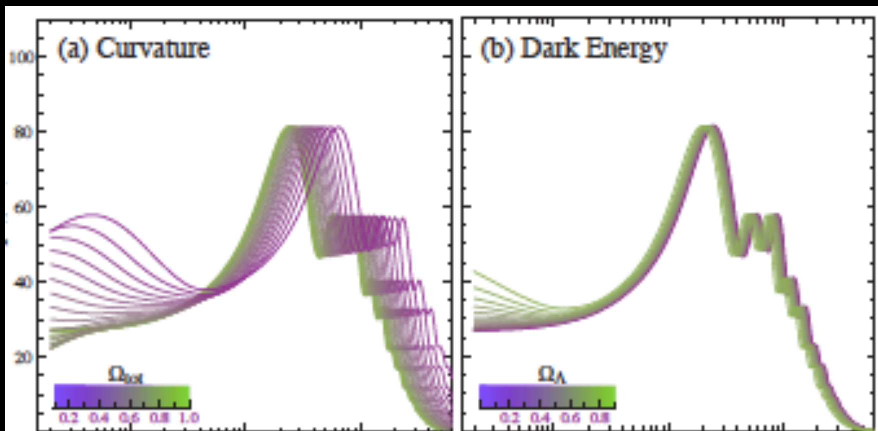


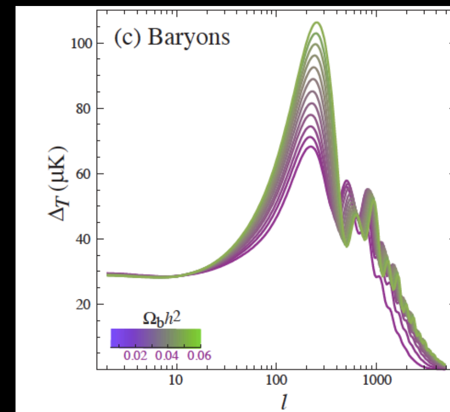
- ...Translate into fluctuations in the blackbody photon temp at ~1/100,000 level

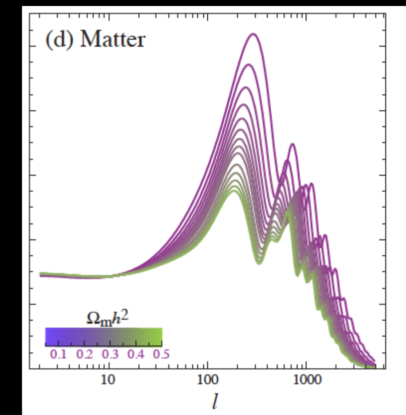


Cosmic ingredients

(Hu, Dodelson, astro-ph/0110414)



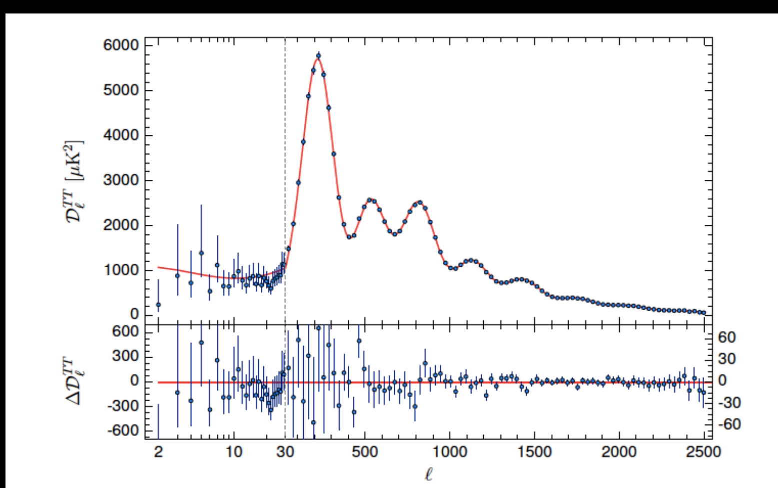




$\Omega_0 = 1.005 \pm 0.005$ $\quad \Omega_{\Lambda 0} = 0.685 \pm 0.013$ $\quad \Omega_{B0} h^2 = 0.02222 \pm 0.00023$ $\quad \boxed{\Omega_{CDM,0} h^2 = 0.1198 \pm 0.0015 \sim 5\Omega_{B,0} h^2}$

$$h \equiv \frac{H_0}{100 km\, s^{-1}\ Mpc^{-1}} = 0.67 \pm 0.1$$

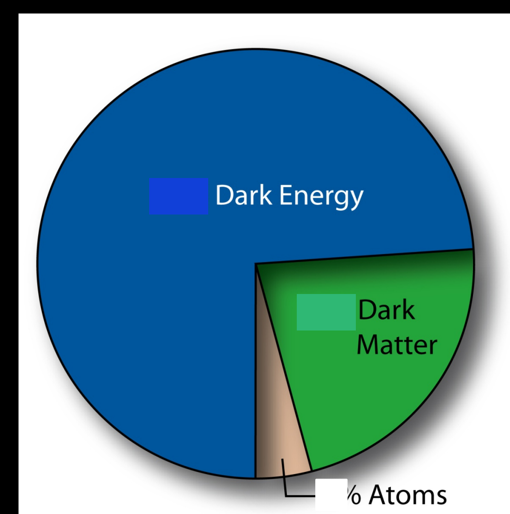
(Planck 2015, 1502.01589)



$$\Omega_{B0} \simeq 0.048$$

$$\Omega_{CDM,0} \simeq 0.26$$

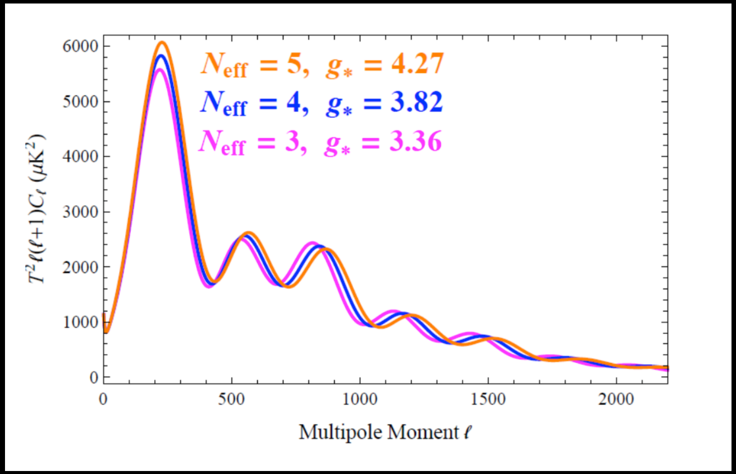
$$\Omega_{M,0} \simeq 0.308$$

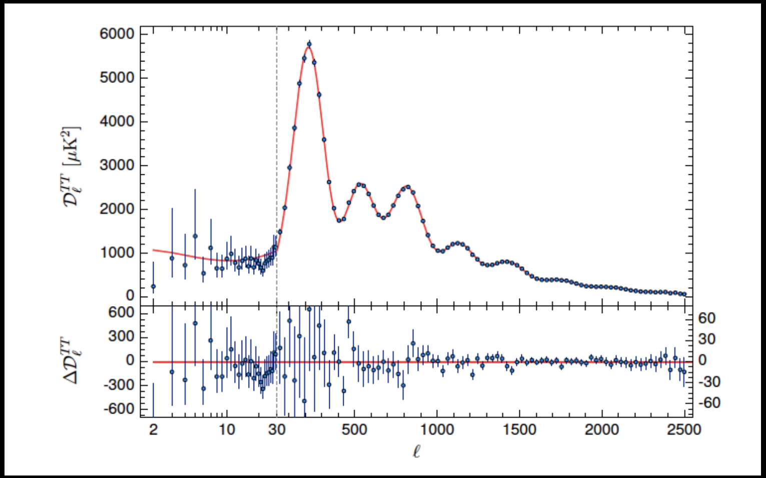


Radiation at CMB decoupling

$$\Omega_{R0} = \Omega_{\gamma 0} + \Omega_{\nu 0} = g_{R0} \frac{\pi^2}{30} \frac{T_0^4}{\varepsilon_{c0}} \simeq 0.27\, g_{R0} \times 10^{-4}$$

$$g_{R0} = 2 + N_\nu^{dec} \frac{7}{4} \left(\frac{T_{\nu 0}}{T_0} \right)^4 \simeq 3.36 + \frac{7}{4} (N_\nu^{dec} - 3) \left(\frac{T_{\nu 0}}{T_0} \right)^4$$





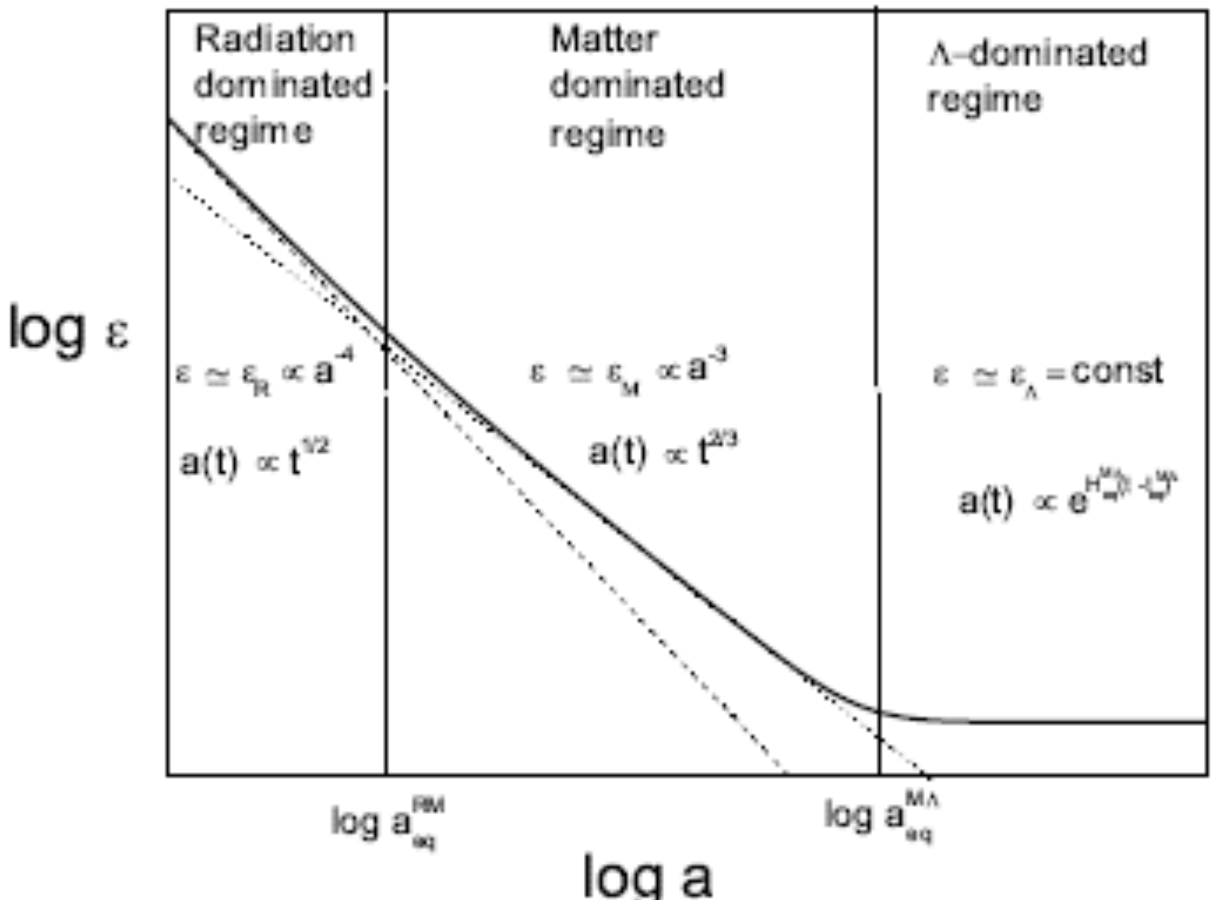
(Planck 2015, 1502.10589)

TT+TE+EE+lensing $\longrightarrow$ $\boxed{N_\nu^{rec} = 2.94 \pm 0.38}$

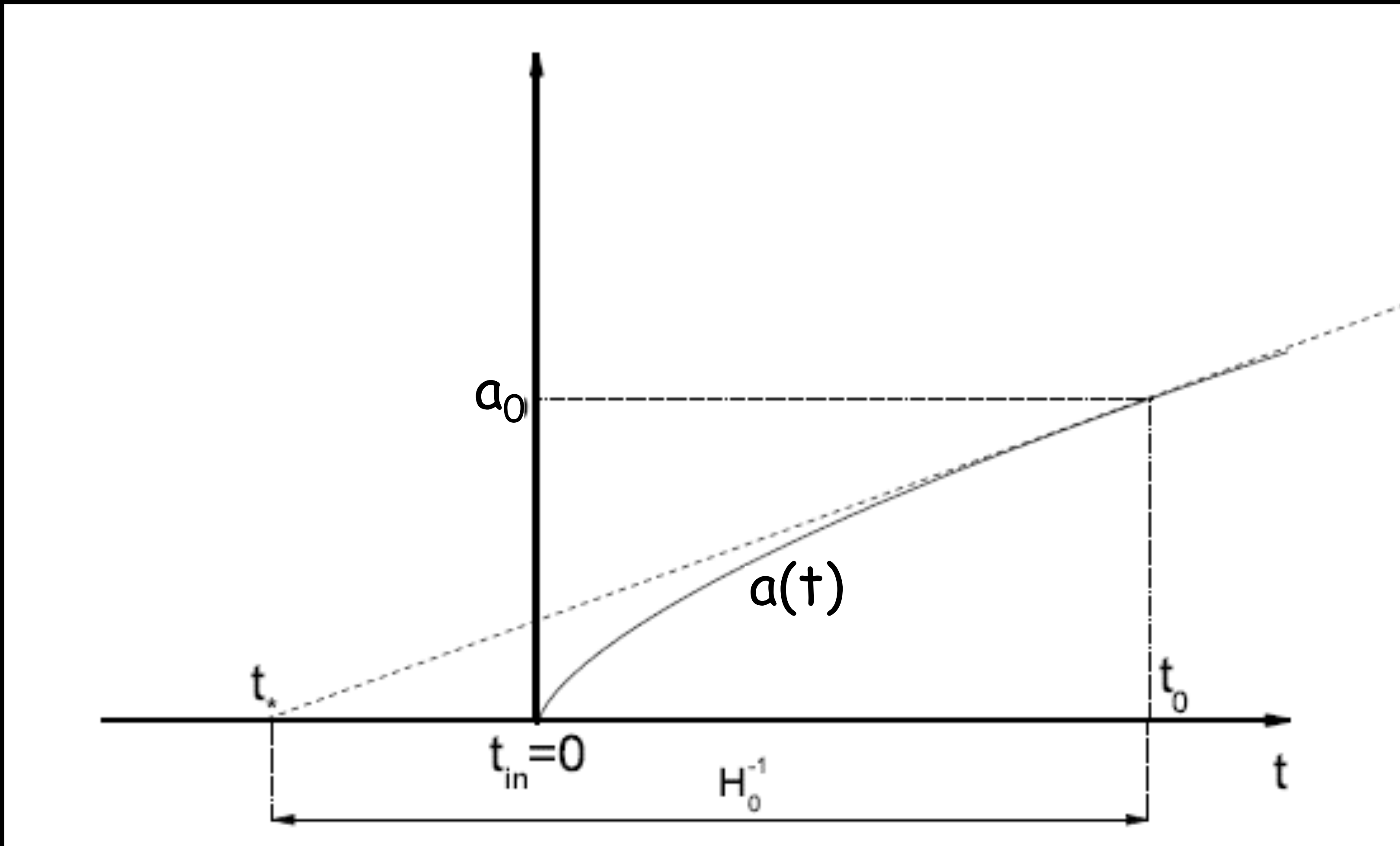
This proves the presence of neutrinos at recombination and also places a stringent upper bound on the amount of dark radiation ⇒ strong constraints on BSM models

But what is the condition for neutrinos to be thermalised?

Flat Radiation-Matter-Λ model



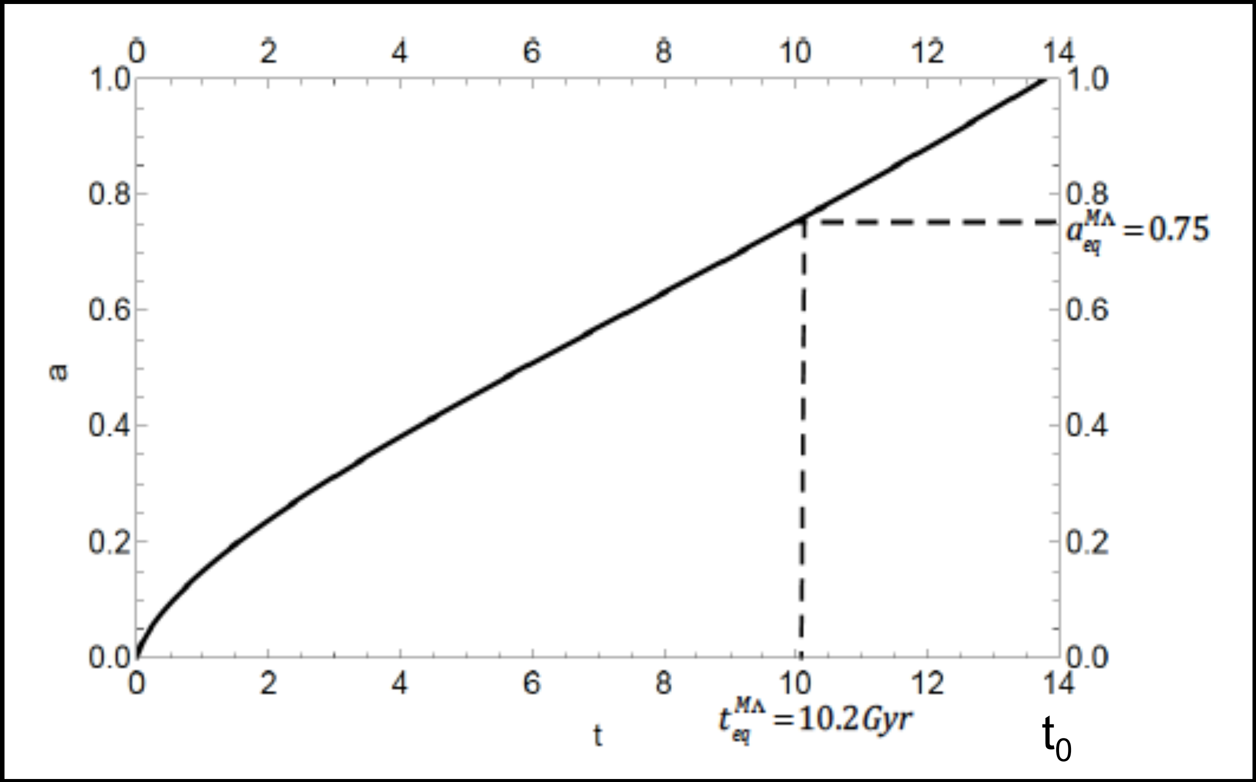
Age of the Universe: in general



$$a_0 = \dot{a}_0 \, (t_0 - t_\star)$$

$$H_0^{-1} = \, (a_0/\dot{a}_0) = t_0 - t_\star$$

Age of the universe in the ΛCDM model



$$\Omega_{\Lambda 0} = 0.692$$

$$\Omega_{M0} = 0.308$$

$$H_0^{-1} = 14.4 Gyr$$

$$t_0 = \frac{2H_0^{-1}}{3\sqrt{\Omega_{\Lambda 0}}} \ln\left[\frac{1+\sqrt{\Omega_{\Lambda 0}}}{\sqrt{1-\Omega_{\Lambda 0}}}\right] \simeq 13.8 Gyr$$

Matter-radiation equality

$$p = p_M + p_R, \quad \varepsilon = \varepsilon_M + \varepsilon_R$$

with equations of state:

$$p_M = 0, \; p_R = \frac{1}{3}\varepsilon_R \,,$$

That, from the fluid equation, lead to :

$$\varepsilon_M = \frac{\varepsilon_{M,0}}{a^3}, \quad \varepsilon_R = \frac{\varepsilon_{R,0}}{a^4}$$

The equality matter-radiation time is defined as:

$$\frac{\varepsilon_{M0}}{a_{eq}^3} = \frac{\varepsilon_{R0}}{a_{eq}^4} \Rightarrow a_{eq} = \frac{\varepsilon_{R0}}{\varepsilon_{M0}} = \frac{\Omega_{R0}}{\Omega_{M0}}$$

Baryon-to-photon number ratio and recombination

Fractional ionization

$$X \equiv \frac{n_p}{n_p + n_H} = \frac{n_e}{n_B}$$

$$\frac{1-X}{X^2} \simeq 3.84\,\eta_B \left(\frac{T}{m_e c^2}\right)^{3/2} e^{Q/T}$$

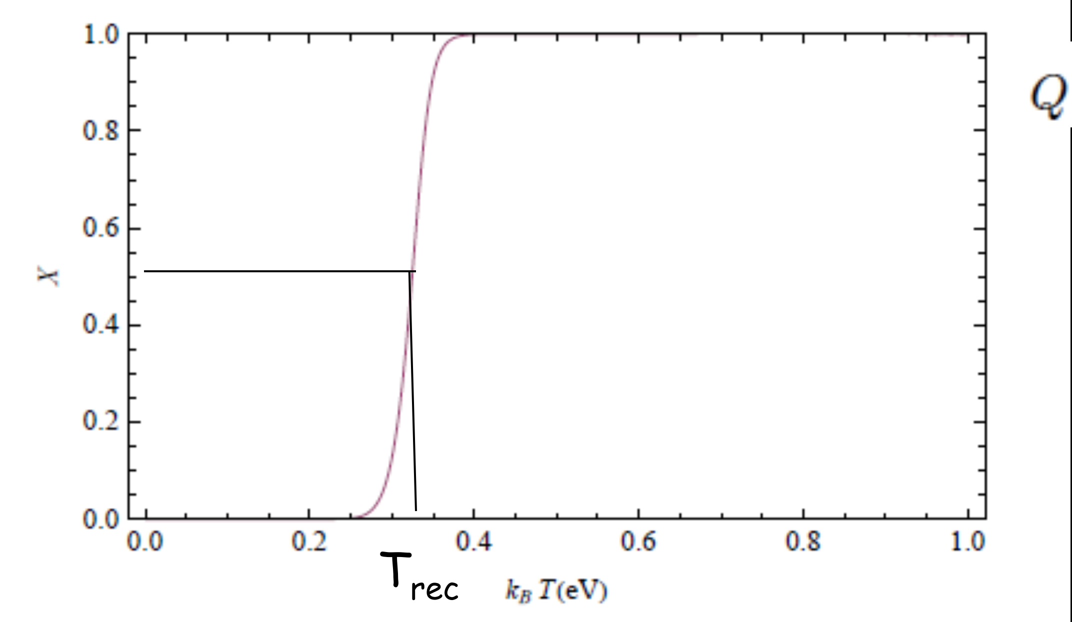
Saha equation

Baryon-to-photon number ratio

$$\eta_B \equiv \frac{n_B}{n_\gamma} \simeq \frac{\Omega_B\,\varepsilon_c}{m_p\,c^2\,n_\gamma}$$

$$\eta_{B,0} \simeq 273.5\,\Omega_{B,0} h^2 \times 10^{-10}$$

$$\eta_{B,0}^{(CMB)} = (6.08 \pm 0.06) \times 10^{-10}$$



$$Q \equiv (m_p + m_e - m_H)\,c^2 \simeq 13.6\,\mathrm{eV}$$

$$T_{\rm rec} \simeq \frac{Q}{42} \simeq 0.32\,\mathrm{eV}$$

Decoupling and recombination

Matter and Radiation are coupled until the Thomson scatterings

$$\gamma + e^- \leftrightarrow \gamma + e^-$$

of photons on free electrons are fast enough: $\Gamma \gtrsim H$

$$\Gamma \equiv \tau^{-1} = n_e \, \sigma_e \, c$$

$$n_e(a) = \frac{n_{B,0}}{a^3} X(a)$$

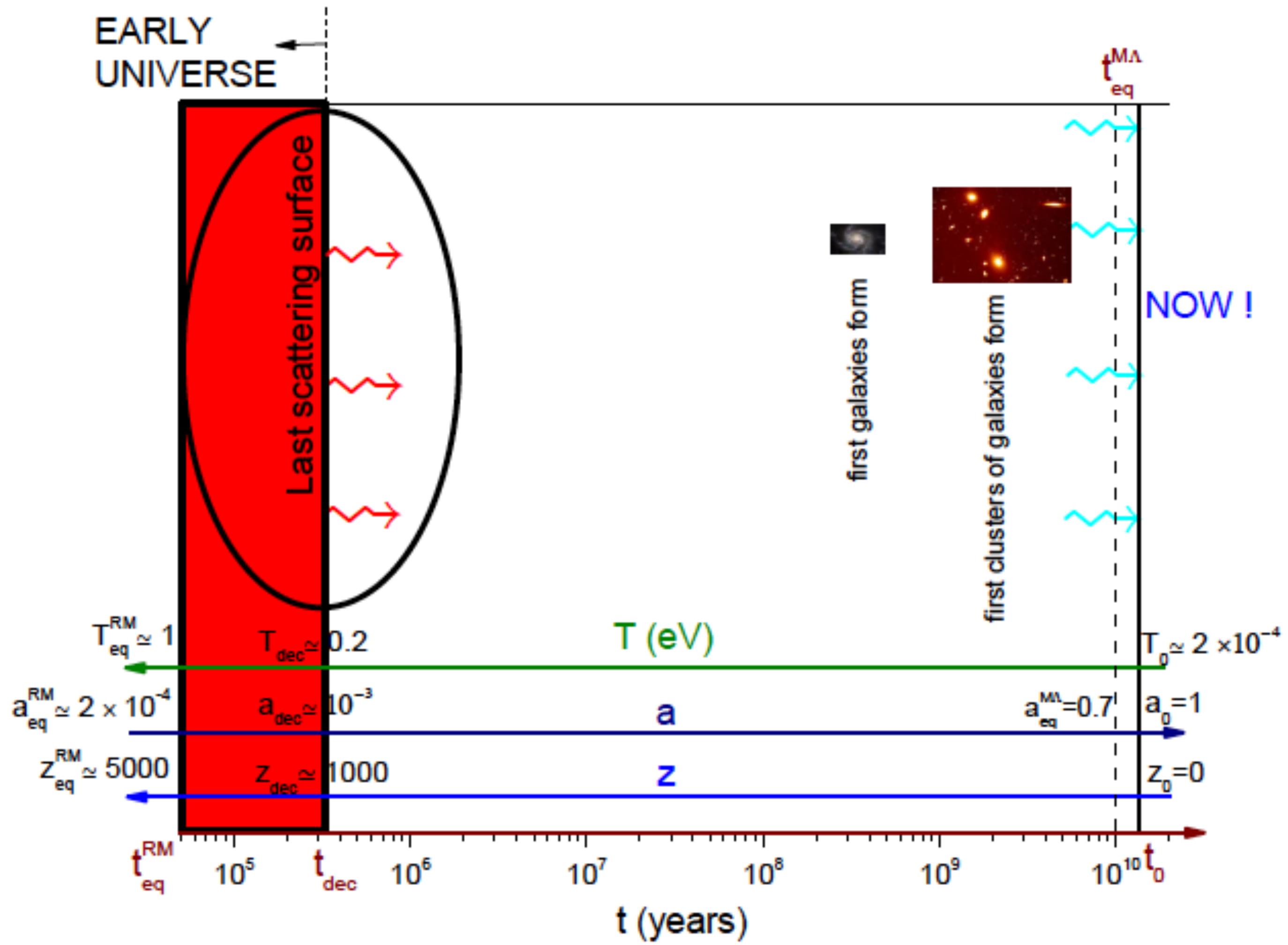
Expansion of the Universe

It takes into account the "recombination" of electrons with protons to form Hydrogen athoms,

$$\Gamma \gtrsim H \Rightarrow n_e(a) \, \sigma_e \, c \gtrsim H$$

$$a \lesssim a_{\rm dec} \simeq 8.8 \times 10^{-4} , \quad z \gtrsim z_{\rm dec} \simeq 1130 , \quad k_B T \gtrsim k_B T_{\rm dec} \simeq 0.26 \, {\rm eV}$$

$$t_{\rm dec} \simeq \frac{t_{\rm eq}^{M\Lambda}}{(a_{\rm eq}^{M\Lambda} \, z_{\rm dec})^{\frac{3}{2}}} \simeq 400,000 \, {\rm yr}$$


EARLY
UNIVERSE
Last scattering surface
first galaxies form
first clusters of galaxies form
NOW !
$t_{eq}^{M\Lambda}$
T (eV)
$T_{eq}^{RM} \simeq 1$
$T_{dec} \simeq 0.2$
$T_0 \simeq 2 \times 10^{-4}$
a
$a_{eq}^{RM} \simeq 2 \times 10^{-4}$
$a_{dec} \simeq 10^{-3}$
$a_{eq}^{M\Lambda}=0.7$
$a_0=1$
z
$z_{eq}^{RM} \simeq 5000$
$z_{dec} \simeq 1000$
$z_0=0$
t_{eq}^{RM} 10^5 t_{dec} 10^6 10^7 10^8 10^9 10^{10} t_0
t (years)

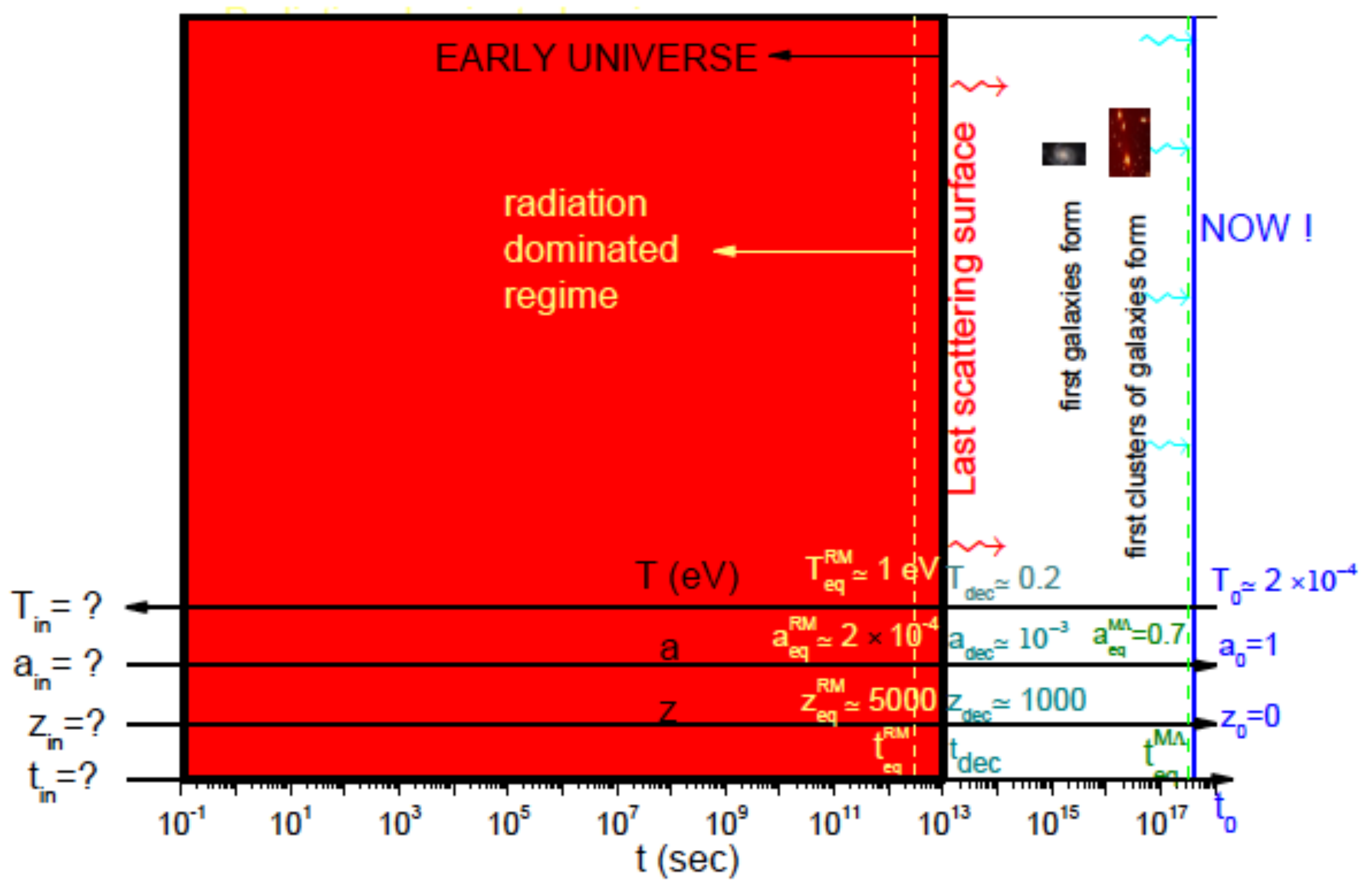
Matter-radiation equality in numbers

$$\frac{\varepsilon_{M0}}{a_{eq}^3} = \frac{\varepsilon_{R0}}{a_{eq}^4} \Rightarrow a_{eq} = \frac{\varepsilon_{R0}}{\varepsilon_{M0}} = \frac{\Omega_{R0}}{\Omega_{M0}} = \frac{0.90 \times 10^{-4}}{0.31} \simeq 2.9 \times 10^{-4}$$

$$z_{eq} = \frac{1}{a_{eq}} - 1 \simeq 3400$$

$$t_{eq} \simeq t_{eq}^{M\Lambda} \left(\frac{a_{eq}}{a_{eq}^{M\Lambda}} \right)^{\frac{3}{2}} \approx 50{,}000\, yr$$

History of The Early Universe



The Early Universe is mainly in a radiation dominated regime

$$H^2 = \frac{8\pi G}{3}\varepsilon_R\,,$$

$$\varepsilon_R = g_R\,\frac{\pi^2}{30}\,\frac{(k_B T)^4}{(\hbar c)^3}$$

$T \gg m_{X_{\mathrm{b,f}}} c^2/2,$

$$g_R(T) \simeq \sum_{X_{\mathrm{b}}} g_{X_{\mathrm{b}}} + \frac{7}{8}\sum_{X_{\mathrm{f}}} g_{X_{\mathrm{f}}}$$

Number of ultra-relativistic degrees of freedom

$$H(T) = \sqrt{g_R}\sqrt{\frac{8\,\pi^3\,G}{90}}\,T^2 \simeq 0.21\,\sqrt{g_R}\left(\frac{k_B\,T}{\mathrm{MeV}}\right)^2 s^{-1}\,.$$

$$t = \frac{1}{2\,\sqrt{g_R}\,T^2}\sqrt{\frac{90}{8\,\pi^3\,G}} \simeq \frac{2.4\,\mathrm{s}}{\sqrt{g_R}}\left(\frac{\mathrm{MeV}}{k_B\,T}\right)^2\,.$$

T^{ν}_{dec}

$$g_R(T \gtrsim 1\,\mathrm{MeV}) = g_R^{\gamma+e^{\pm}+3\nu} = 2 + \frac{7}{8}(4 + 2\times 3) = \frac{43}{4} = 10.75\,.$$

$\sim m_e$

$$g_R(k_B\,T \lesssim 0.5\,\mathrm{MeV}) \simeq 2 + 3\,\frac{7}{4}\left(\frac{4}{11}\right)^{4/3} \simeq 3.36\,.$$

Neutrino decoupling ⇒ relic neutrinos

$$\Rightarrow n_{\nu_\alpha}(T) = \frac{3}{4}\,\frac{\xi(3)}{\pi^2}\,g_\nu\,T^3 \;\text{ for } T \gtrsim T_{\nu_\alpha}^{\rm dec}$$

$$\frac{\Gamma_{\nu_{\rm weak}}}{H}\Big|_{T_{\nu_\alpha}^{\rm dec}} \sim \frac{n_{\nu_\alpha}\langle\sigma\, v\rangle}{H}\Big|_{T_{\nu_\alpha}^{\rm dec}} \sim \frac{y_\alpha\, G_F^2\, T^5}{\sqrt{2.8 g_R}\,\frac{T^2}{M_{\rm Pl}}}\Bigg|_{T_{\nu_\alpha}^{\rm dec}} = 1 \Rightarrow T_{\nu_\alpha}^{\rm dec} \simeq y_\alpha^{-\frac{1}{3}}\, 1.5\,{\rm MeV}$$

For $T < T_\nu^{dec}$ neutrinos are decoupled and their number in the comoving volume remains constant and one expects that at present there is a relic neutrino background together with CMBR with a temperature::

$$T_{\nu 0} \simeq \left(\frac{4}{11}\right)^{\frac{1}{3}}\, T_{\gamma 0} \simeq 1.96\,^0 K$$

Big Bang Nucleosynthesis

George Gamow
(1904-1968)

$$
\begin{aligned}
n &\leftrightarrow p + e^- + \bar{\nu}_e \\
n + e^+ &\leftrightarrow p + \bar{\nu}_e \\
n + \nu_e &\leftrightarrow p + e^- \,.
\end{aligned}
$$

Neutrons-protons inter-converting processes

At the equilibrium:

$$\left(\frac{n_n}{n_p}\right) \simeq \left(\frac{n_n}{n_p}\right)_{\rm eq} \simeq e^{-\frac{Q_n}{k_B T}}$$

$$Q_n \;=\; (m_n - m_p)\, c^2 \;\simeq\; 1.29\,{\rm MeV}$$

Equilibrium holds until

$$\Gamma_{n\leftrightarrow p} \simeq G_F^2\, T^5 \gtrsim H \;\Rightarrow\; T \gtrsim T_{\rm fr} = \frac{\sqrt{2.4}}{g_R^{1/4}} \left(\frac{\rm sec}{t_{\rm fr}}\right)^{1/2} {\rm MeV} \simeq 0.85\,{\rm MeV}$$

Freeze-out temperature

At the freeze-out:

$$\frac{n_n}{n_p}(T_{\rm fr}) = e^{-\frac{Q_n}{T_{\rm fr}}} \simeq e^{-\frac{1.29}{0.85}} \simeq 0.22$$

$$t_{\rm fr} \simeq 1.0\,{\rm sec}\,.$$

After the freeze-out neutrons start to decay prior to nucleosynthesis at $t_{\rm nuc} \simeq 310\, s\,.$

$\tau_n \simeq 885\,{\rm s}.$ Life time of neutrons

$$\frac{(n_n/n_p)_{\rm nuc}}{(n_n/n_p)_{\rm fr}} = e^{-\frac{t_{\rm nuc}}{\tau}} = e^{-\frac{310}{885}} \simeq 0.7 \Rightarrow (n_n/n_p)_{\rm nuc} \simeq 0.154\,. \Rightarrow Y_p = 2\,\frac{(n_n/n_p)_{\rm nuc}}{1 + (n_n/n_p)_{\rm nuc}} \simeq 0.267$$