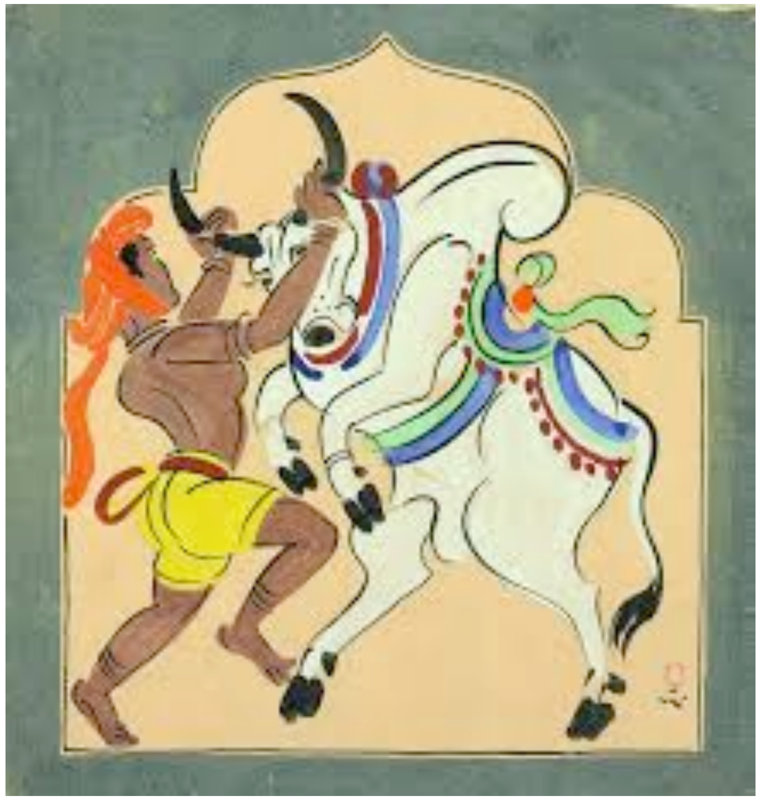


Neutrino Sources

Milind Diwan
(Brookhaven National Lab.)

Sangam at Harish-Chandra Research Institute
Instructional Workshop in Particle Physics
Updated 3/5/2018

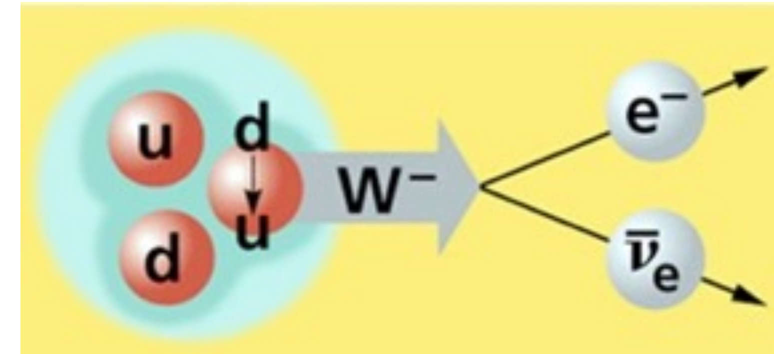
BROOKHAVEN NATIONAL LABORATORY

Outline

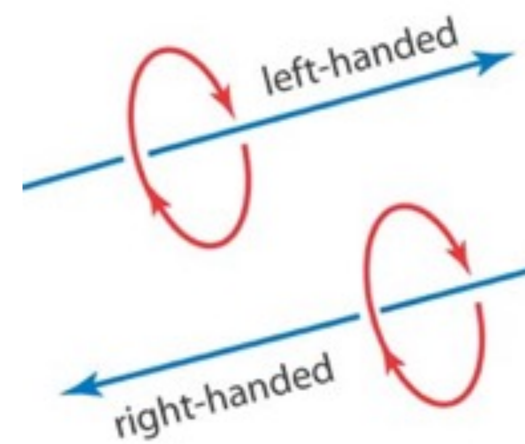
- **Brief review of natural and manmade sources and detectors.**
- **Emphasis on conceptual understanding of the sources and their intensity.**
- **Reference: The recent review of Long-Baseline for annual reviews (2016) by Diwan, Galymov, Qian, Rubbia**

What are neutrinos ?

$$n \rightarrow p\ e^-\ \bar{\nu}_e$$



- A particle with no electric charge. Predicted in 1930 by Pauli, and detected in 1957 by Reines and Cowan.
- It is emitted in radioactive decay. And has no other types of interactions.
- It has 1/2 unit of spin, and therefore is classified as a Fermion (or particle of matter.)
- Neutrino is extremely light.
- Neutrino comes in flavors !
- Neutrino is left handed ! Or has no mirror image !
- Neutrinos are as numerous as photons in the Universe.
- Important component of dark matter. May be responsible for matter/antimatter asym.



From the Sun:
10^{11} neutrinos/cm^2/sec

What is the scientific interest in neutrinos ?

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

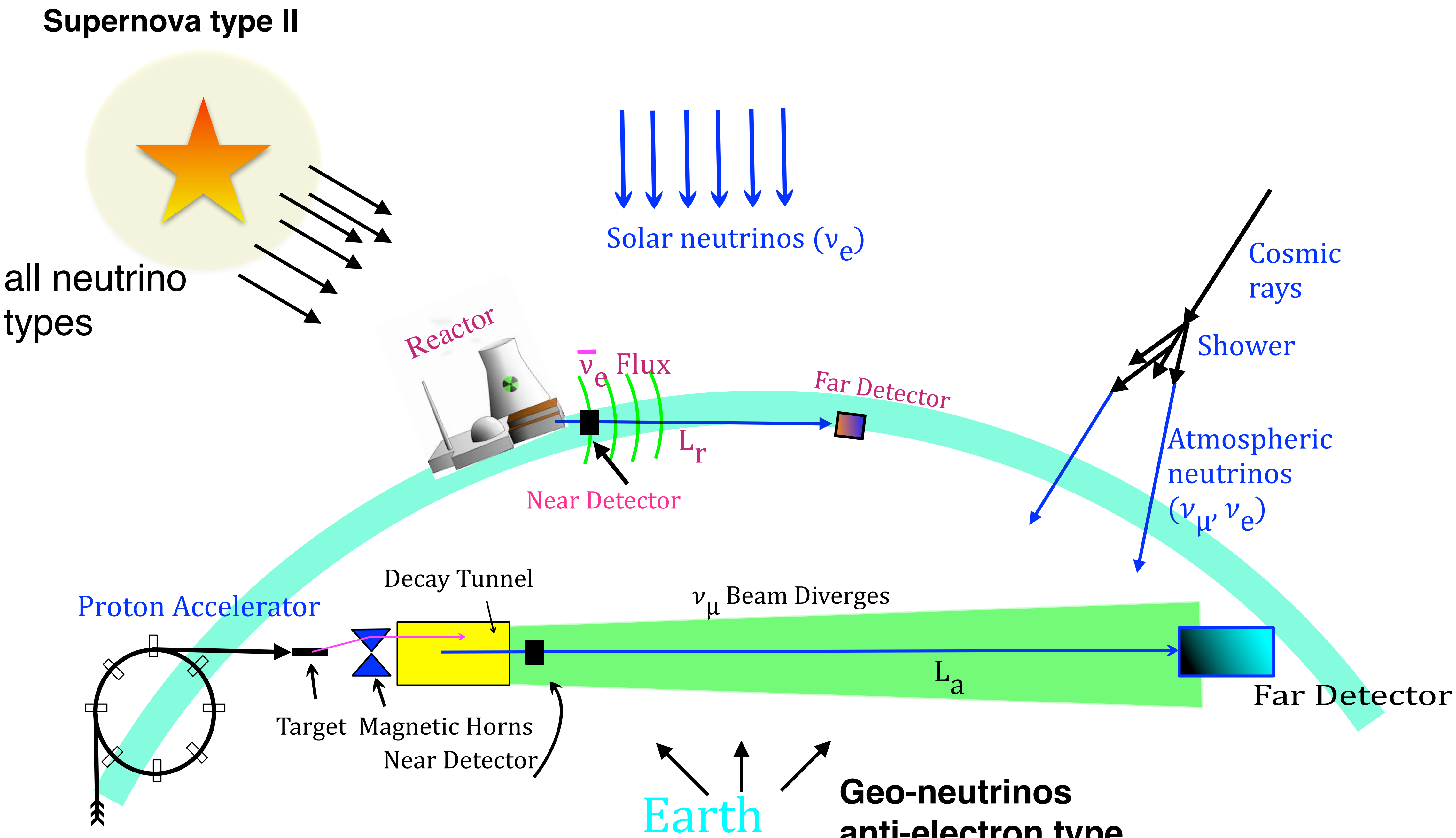
Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

- ~20 yrs ago all neutrino masses were thought to be 0 and all neutrino flavors distinct.
- With new discoveries a distinct, unexpected pattern appears to be emerging.

- Science of neutrinos is has deep connections to understanding of matter, cosmology, and astrophysics.
- Existence of neutrino mass itself is physics beyond the standard model because in the standard model there is no interaction with right handed neutrinos. And so by definition the mass is zero.
- A new mechanism for mass generation may be needed in which neutrinos are their own anti-particles.

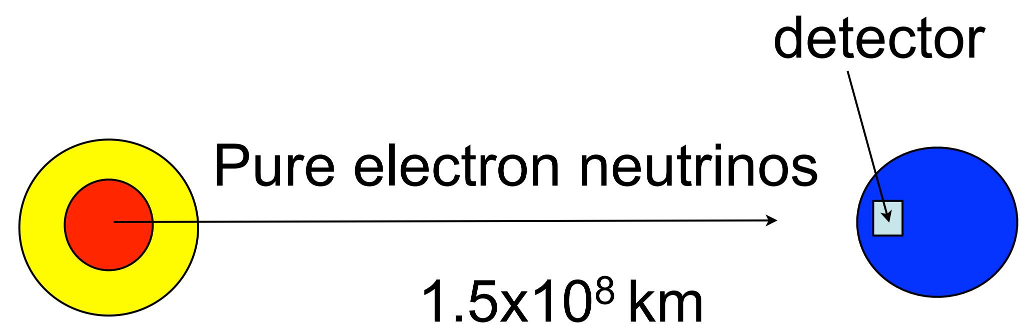
Neutrino Sources



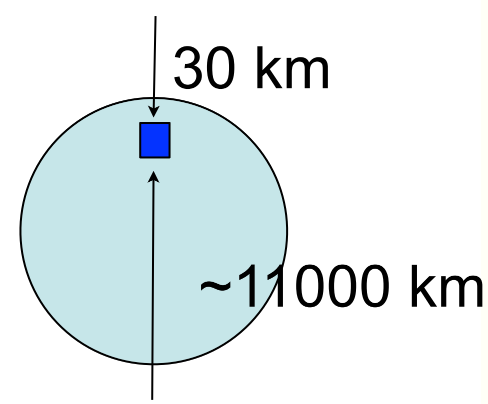
Natural and manmade sources of led us to understand the properties of neutrinos in much greater detail.

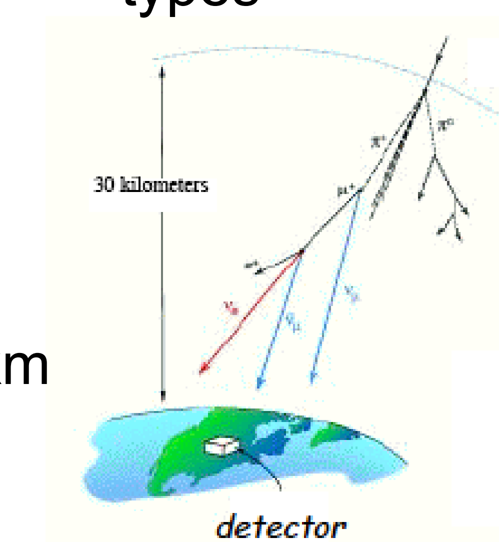
Natural Neutrino Sources on Earth's surface

- **The Sun**
  - **<0.5 MeV, 10^{11} /cm^2 s**
  - **3-14 MeV, 3 x 10^6 /cm^2 s**
- **Cosmic rays hitting Atmosphere**
  - **~1 GeV, ~5000/m^2/sec**
- **Radioactive decays in the Earth**
  - **<3 MeV, 10^6-10^7 /cm^2/sec**
- **Supernova. 99% of the energy of the explosion goes into neutrinos of all types. ~10 MeV, 20 seen in 1987.**
- **CvB nus. 300 cm^{-3} @ 1.9 K. (not detectable as yet).**
- **Very high energy sources from deep space.**

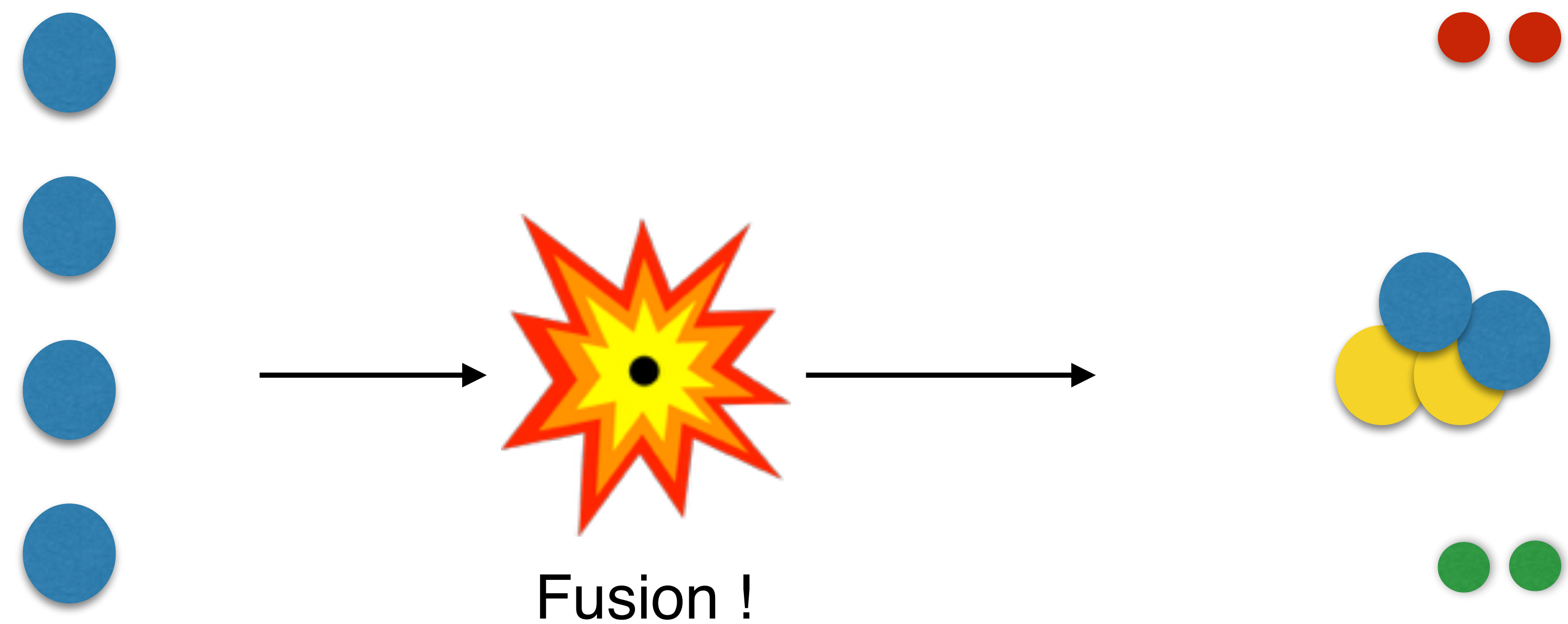


mix of muon
and electron
types





The Sun



Charge = +4
mass = 4 x 1.007276 u
= 4.029104

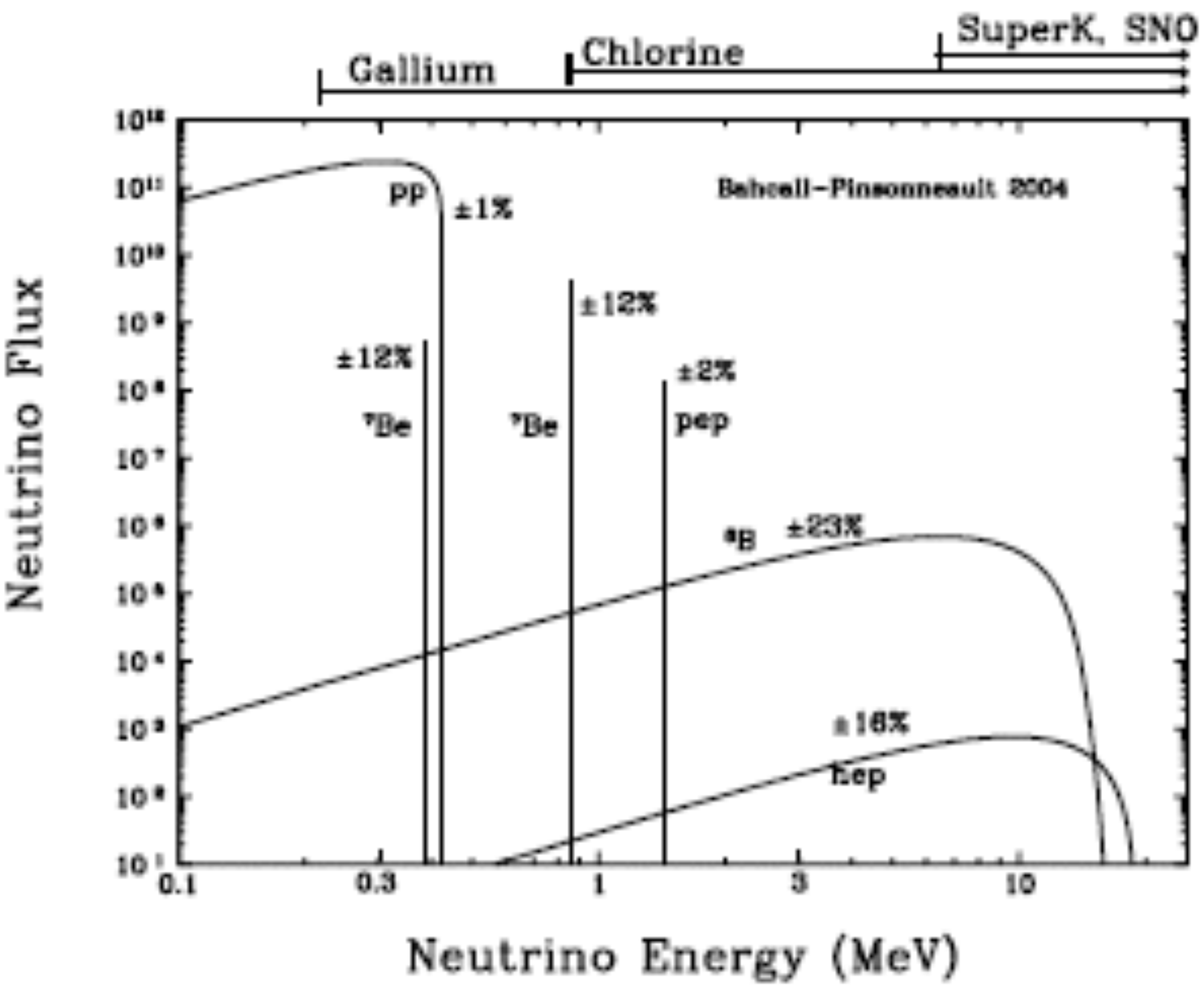
Helium 4
Charge = +2
mass = 4.002602 u
Positron mass = 2 x 0.00054 u

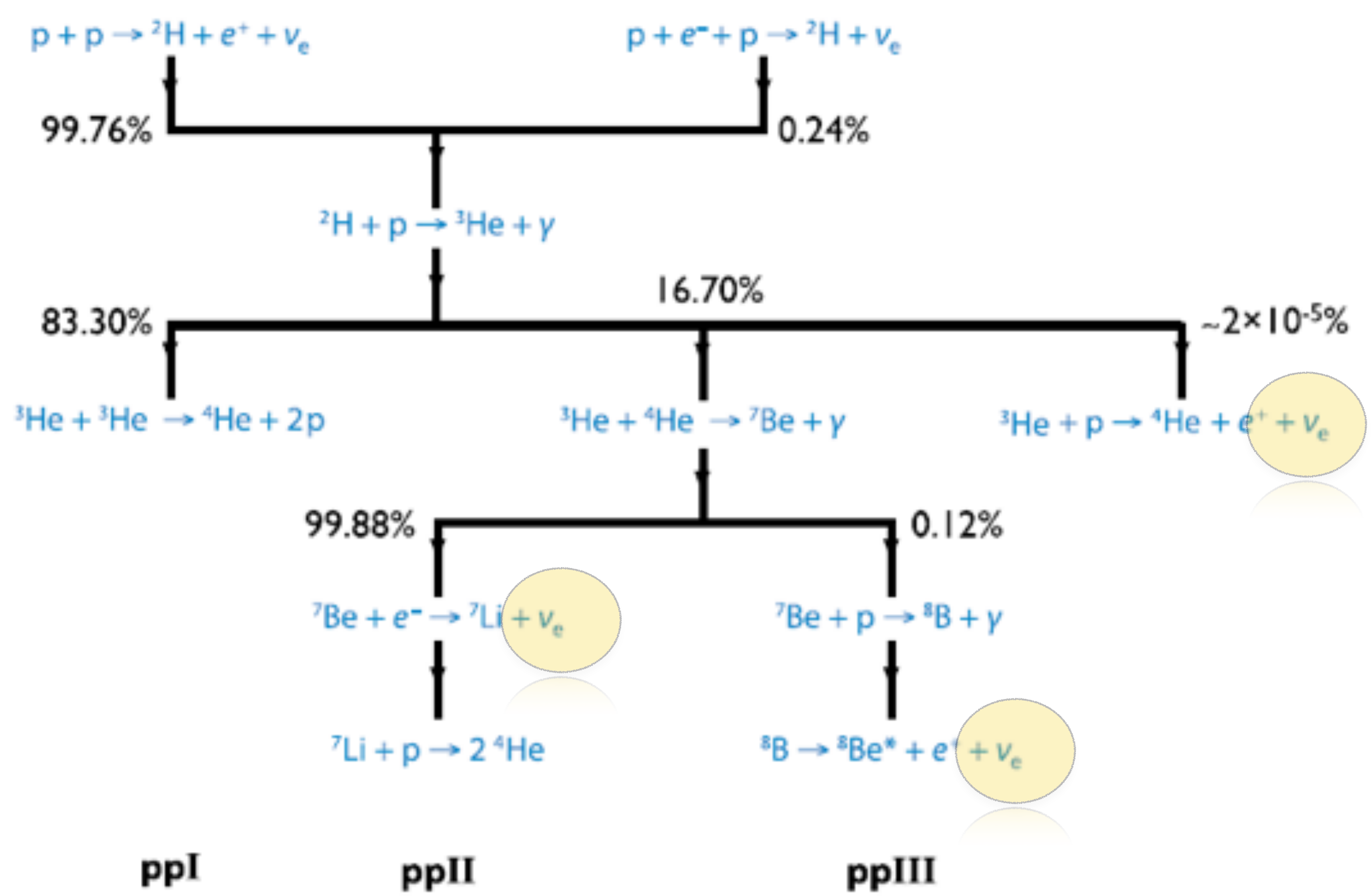
Energy released = 0.02542 u = 23.68 MeV/c^2 ~ 3.8 x 10^{-12} Joules

4 Hydrogen nuclei (protons) fuse into a Helium4 nucleus and release energy and neutrinos. Total energy from Sun is 3.8 x 10^{26} Watts !

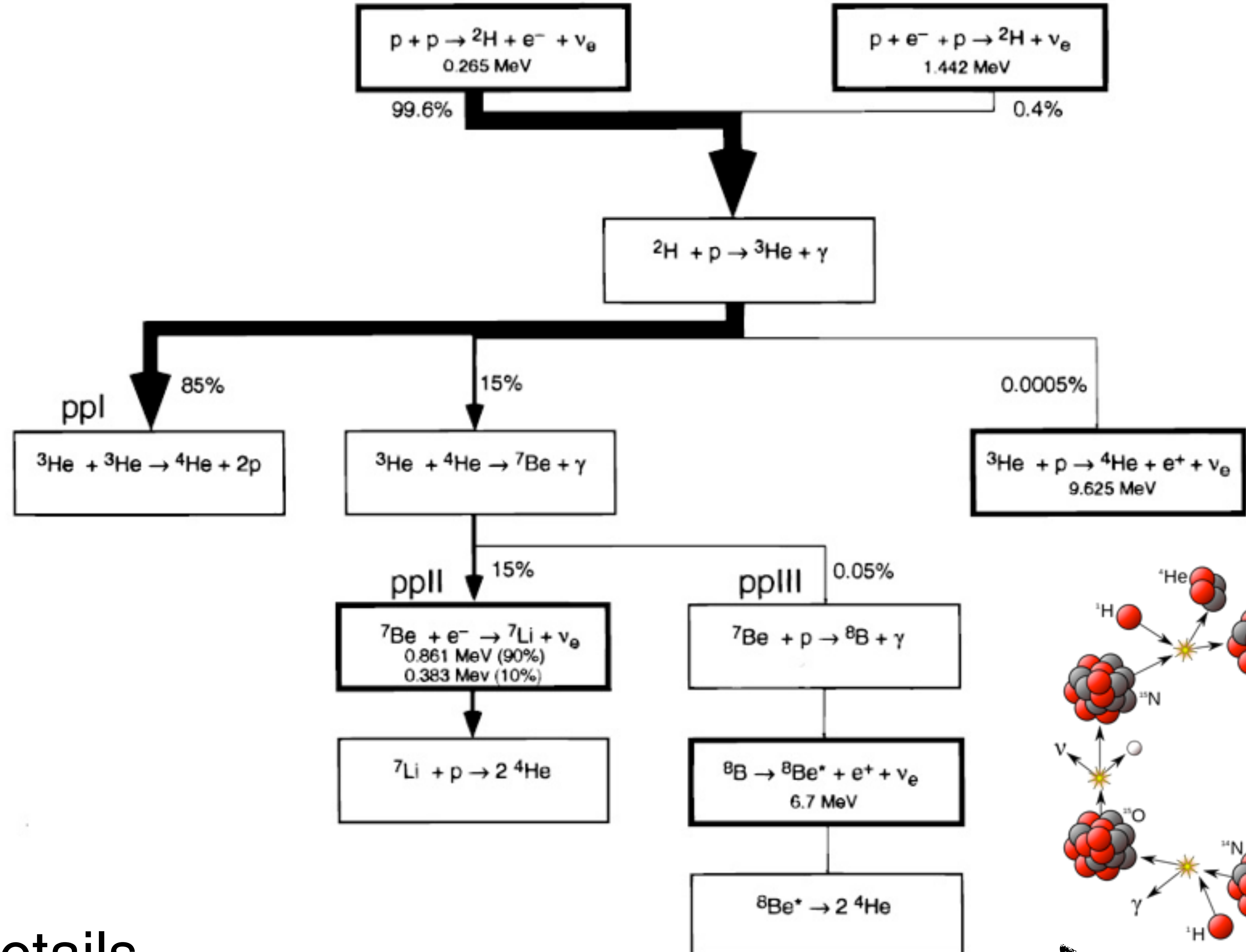
680 million tons of hydrogen burns each second ! How many neutrinos are produced each second ?
There are many other channels of solar neutrinos and so the spectrum is quite complicated and goes up to ~15 MeV. (Standard Solar model)

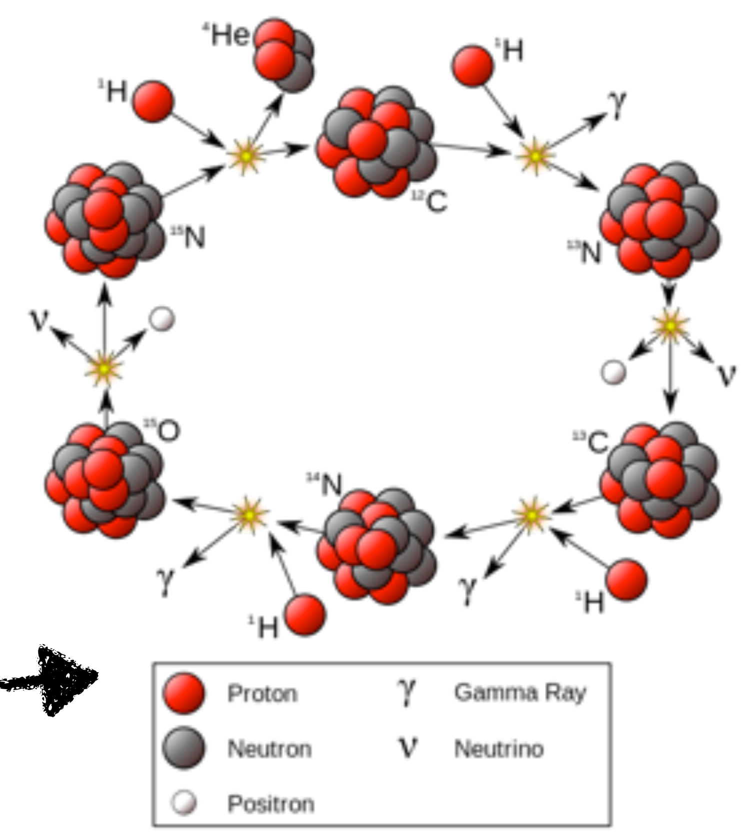
Standard Solar Model (coupled DE) see http://www.sns.ias.edu/~jnb/





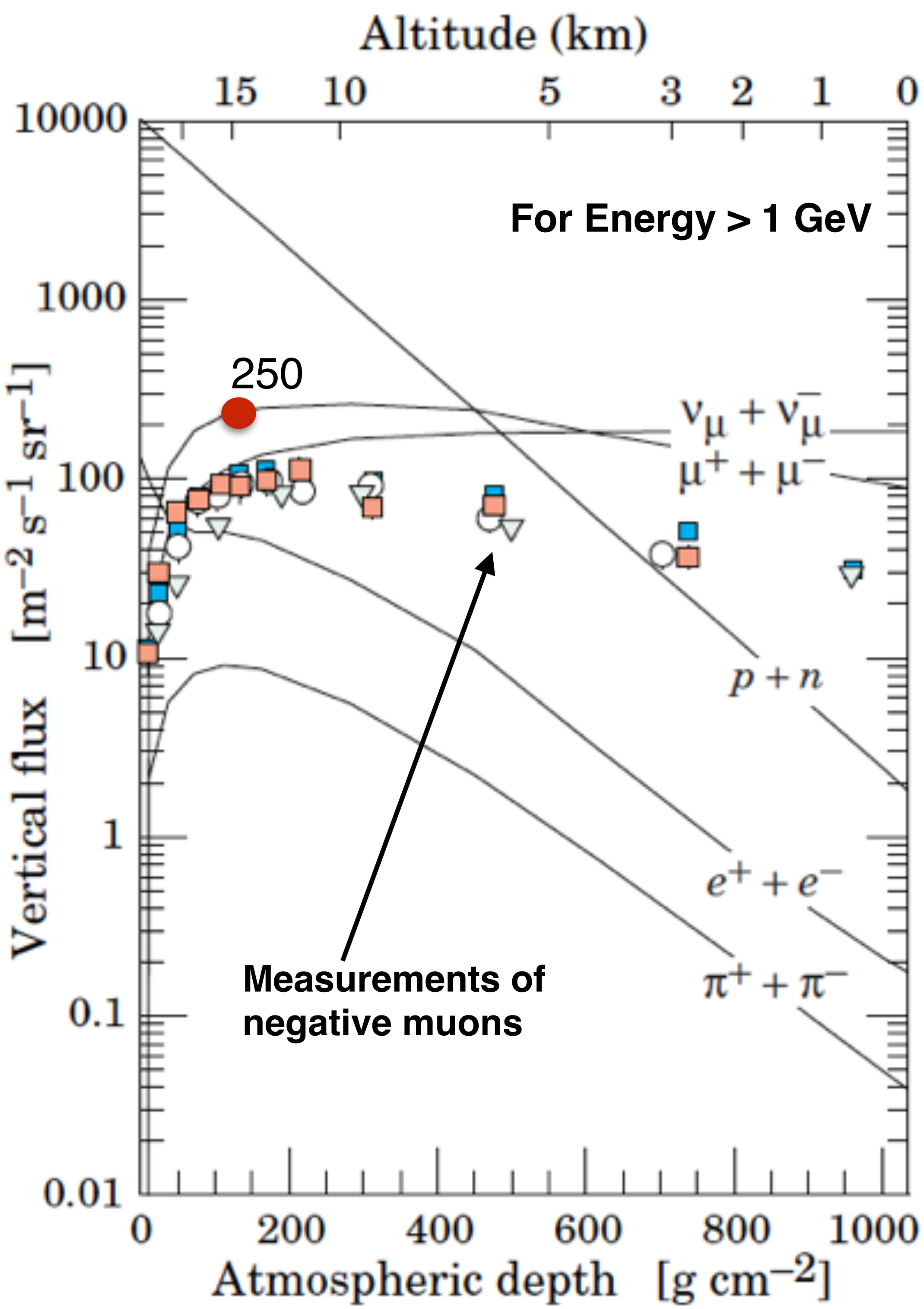
- The conversion of H to He happens in many steps.
- The cross sections are extremely small and eat up hydrogen that fuses through the coulomb barrier; average proton survival time ~ few billion yrs.
- There is another cycle with C-N-O catalyzing H to He conversion. (1.6%)
- No obvious path to heavier elements, carbon, oxygen, etc. ? Special rare reaction: ${}^4He + {}^4He$ to ${}^8Be$ and ${}^4He$ + ${}^8Be$ to ${}^{12}C^*$. (no life without excited state of C12 !) This must happen before 8Be decays away.



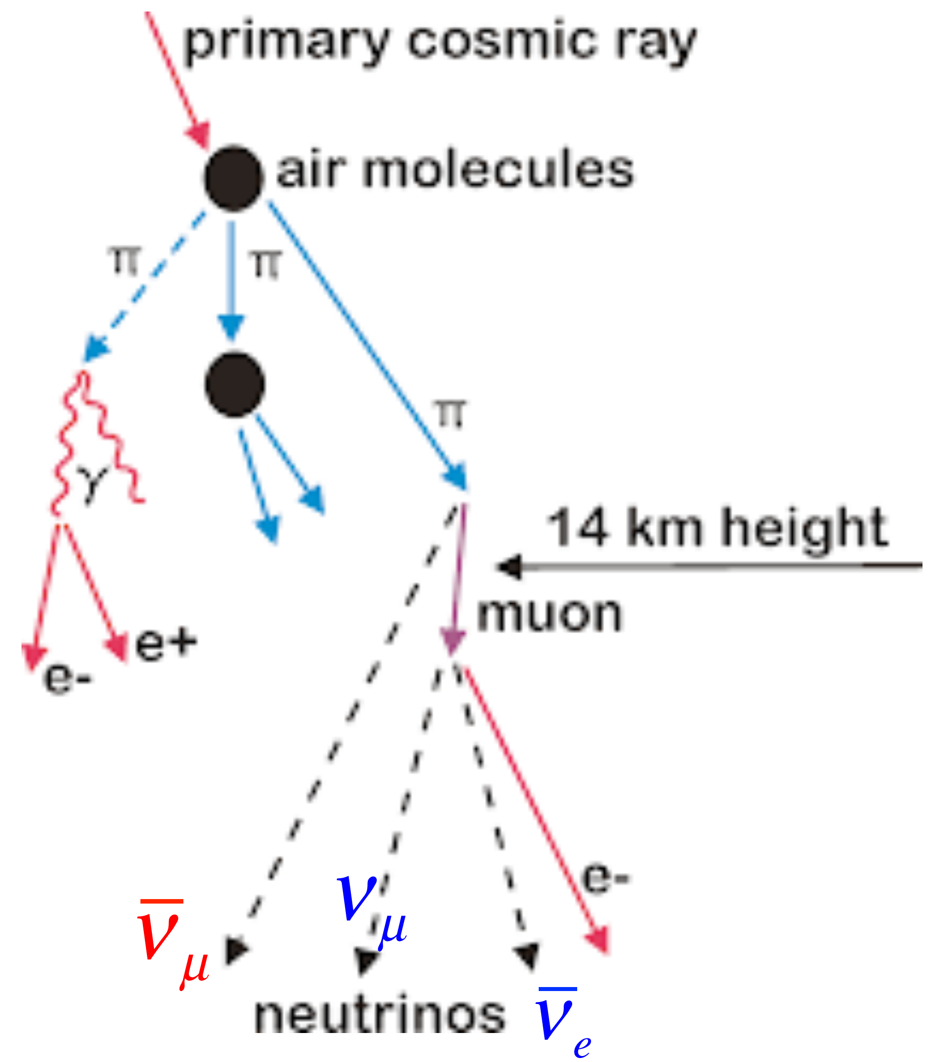


Details
CNO catalytic cycle

Cosmic Rays



From PDG 2015



There should be 3 neutrinos for each muon of either charge).
Ratio of neutrinos (e-type/mu-type) = 1/2

From high altitude muon data one can roughly calculate neutrino flux ~ 5000 /m²/sec
~250 * 2 Pi * 3

Atmospheric neutrinos

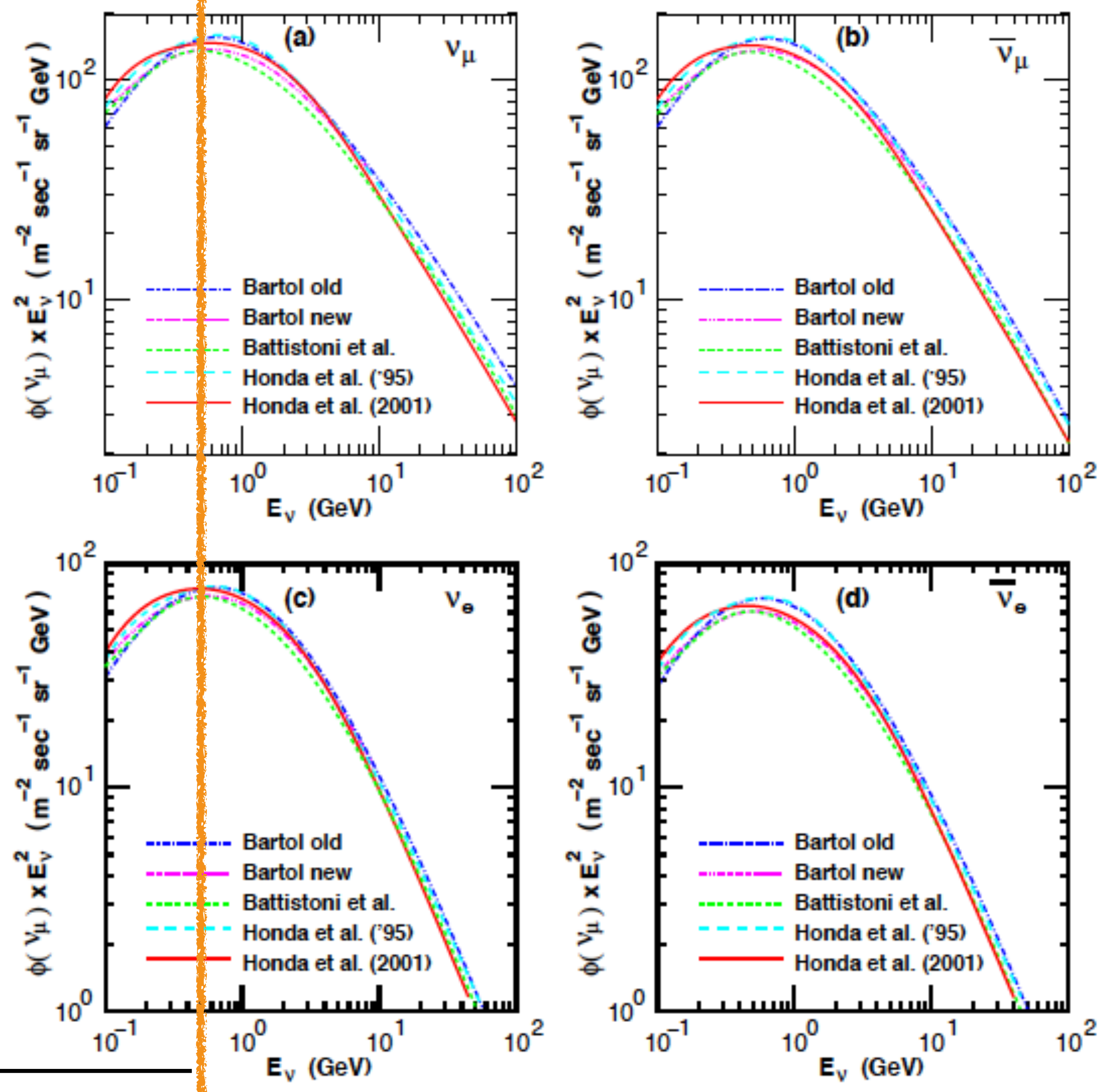
Convolution: $\phi_i = \phi_p \otimes R_p \otimes Y_{p\to\nu_i}$

$$+\sum_A \phi_A \otimes R_A \otimes Y_{A\to\nu}$$

$\phi_{p,A}$ – flux of primary protons and nuclei

R – effect of geomagnetic field

Y – effect of getting neutrino of flavor from interaction. Must include production of pions, kaons, and decays to muons



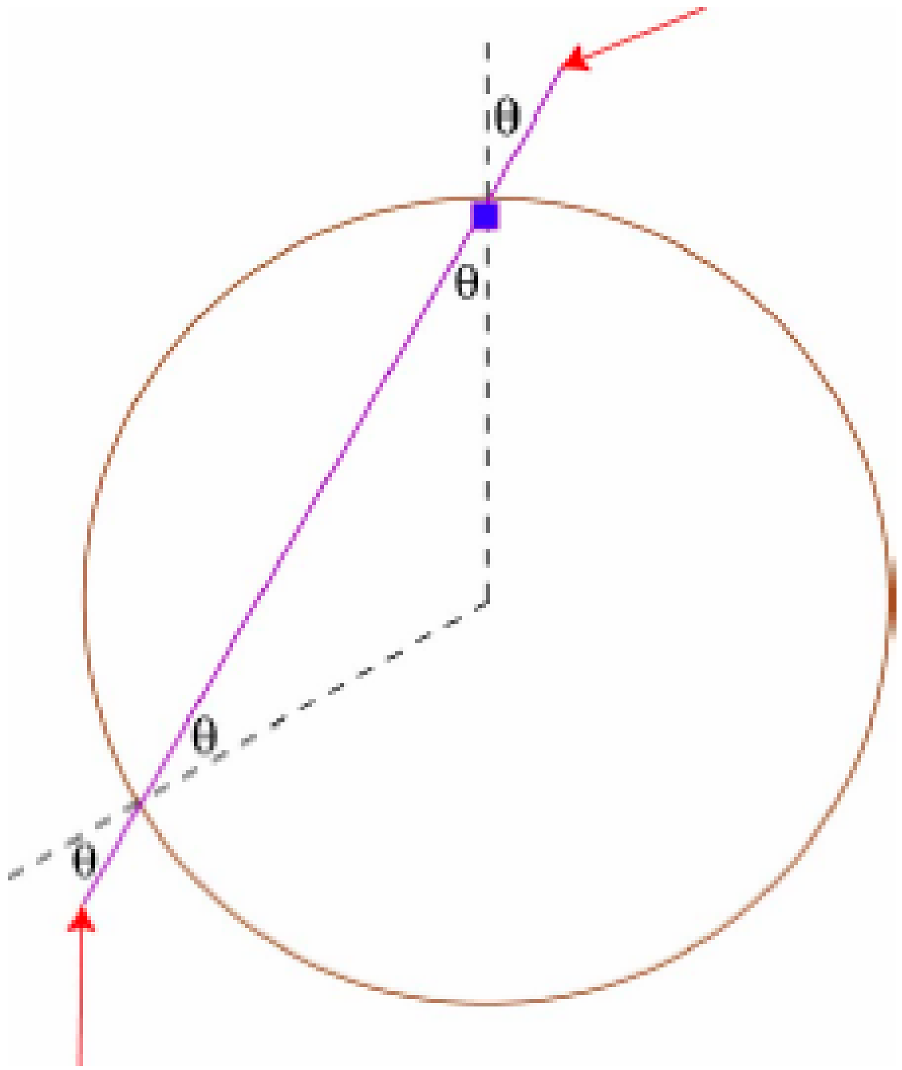
At low energies dominated by geomagnetic cutoff and location of the calculation.

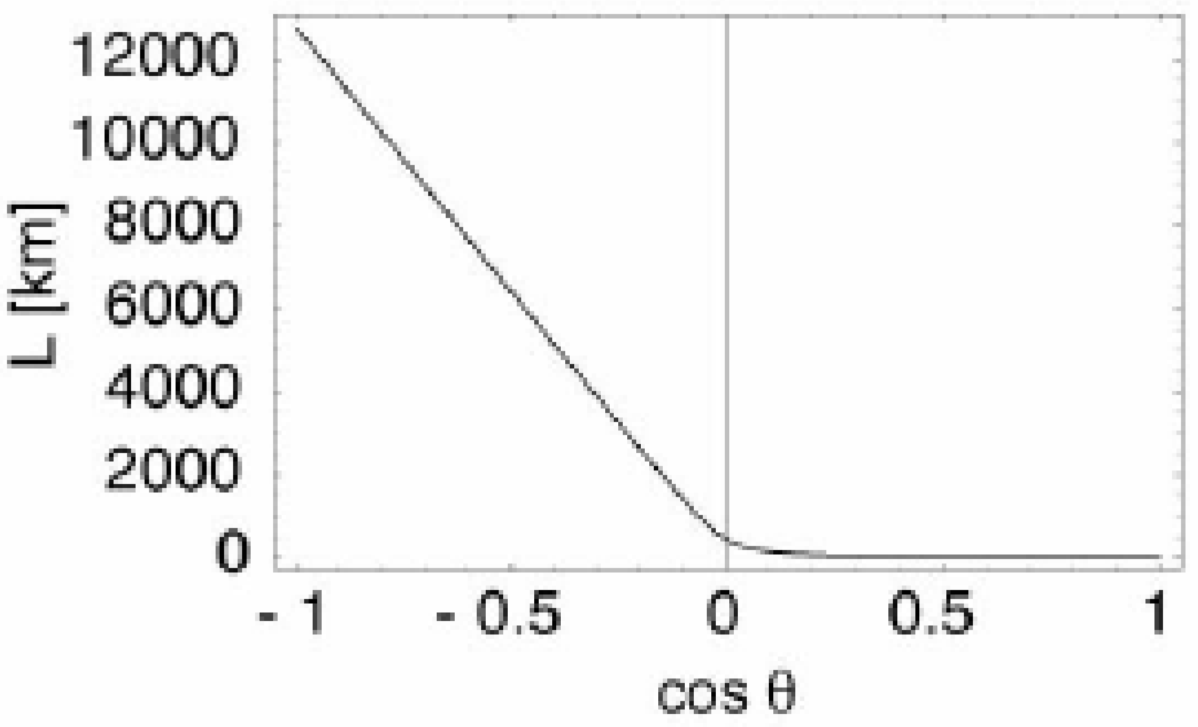
At high energies, the power law of primary cosmic rays dominates

Ratio of electron and muon types changes with energy because of muon decay

Reference: Gaisser and Honda hep-ph/0203272
Annual Review of Nuclear & Particle Science Vol. 52

Geometry of atmospheric neutrinos





Flux inside uniformly illuminated spherical shell is constant and isotropic everywhere in the volume, yet neutrinos coming from below have traveled very long. Prove this with a simple geometric argument.

Neutrinos from the sky

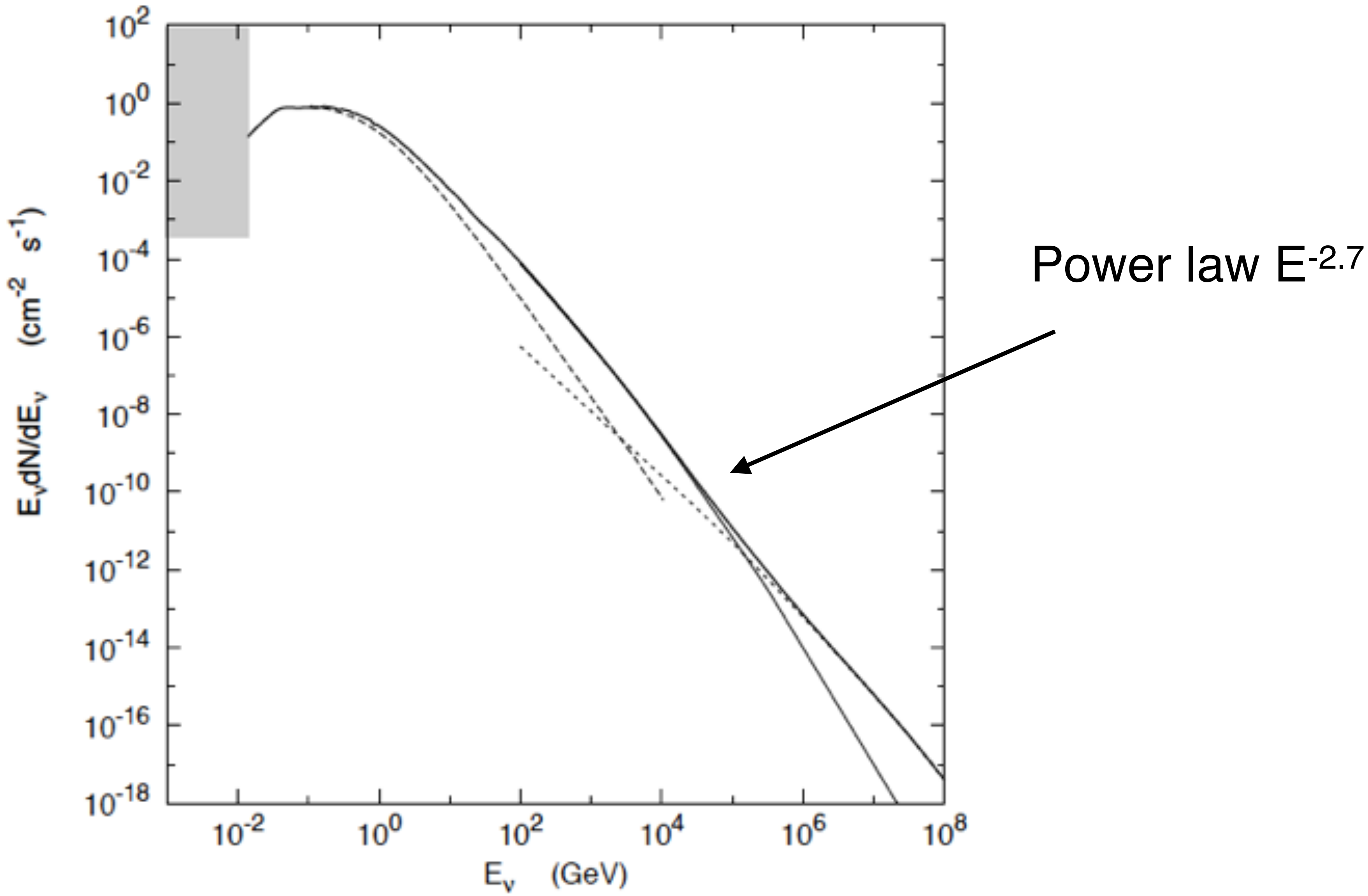
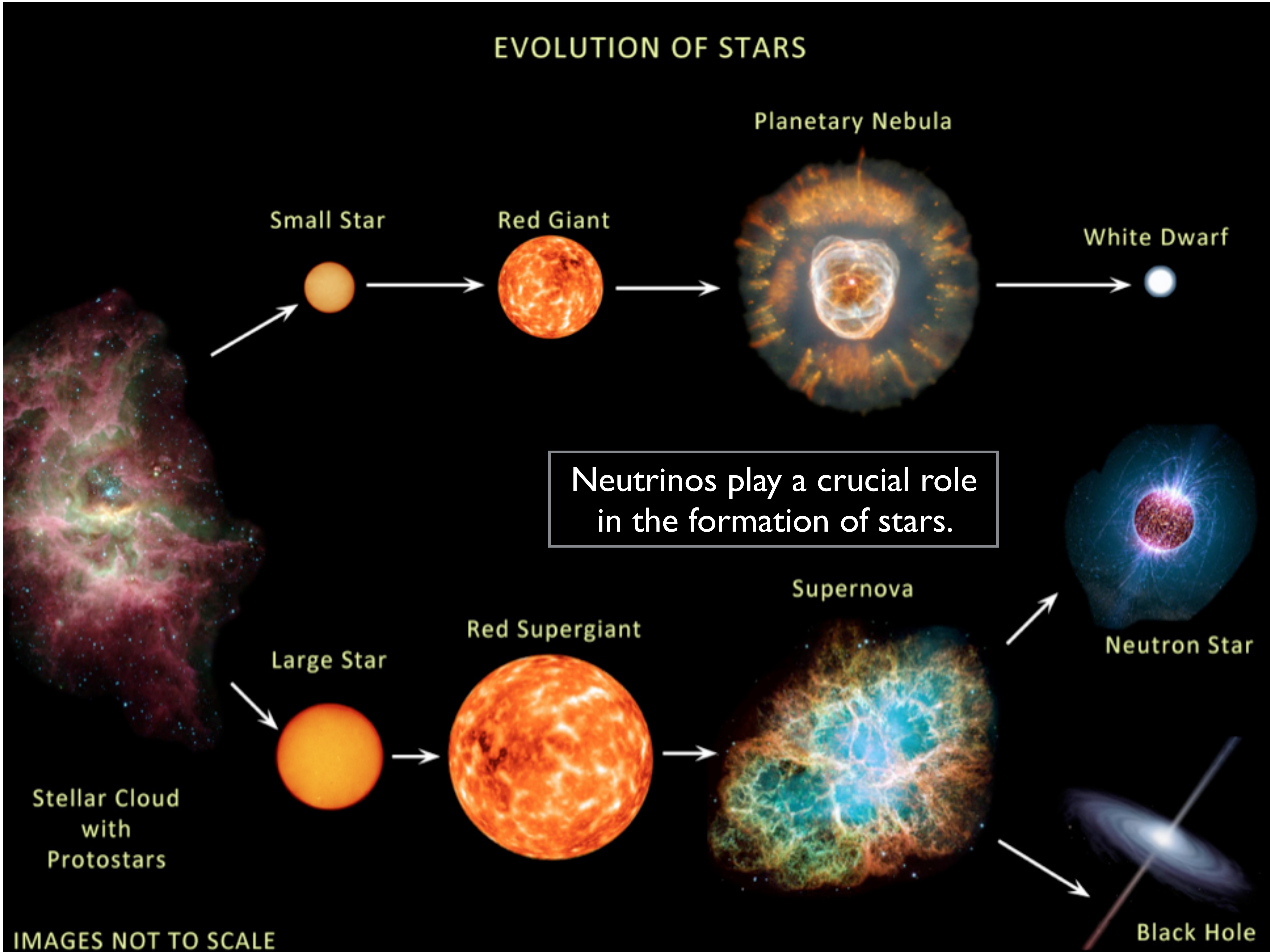


Figure 25 Global view of the neutrino spectrum: vertical flux of $\nu_\mu + \bar{\nu}_\mu$ (heavy solid line); $\nu_e + \bar{\nu}_e$ (dashed line); prompt neutrinos (dotted line); $\nu_\mu + \bar{\nu}$ from pions and kaons (thin solid line at high energy). The shaded region is dominated by solar neutrinos.

From Gaisser and Honda (2002)



Neutrinos play a crucial role in the formation of stars.

Physics of collapse and Chandrasekhar mass

This calculation has a lot of physics, and so let's do this step by step.

Assume a cube (volume V) with side length of L. Quantum momentum states

$\vec{p} = \hbar \vec{k}$ (Use natural units otherwise you will just confuse yourselves with π's)

$\vec{k} = (k_x, k_y, k_z)$ is the wave vector with $k_i = 2\pi n_i / L$; (n_i are integers)

assume the maximum momentum is p_{fermi}(radius of a momentum sphere).

$$N = \text{total number of states} = \frac{\frac{4}{3}\pi p_f^3}{(2\pi / L)^3} = \frac{p_f^3}{6\pi^2} V$$

$$n_e = \text{ electron density} = 2 \times \frac{N}{V} \text{ to account for the two spin states}$$

We arrive at an important result in condensed matter physics.

$$p_{fermi} = (3\pi^2 n_e)^{1/3}$$

When all states are occupied, this defines the minimum density of the electron gas.

If we squeeze the gas more then the electrons will have to be at higher momentum.

From this result we can calculate the pressure of such a gas as we proceed.

Pressure and star stability

The star gas must have a pressure gradient to keep the star from falling in.

First calculate this average required pressure for mass M, radius R, density ρ.

Use $\rho = n_e \times \frac{A}{Z} \times m_N$ A, Z are the average atomic mass and number. m_N nucleon mass

$$M = \rho \ \frac{4}{3}\pi R^3 \quad \rightarrow dM = \rho 4\pi R^2 dR$$

$$E_{Gravity} = \int_0^R \frac{GMdM}{R} = \frac{3}{5}\frac{G}{R}M^2$$

What is the pressure inside a stable star ? It must vary as radius so that each slab of matter is held against gravity. Let $P(r)$ be the pressure versus radius.

Then a slab of thickness dr and mass dM experiences an outward force of $area \bullet dr \bullet dP/dr$

$G\frac{M \bullet dM}{r^2} = 4\pi r^2 \frac{dP(r)}{dr}dr$ ..... Multiply by r and integrate RHS by parts, use P(R)=0

$E_{gravity} = -3\int P(r)dV \equiv -3P_{average} \bullet V$ .... Integral has been turned into a volume integral

$P_{average} = \frac{G}{5}\left(\frac{4\pi}{3}\right)^{1/3}\left(\frac{m_N A}{Z}\right)^{4/3} M^{2/3} n_e^{4/3}$ ... Ave. pressure for stability needs to rises as $n_e^{4/3}$

Pressure

First we review a standard result from kinetic theory.

We are interested in an electron gas. Imagine a cube of side length L.

Such a cube inside isotropic gas has no net force on its surfaces, but there is a force from the inside due to particles hitting the surface which is perfectly balanced by the force from the outside. The average number hitting one side surface per unit time is

$= N_e \bullet \frac{v_x}{2L}$ where v_x is the component of velocity normal to the side. 2L is the roundtrip.

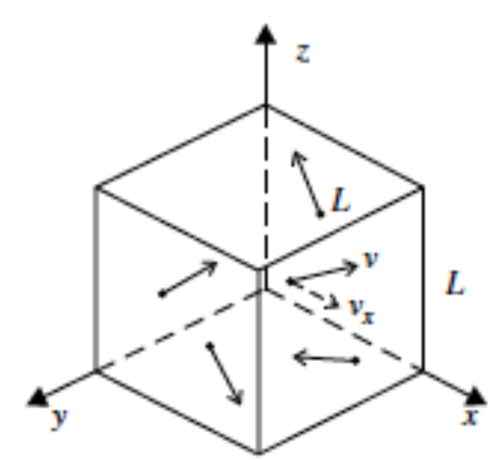
Each hit causes momentum change of $2p_x$, therefore the average force on the side is

$$F = 2p_x N_e v_x / 2L = \frac{N_e \langle p_x v_x \rangle}{L}$$

$$\text{Pressure} = \text{F} / L^2 = n_e \langle p_x v_x \rangle$$

Given p and v are in the same direction. $\langle p.v \rangle = \langle p_x v_x \rangle + \langle p_y v_y \rangle + \langle p_z v_z \rangle$

$$\Rightarrow \langle p_x v_x \rangle = \frac{1}{3} \langle p.v \rangle$$



we can shrink the volume to zero to obtain that pressure in a static gas is a scalar quantity

$$P = \frac{1}{3} n_e \langle pv \rangle$$

Pressure due to degeneracy

We now start with the pressure inside an isotropic gas.

$$P_{gas} = \frac{1}{3} n_e \langle pv \rangle$$

Define energy density $\varepsilon = n_e \langle E \rangle$

For non-relativistic gas $p = mv$, $\mathrm{E} = \frac{1}{2} mv^2 \Rightarrow P_{gas} = \frac{2}{3}\varepsilon$

For relativistic gas $E \approx pc \Rightarrow P_{gas} = \frac{1}{3}\varepsilon$

We now calculate the energy density for a degenerate gas. Integrating over all states one gets.

$$\varepsilon = \begin{cases} \int_0^{P_f} 2(\frac{p^2}{2m_e})\left(\frac{4\pi p^2 dp}{(2\pi)^3}\right) = \frac{p_f^5}{10 m_e \pi^2} = \boxed{\frac{3^{5/3}}{10 m_e}\pi^{4/3} n_e^{5/3}} & \text{For non-relativistic gas} \\ \int_0^{P_f} 2(p)\left(\frac{4\pi p^2 dp}{(2\pi)^3}\right) = \frac{p_f^4}{4\pi^2} = \frac{3^{4/3}}{4}\pi^{2/3} n_e^{4/3} & \text{For relativistic gas} \end{cases}$$

Pressure inside a relativistic gas rises slower with density ! It is barely stable against gravity.

Approximate Chandrasekhar Mass

To add an electron to a given volume of degenerate gas requires higher energy level. This energy gain to compress an additional electron into a given volume is the degeneracy pressure. This pressure must exceed the pressure needed for gravitational stability.

$$P_{\text{degeneracy}} = P_{gravity}$$

$$\frac{2}{3}\frac{3^{5/3}}{10}\pi^{4/3} n_e^{5/3} = \frac{G}{5}\left(\frac{4\pi}{3}\right)^{1/3}\left(\frac{m_N A}{Z}\right)^{4/3} M^{2/3} n_e^{4/3}$$

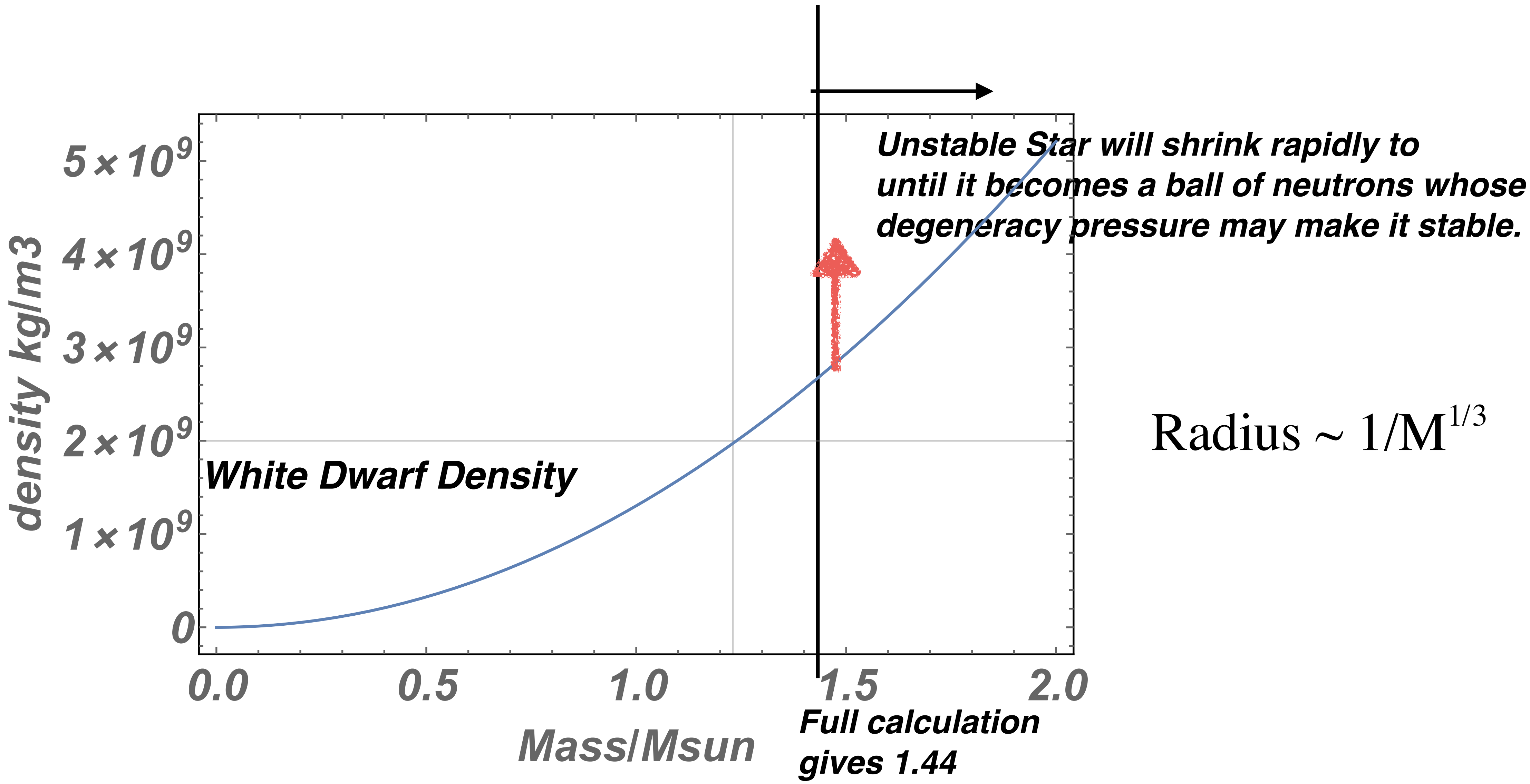
$$n_e = \frac{4G^3 m_e^3}{27\pi^3}\left(\frac{m_N A}{Z}\right)^4 M^2 = 3.9\times 10^{35}\left[\frac{M}{M_\odot}\right]^2 (m^{-3})$$

As a degenerate star increases in mass, the density must increase rapidly and the radius actually decreases, but at some point density is so high that the electrons are relativistic $p_{fermi} \sim m_e \Rightarrow n_e = \frac{m_e^3}{3\pi^2} = 5.9\times 10^{35}(m^{-3}) \Rightarrow 2\times 10^9 (kg/m^3)$

2 tons/cc, Density when star become soft and not stable against gravity.

$$M_{chandra} \approx \frac{3\sqrt{\pi}}{2}\left(\frac{Z}{A}\right)^2\left(\frac{M_{PL}^3}{m_p^2}\right) = 1.2 M_{SUN}$$

Star stability versus mass



Radius ~ $1/M^{1/3}$

- When $P_F \geq m_e$ degeneracy pressure is too low.
- The collapse goes until density reaches nuclear
- The energy release is from gravity with R~15km.

$$E_{collapse} = \left(\frac{3}{5}\right) G m_n^2 \frac{A_\odot^{5/3}}{r_0}$$

$$E_{collapse} \approx 10^{59}\, MeV!$$

$$r_0 = 1.2\, fm \quad A_\odot \approx 2 \times 10^{57}$$

The southern sky

Center Milky Way

SMC

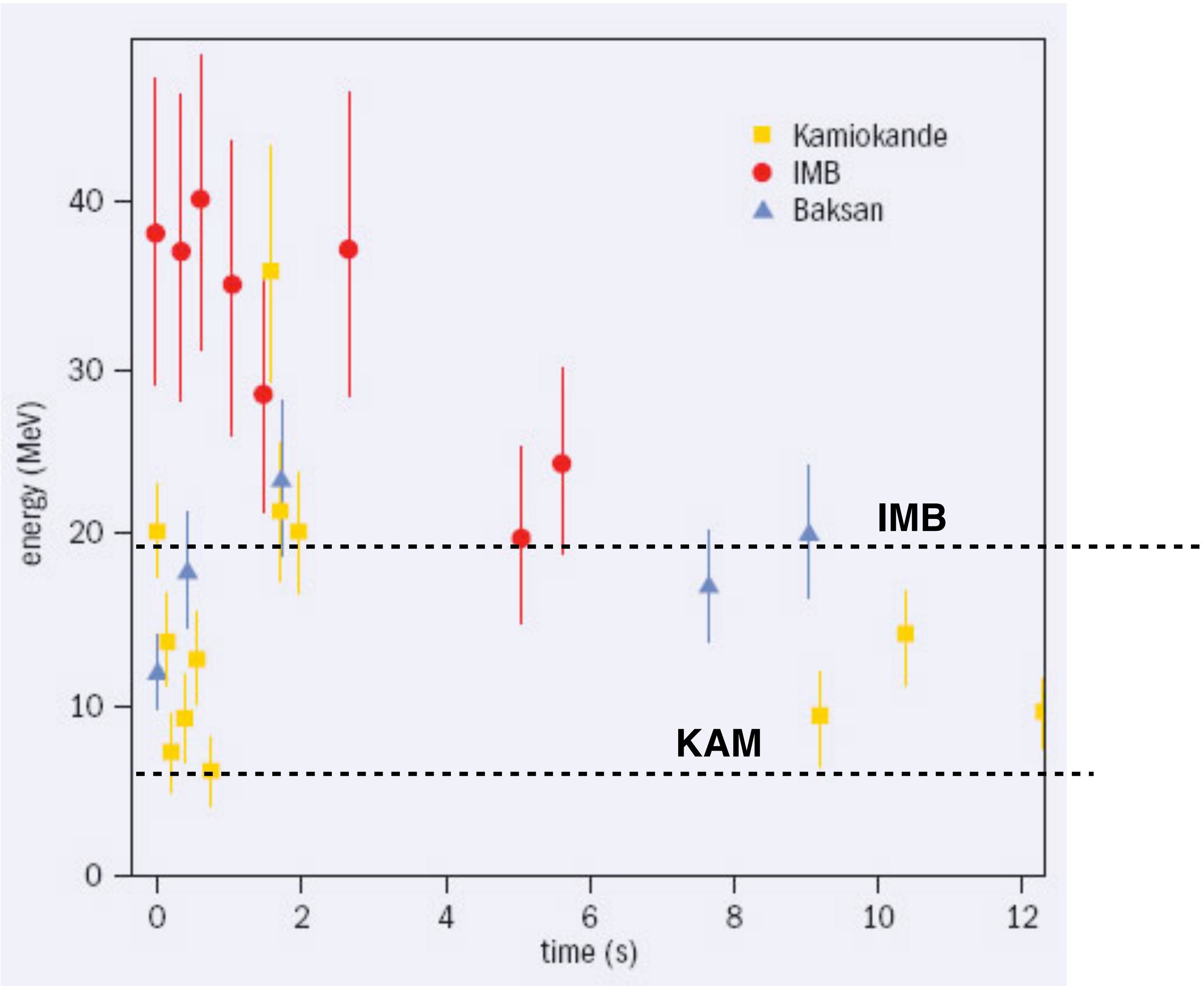
Large Magellanic Cloud
1/100 of Milky Way
160000 LY away (50 kpc)

Supernova 1987a

Feb 23, 1987

In early 1987 a supernova in the LMC (arrowed, near the Tarantula Nebula) rose to naked-eye visibilty: the brightest supernova observed since 1604. This image was made with an SLR camera and colour transparency film.

Data from SN1987a



- Kamioka
  - 2 kton,
  - Ethres > 6 MeV
- Irvine-Michigan-Brookhaven (IMB)
  - 7 kton
  - Ethres>20 MeV
- Baksan
  - Liquid scint. 0.2 kton
- 7 hours before optical observation.

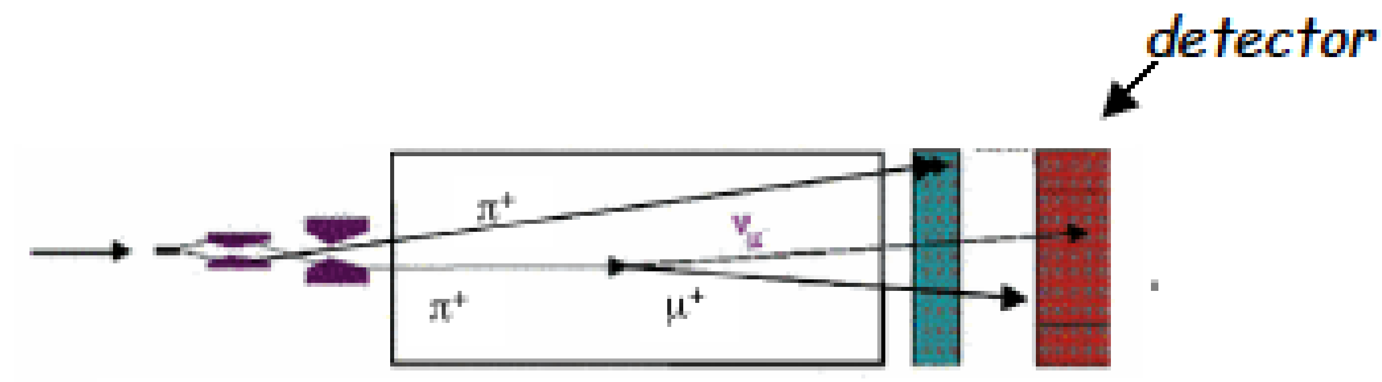
$$\overline{\nu}_e + p \rightarrow n + e^+$$

$$N_\nu^{Total} \approx 6 \times 20 evts \times \frac{4\pi(60kpc)^2}{\sigma_{IBD}(10MeV) \times 3 \times 10^{32}(p/9kTon)} = 10^{57} - 10^{58}$$

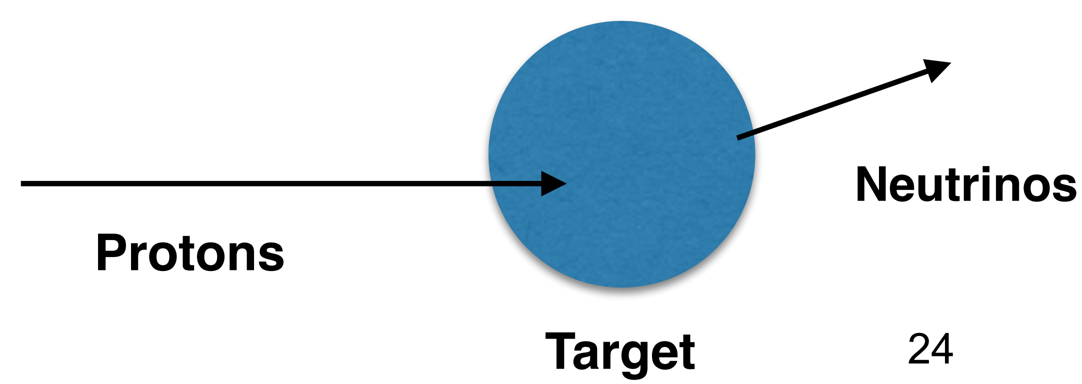
Human-made Neutrino Sources

- Nuclear reactors.
  - <10 MeV, 6×10^{20} /3GW(th)
- Accelerator Beam (10-120 GeV proton)
  - 1-100 GeV, $10^{17}/m^2/GeV/MW*yr$ @1 km.
- Accelerator beam with decay at rest.
  - Pions and muons stop and decay in the target.

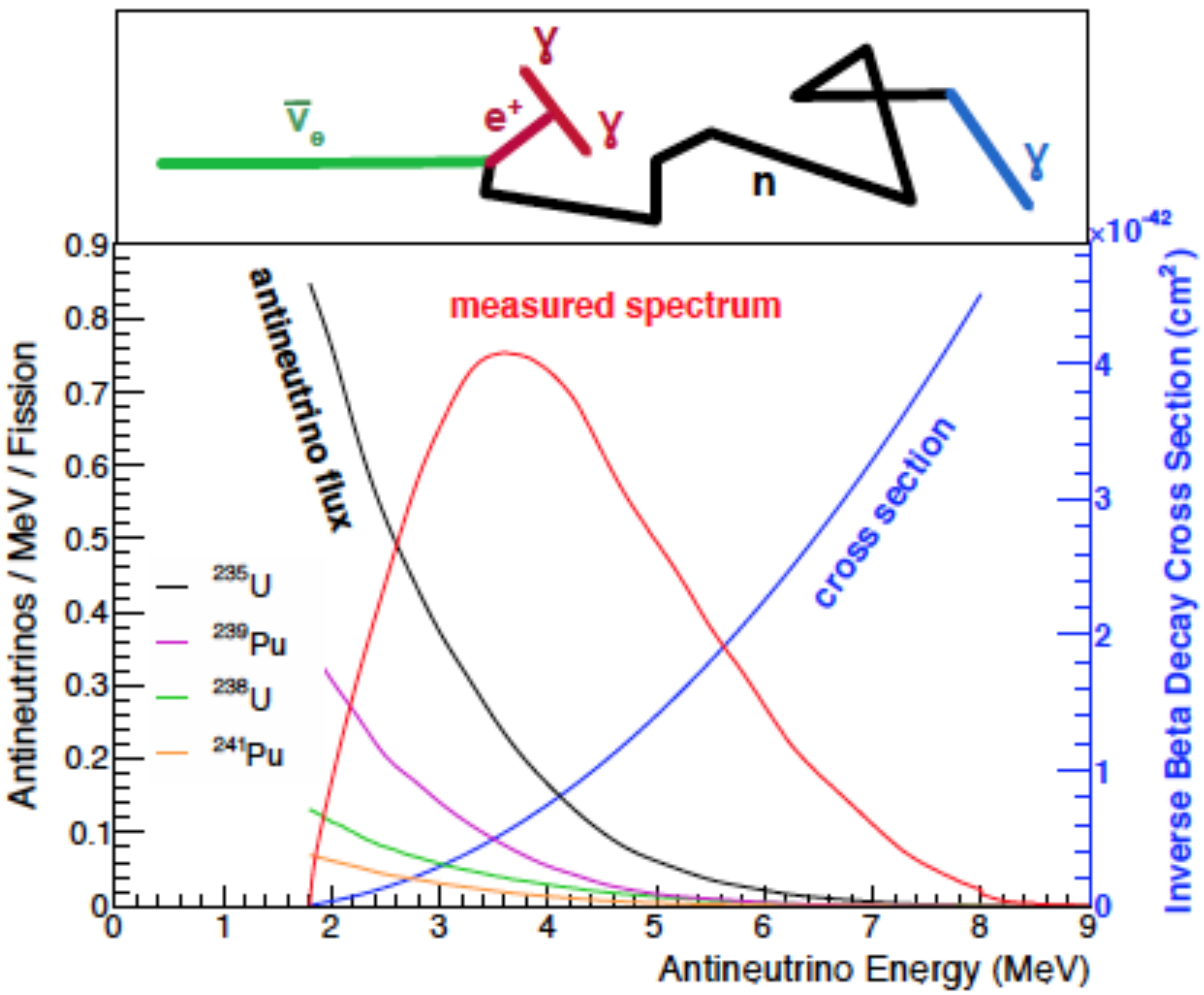
Pure electron anti-neutrino source. Isotropic (4Pi) beam.



Pure muon neutrino (antinu) source, pulsed, directed



Reactor Spectra



Power reactors produce 3 GW of thermal energy.

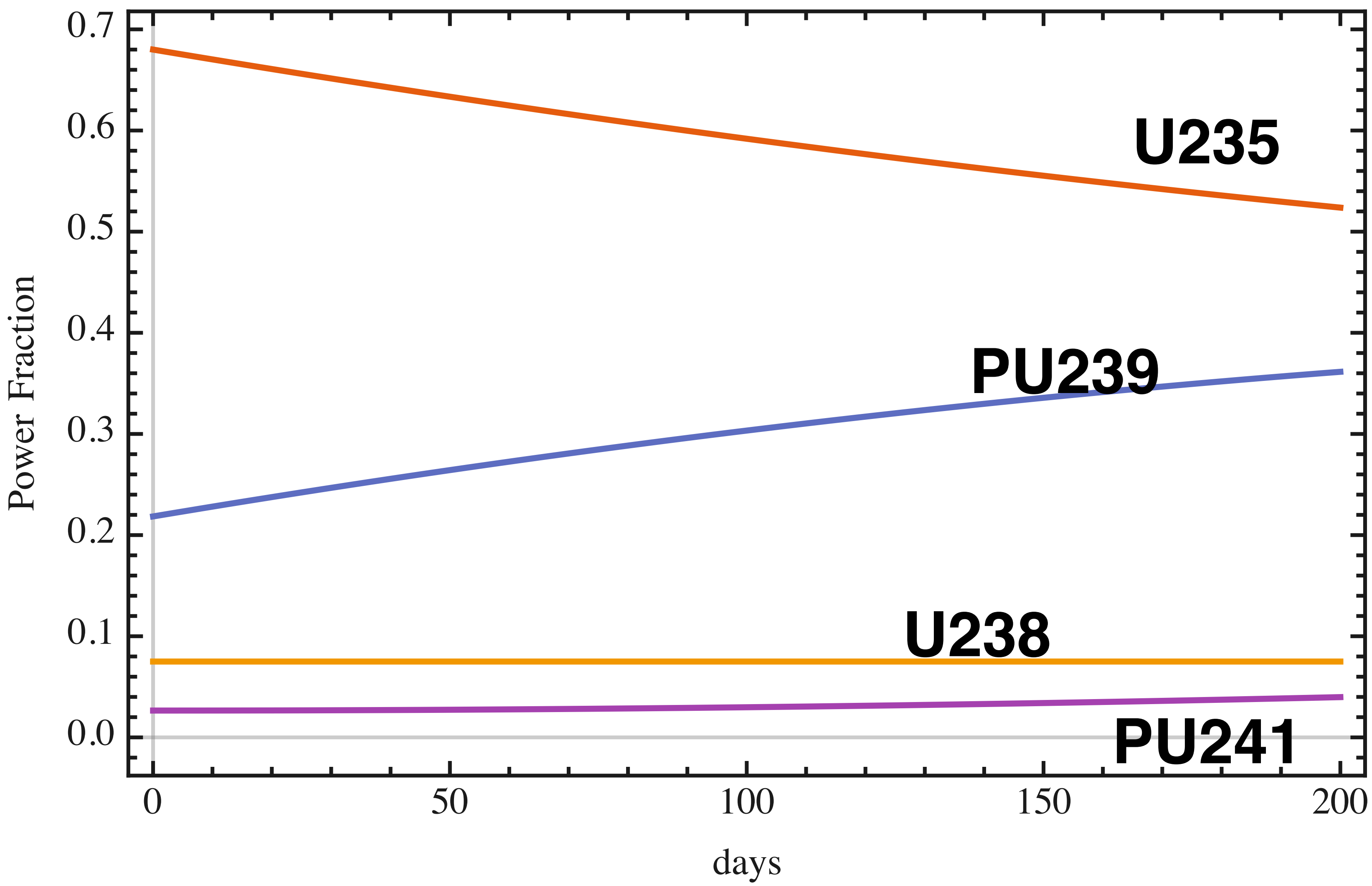
Each fission has ~200 MeV.

Each fission leads to 6 beta decays.

$$Neutrinos/\sec = 6\frac{3\times10^{9}\,J/\sec}{1.6\times10^{-13}\,J/MeV\bullet200\,MeV}\approx 6\times10^{20}$$

Find how to calculate the spectrum from literature. (P. Vogel et al.)

How to calculate reactor spectra

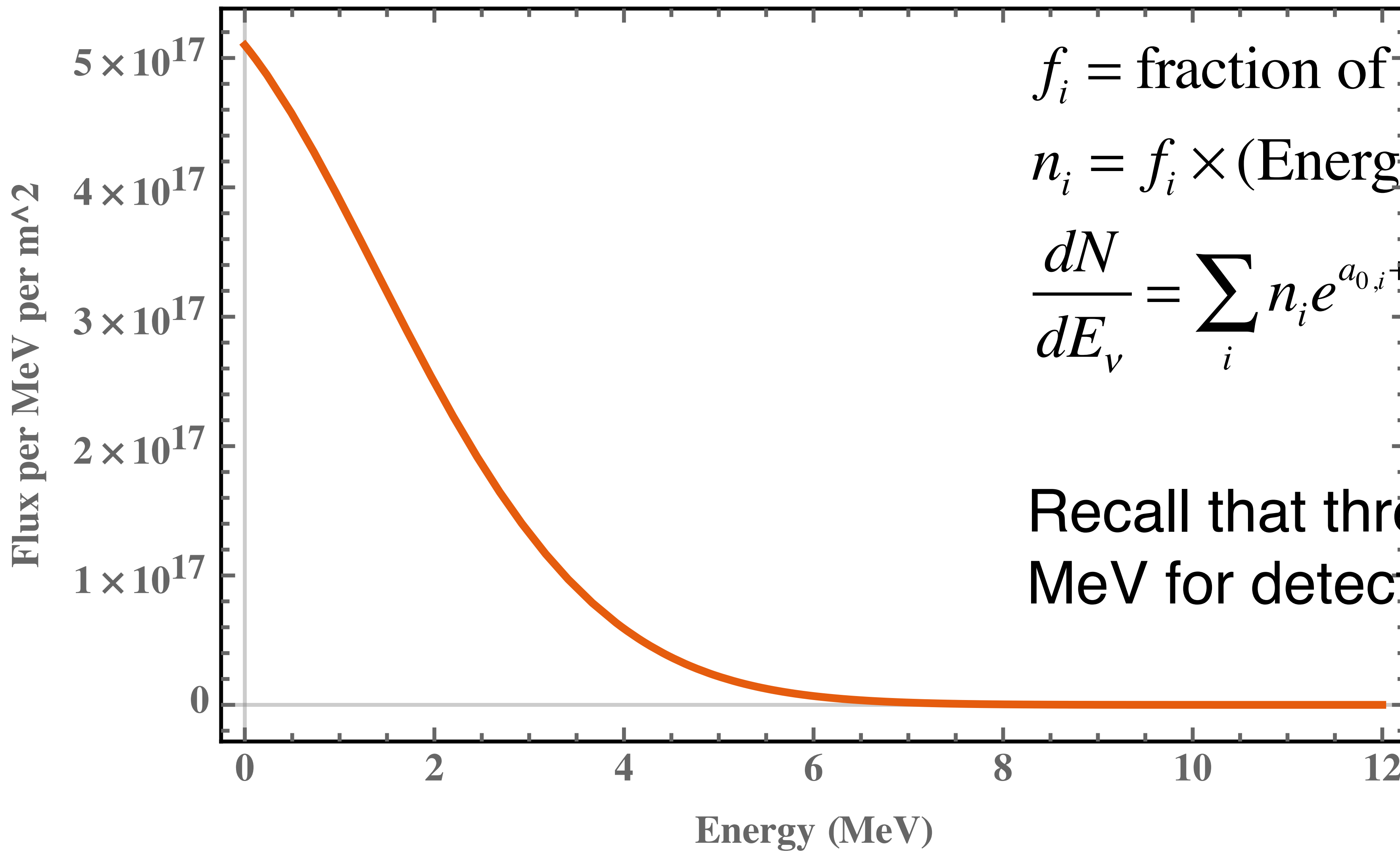


First calculate power produced by each isotope.

In a real experiment the power company will provide this information.

The reactor spectrum parameterization from Vogel and Engel based on data by Schreckenbach. New methods use tabulated beta decay spectra.

Reactor Spectrum



f_i = fraction of energy from isotope i

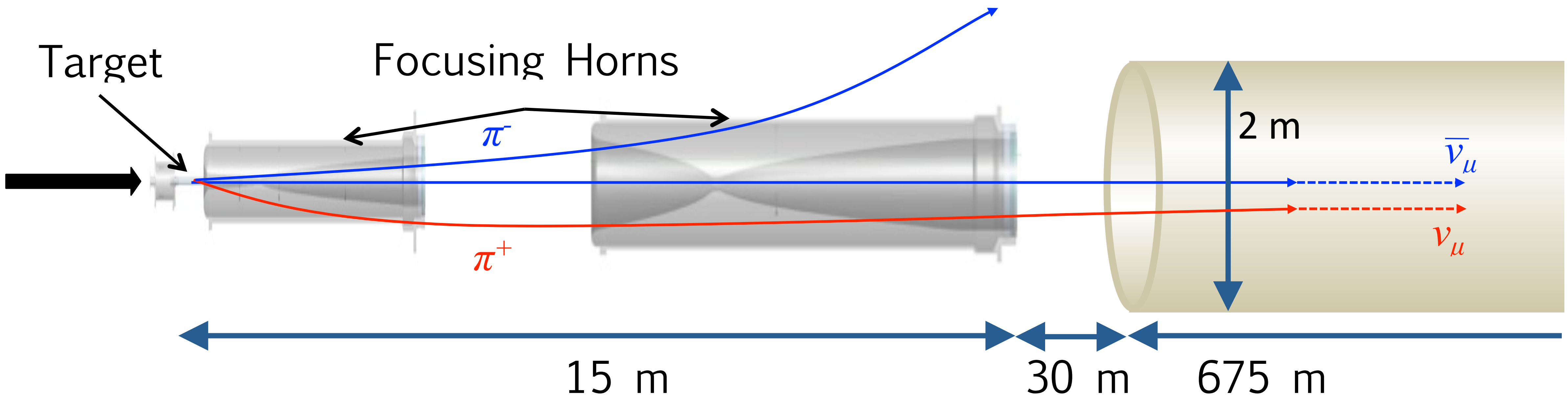
$n_i = f_i \times$(Energy per day)/(Energy per fissio

$$\frac{dN}{dE_\nu} = \sum_i n_i e^{a_{0,i}+a_{1,i}E_\nu+a_{2,i}E_\nu^2}$$

Recall that threshold is 1.8 MeV for detection

Isotope	U235	Pu239	U238	Pu241
Energy (MeV)	201.7	205.0	210.0	212.4
a0	0.870	0.896	0.976	0.793
a1	-0.160	-0.239	-0.162	-0.080
a2	-0.0910	-0.0981	-0.0790	-0.1085

Accelerator Neutrino beam

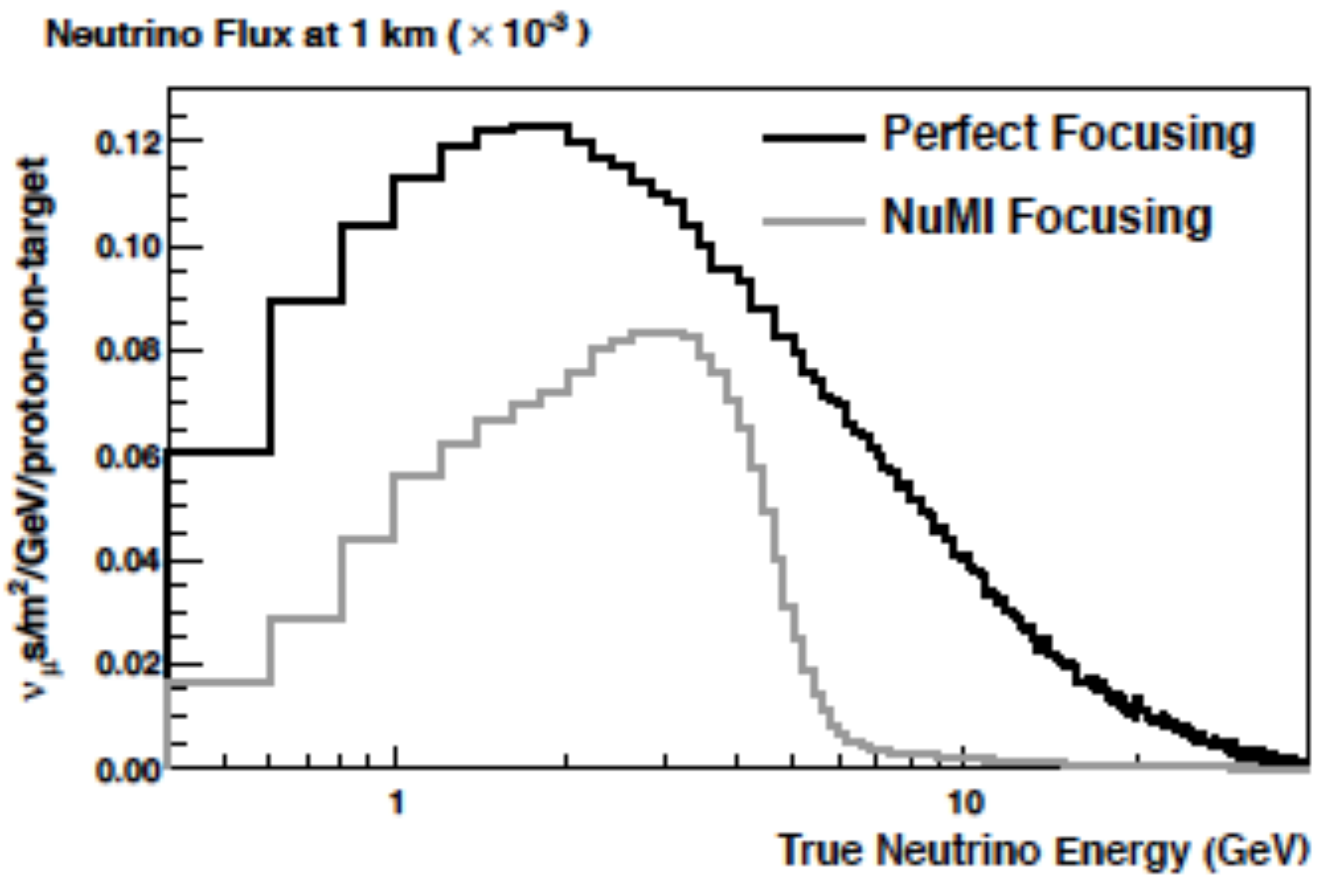


Pions are produced in proton-carbon collisions.

Imagine each pion that emerges from target can be collimated to be forward to get

$$\sim 10^{-4} m^{-2} GeV^{-1} proton^{-1} \quad @1km$$

$$POT(10^{20}) = \frac{Power(MW) \bullet Time(10^{7}\,\text{sec})}{E_{proton}(GeV) \bullet 1.6\times 10^{-3}}$$



From simulation

Characteristics of accelerator neutrino beam

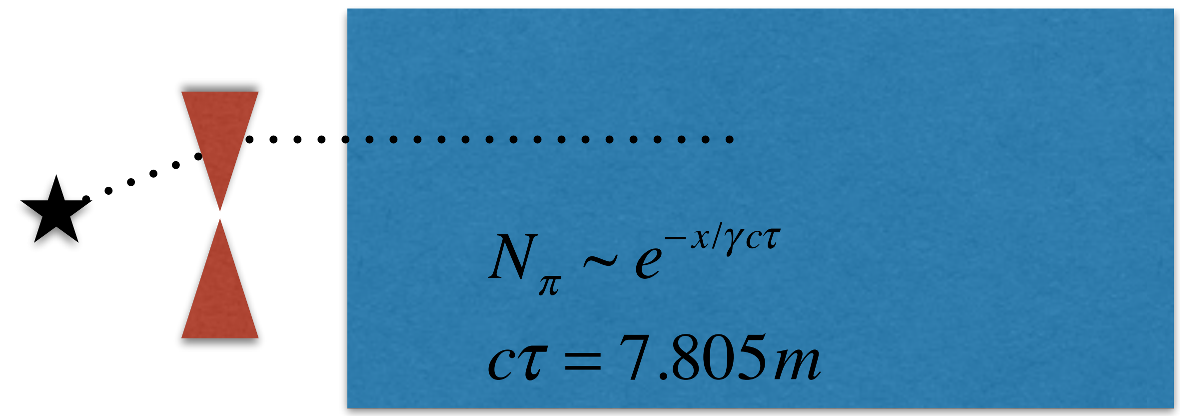
Neutrino beam is produced by the following decays

$\pi^+ \rightarrow \mu^+ + \nu_\mu$

$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$

$K^+ \rightarrow \mu^+ + \nu_\mu$

$K^+ \rightarrow e^+ + \pi^0 + \nu_e$



$\gamma \approx 42 \ for \ E_\pi = 6 \ GeV$

Majority of the flux is from π^+decay >90%

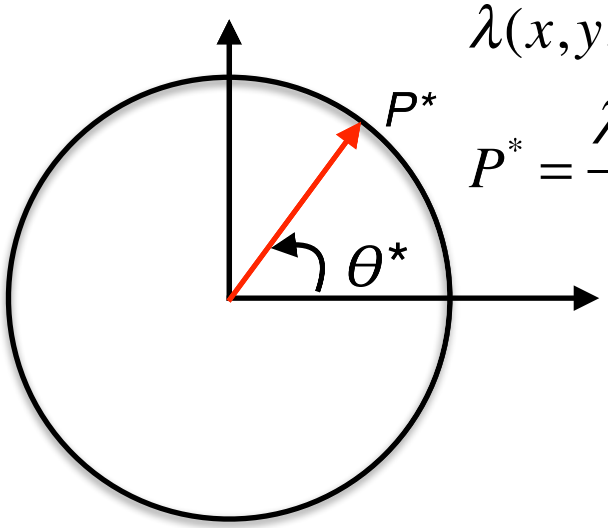
$$\nu_e \text{ cont. } \sim \frac{N_e}{N_\mu} \approx \frac{\tau_\pi}{\tau_\mu} = \frac{26}{2200} = 0.01$$

$$\bar{\nu}_\mu \text{ cont.} \sim \frac{N\pi^-}{N\pi^+} \approx 0.05 \text{ depends on } \mathrm{E}_\nu \text{ and solid angle}$$

About 10% of the neutrino rate is from K^+

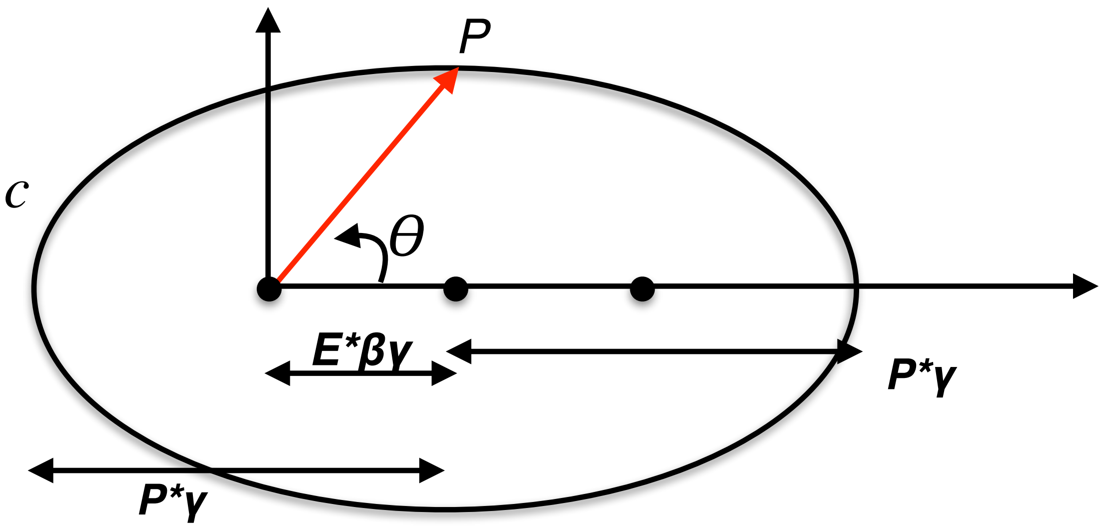
$$\nu_e \text{ from } \mathrm{K}^+ \sim 0.1 \times B(K^+ \rightarrow \pi e \nu) = 0.005$$

Kinematics of accelerator neutrino beam



$$\lambda(x,y,z) = (x-y-z)^2 - 4yz$$

$$P^* = \frac{\lambda^{1/2}(m_\pi^2, m_\mu^2, m_\nu^2)}{2m_\pi} = 29\ MeV/c$$



Boost by γ, β

For a massless particle this is an ellipse with focus at 0

$$P_x = P\sin(\theta) = P^*\sin(\theta^*)$$

$$P_z = P\cos(\theta) = \gamma_\pi P^*\cos(\theta^*) + \beta_\pi\gamma_\pi E^*_\pi$$

$$E = \gamma_\pi E^* + \beta_\pi\gamma_\pi P^*\cos(\theta^*)$$

$$E_{\min} = \gamma_\pi E^* - \beta_\pi\gamma_\pi P^*$$

$E_{\max} = \gamma_\pi E^* + \beta_\pi\gamma_\pi P^*$ E is flat between min/max

For $\theta^* = \pi/2$, $\quad \tan(\theta_C) = \frac{P_x}{P_z} = \frac{1}{\beta_\pi\gamma_\pi}$ for massless particle

$E = \gamma_\pi E^*$ at θ_C

For forward decays in high energy limit: $\theta \to 0,\ \theta^* \to 0$

$$E \approx 2E^* \frac{E_\pi}{m_\pi}\frac{1}{(1+\gamma^2\theta^2)} = \frac{0.42E_\pi}{(1+\gamma^2\theta^2)}$$

Example

$P_\pi = 3$ GeV

$\theta_C = 2.65$ deg

$P_{\nu\text{-}C} = 0.626$ GeV

$E_{\nu MIN} = 0.00067$ GeV

$E_{\nu MAX} = 1.25$ GeV

Neutrino energy versus decay angle

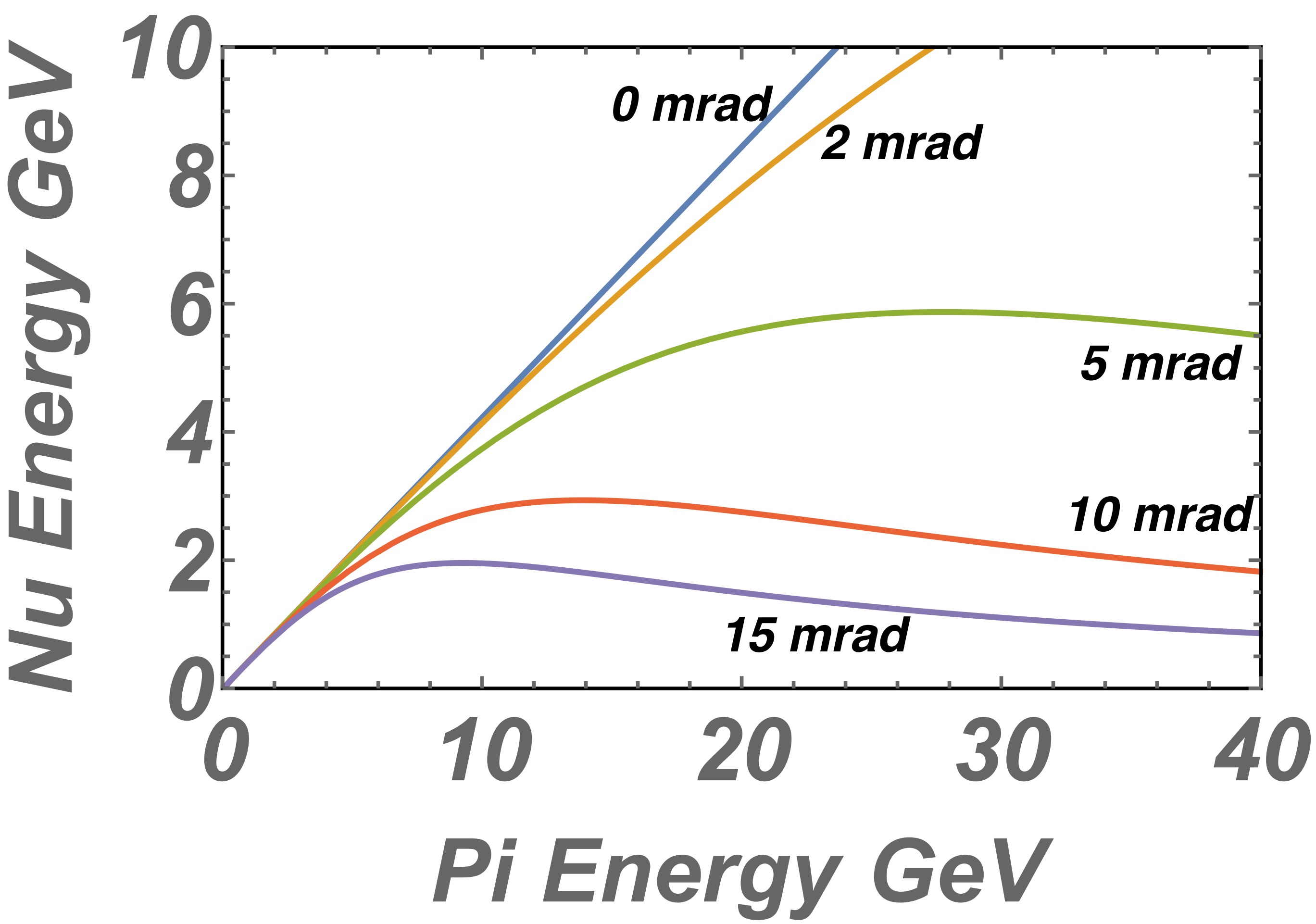
$m_\pi = 139$ MeV and $m_\mu = 105.65$ MeV

and therefore the CM momentum is small $p^* = 29\ MeV/c$

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \mathrm{Cos}(\theta))}$$

As $\theta > 0$, wide range of pion energies contribute to narrow range of neutrino energy giving a sharp peak.

D. Beavis et al., Long Baseline Neutrino Oscillation Experiment, E889, Physics Design Report, BNL 52459 (April 1995), Chap. 3,

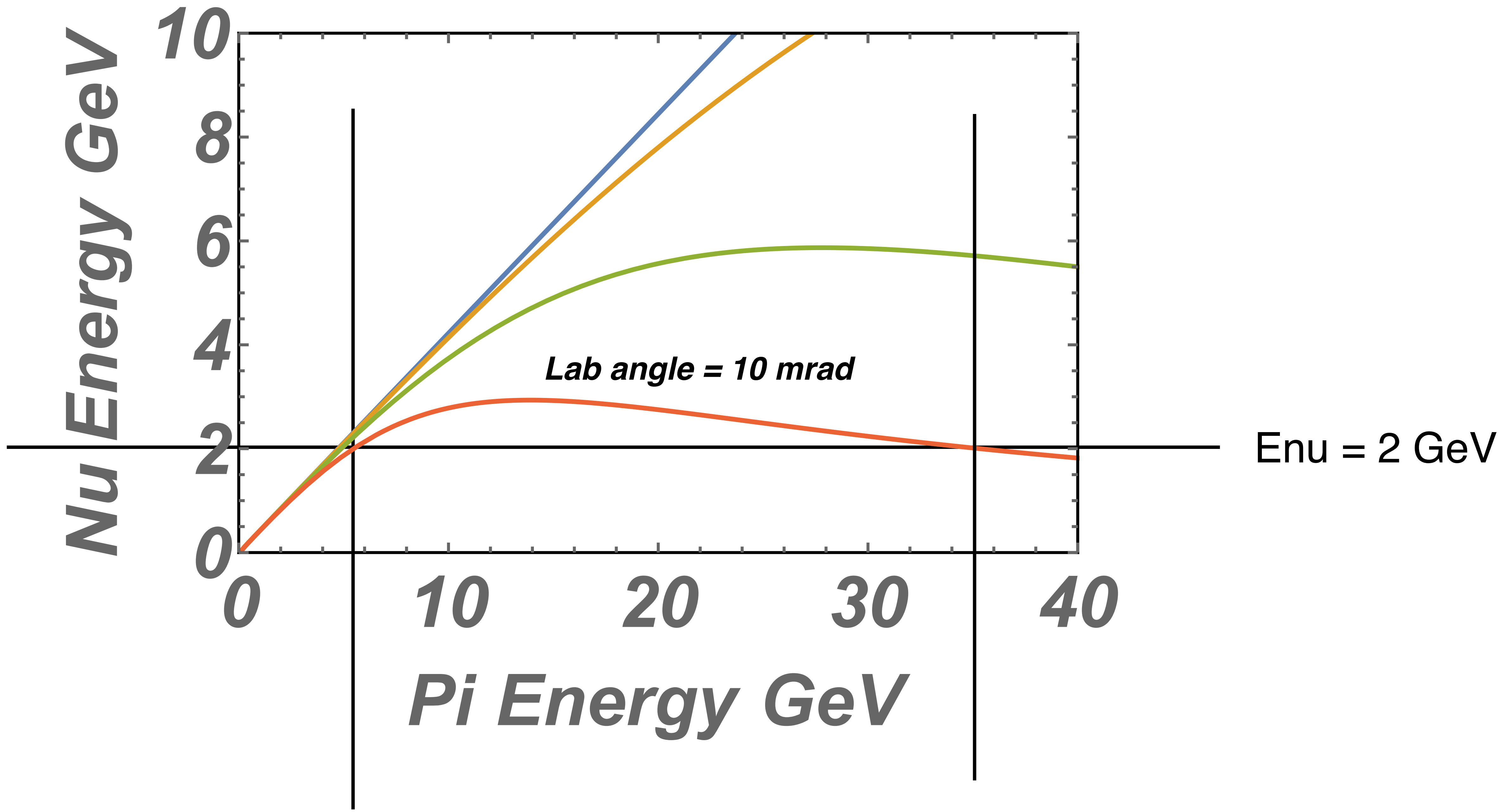


Notice that at any angle > 0 there is a maximum neutrino energy.

$E_\nu \leq \dfrac{E^*}{\mathrm{Sin}(\theta)}$ which does not depend on the pion energy.

For lower E_ν there are two solutions for E_π

Geometry of pion decay that produces a neutrino at an angle. For small angles backwards decays must come from extremely energetic pions



Pion Energy	5.47 GeV	35.5 GeV
CM angle	82.3 deg	91.18 deg
	Forward	Backwards

Nu flux from pion decay

$$P_x = P\sin(\theta) = P^*\sin(\theta^*)$$

$$P_z = \gamma_\pi P^*\cos(\theta^*) + \beta_\pi\gamma_\pi E^*; \quad P_z^* = \gamma_\pi P\cos(\theta) - \beta_\pi\gamma_\pi E$$

$$E = \gamma_\pi E^* + \beta_\pi\gamma_\pi P^*\cos(\theta^*); \quad E^* = \gamma_\pi E - \beta_\pi\gamma_\pi P\cos(\theta)$$

$$\tan(\theta) = \frac{\sin(\theta^*)}{\gamma\cos(\theta^*) + \gamma\beta}; \quad \tan(\theta^*) = \frac{\sin(\theta)}{\gamma\cos(\theta) - \gamma\beta}$$

$$\text{Flux in CM frame: } \frac{dN}{d\Omega_{cm}} = \frac{dN}{d\varphi d(\cos\theta^*)} = \frac{1}{4\pi}$$

$$\cos^2\theta^* = \frac{(\cos\theta - \beta)^2}{(\beta\cos\theta - 1)^2} \Rightarrow \frac{d(\cos\theta^*)}{d(\cos\theta)} = \frac{1}{\gamma^2(1-\beta\cos\theta)^2}$$

$$\frac{dN}{d\Omega_{lab}} = \frac{1}{4\pi}\frac{1}{\gamma^2(1-\beta\cos\theta)^2}$$

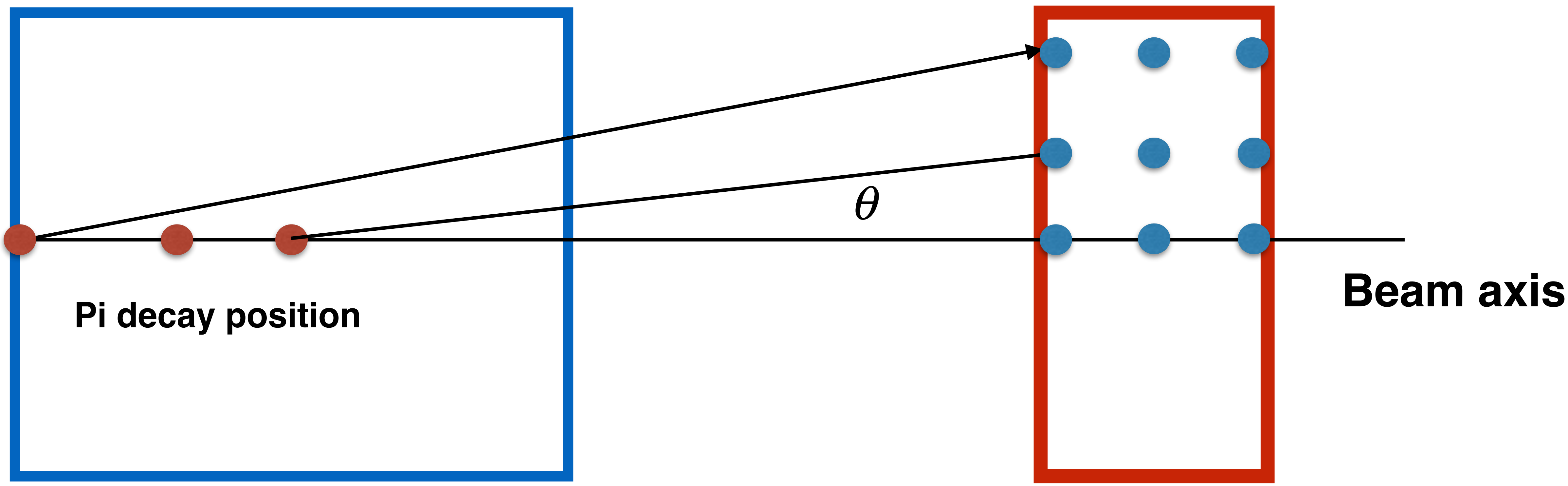
This is angular distribution.

$$\frac{dN}{dA}(r,\theta) = \frac{1}{4\pi r^2}\frac{1}{\gamma^2(1-\beta\cos\theta)^2}$$

This is the flux. It falls as 1/r^2 at fixed angle.

Nu intensity geometrical effects

Let's do some simple calculations for 6 GeV pion



Decay tunnel decay positions 0, 60, 100 m

Detector positions 360, 380, 400 m

Radial positions 0, 1, 2 m

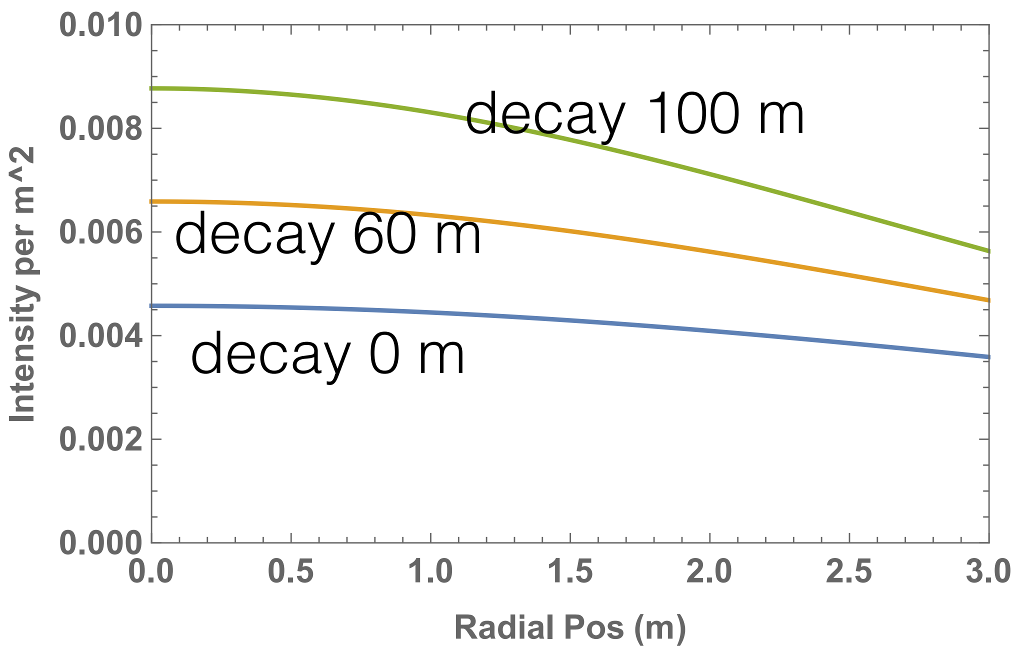
$E_{\pi} = 6\,(GeV)$

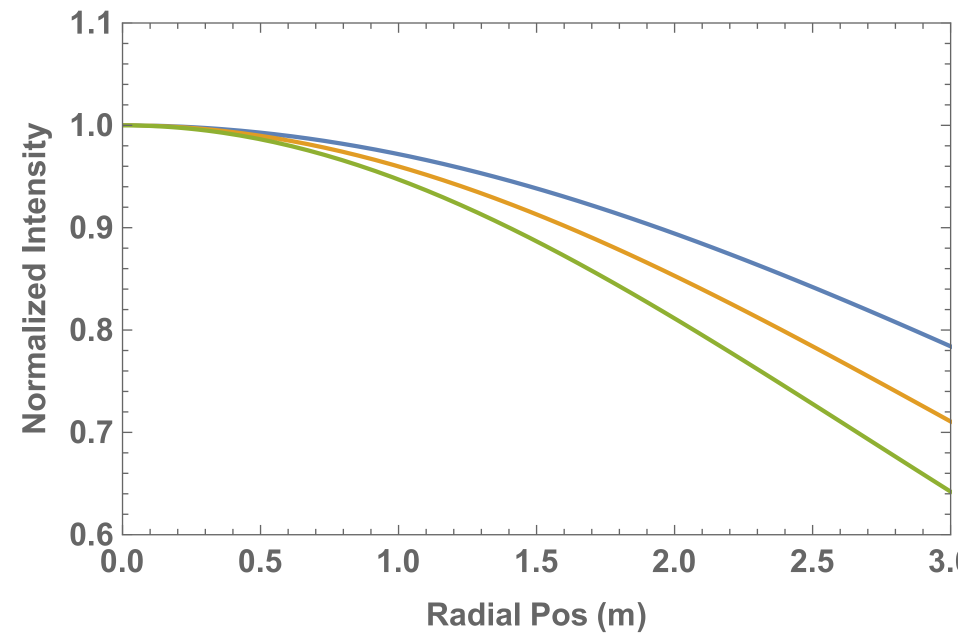
Plot intensity per pion decay versus radial/long pos for selected decay positions

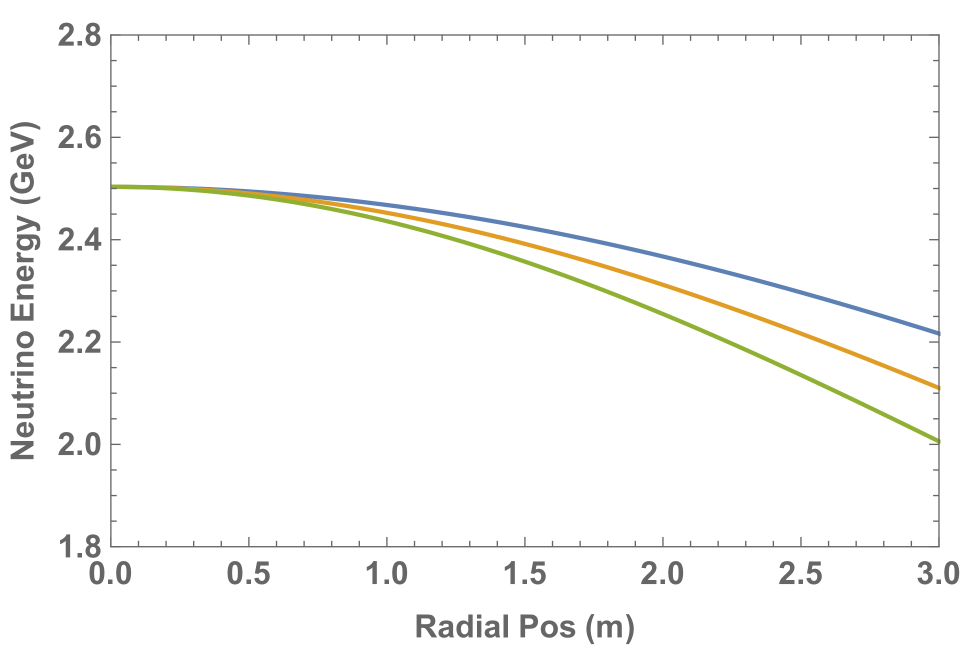
Nu intensity radial dependence

$$\frac{dN}{dA}(r,\theta) = \frac{1}{4\pi(r-r_0)^2}\frac{1}{\gamma^2(1-\beta\cos\theta)^2}$$

Detector at 360 m



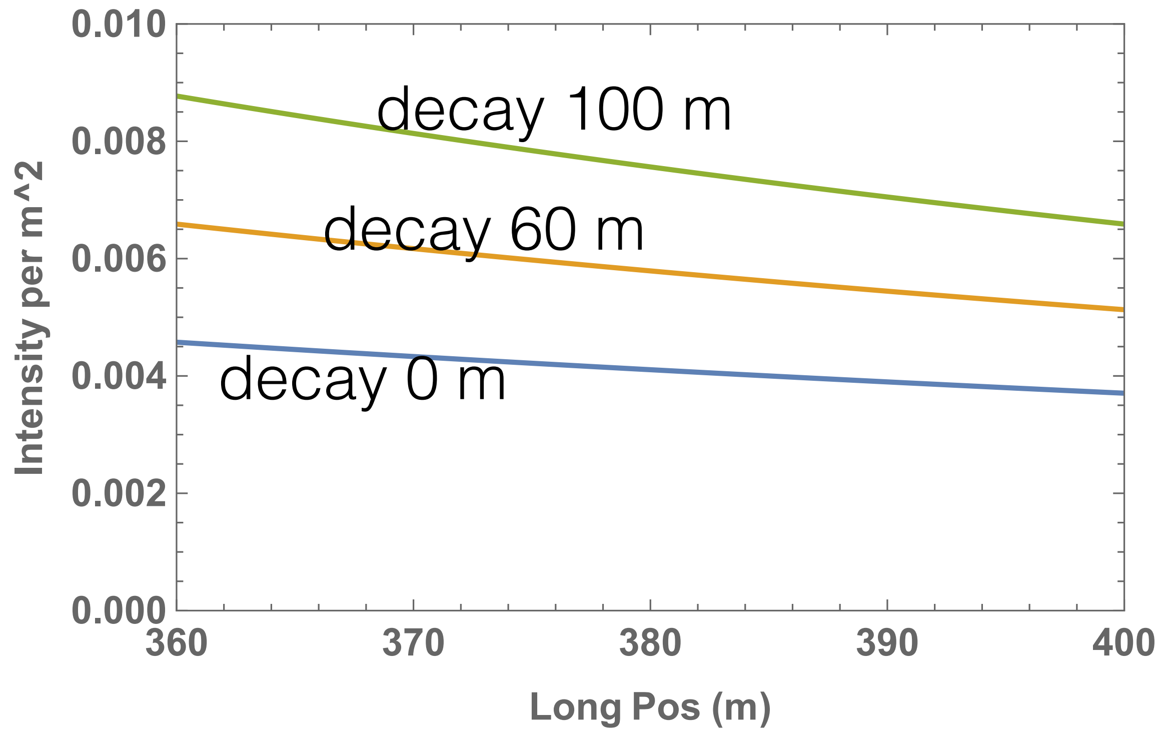


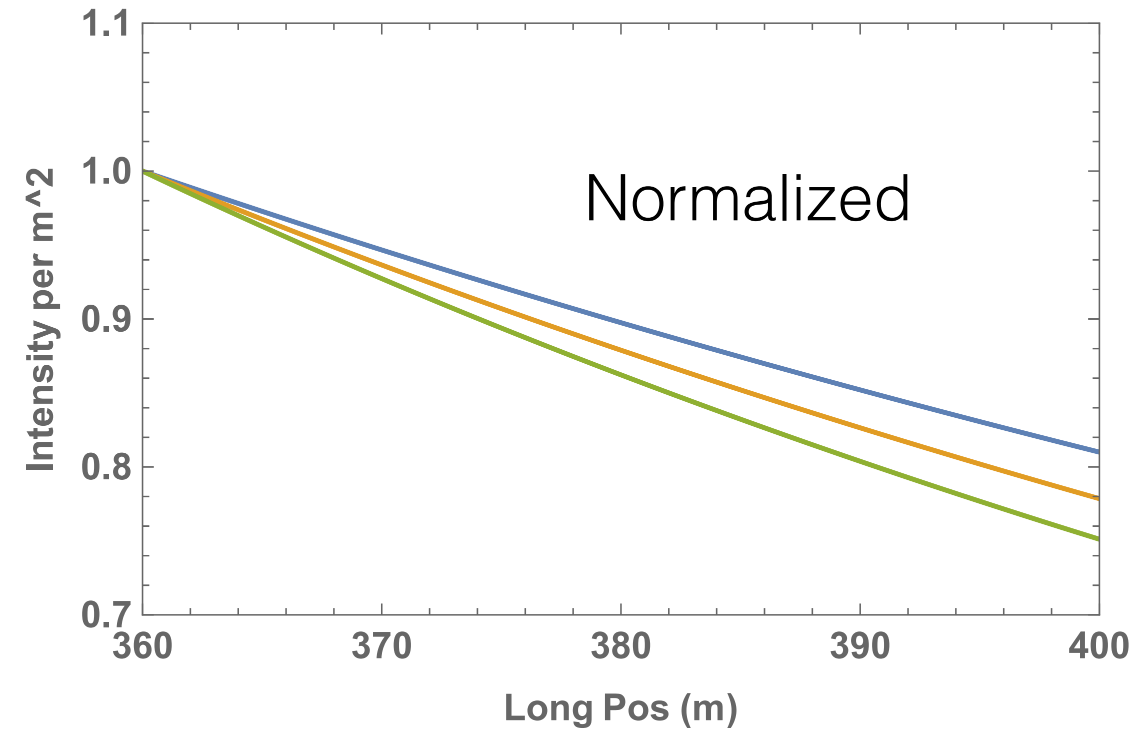


There is ~10% effect in intensity at 2 m radial distance as decay position moves from 0 to 100 m. This comes about from the small angle change. This could get washed out by the beam size.

But Energy changes also.

Nu intensity z-dependence





Intensity on-axis drops about 10 % over ~20 m
The drop varies by ~5% depending on the decay position.

Since the angle is kept constant the neutrino energy and cross section remains the same.

Pion decay spectrum

For proton energy of E_p the pion spectrum is. (assume all are at 0 deg.)

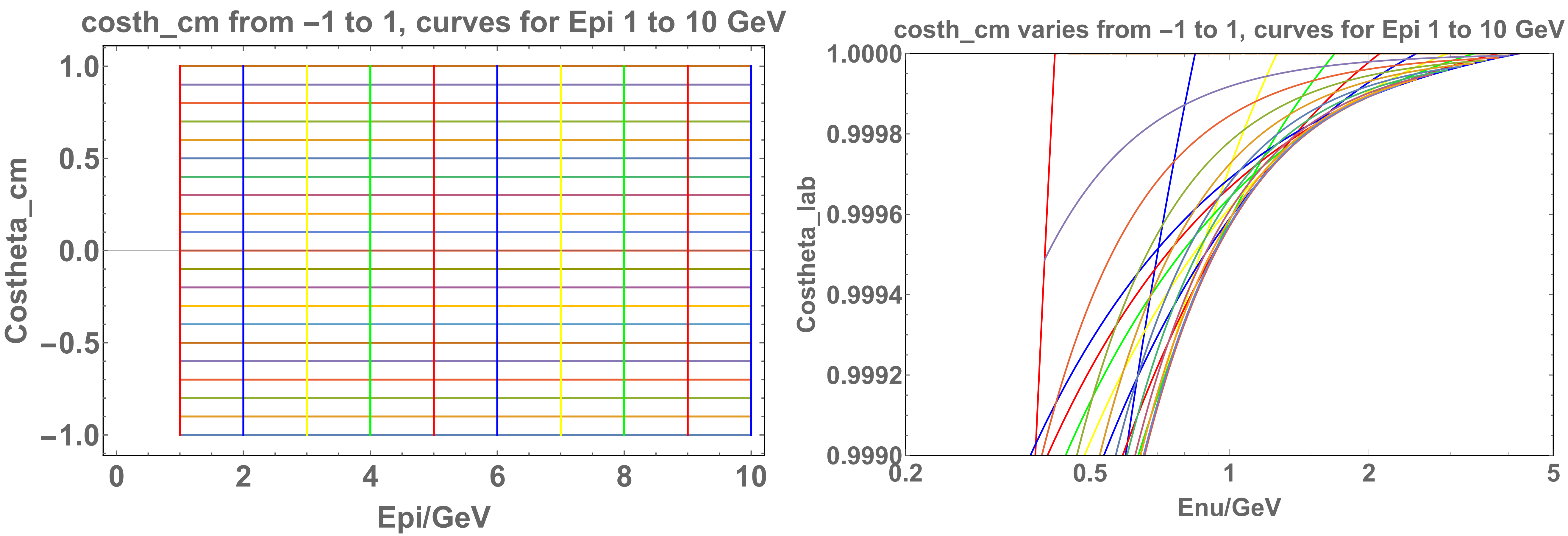
$$\frac{dN}{dE_\pi} \approx K \times (E_\pi - E_p)^\alpha \text{ where K is a constant and } \alpha \approx 5.$$

We ignore transverse momentum dependence for this.

$$\frac{d^2N}{dE_\nu d\Omega_{lab}} = \frac{d^2N}{dE_\pi d\Omega_{cm}} J(E_\pi, \cos\theta^*; E_\nu, \cos\theta)$$

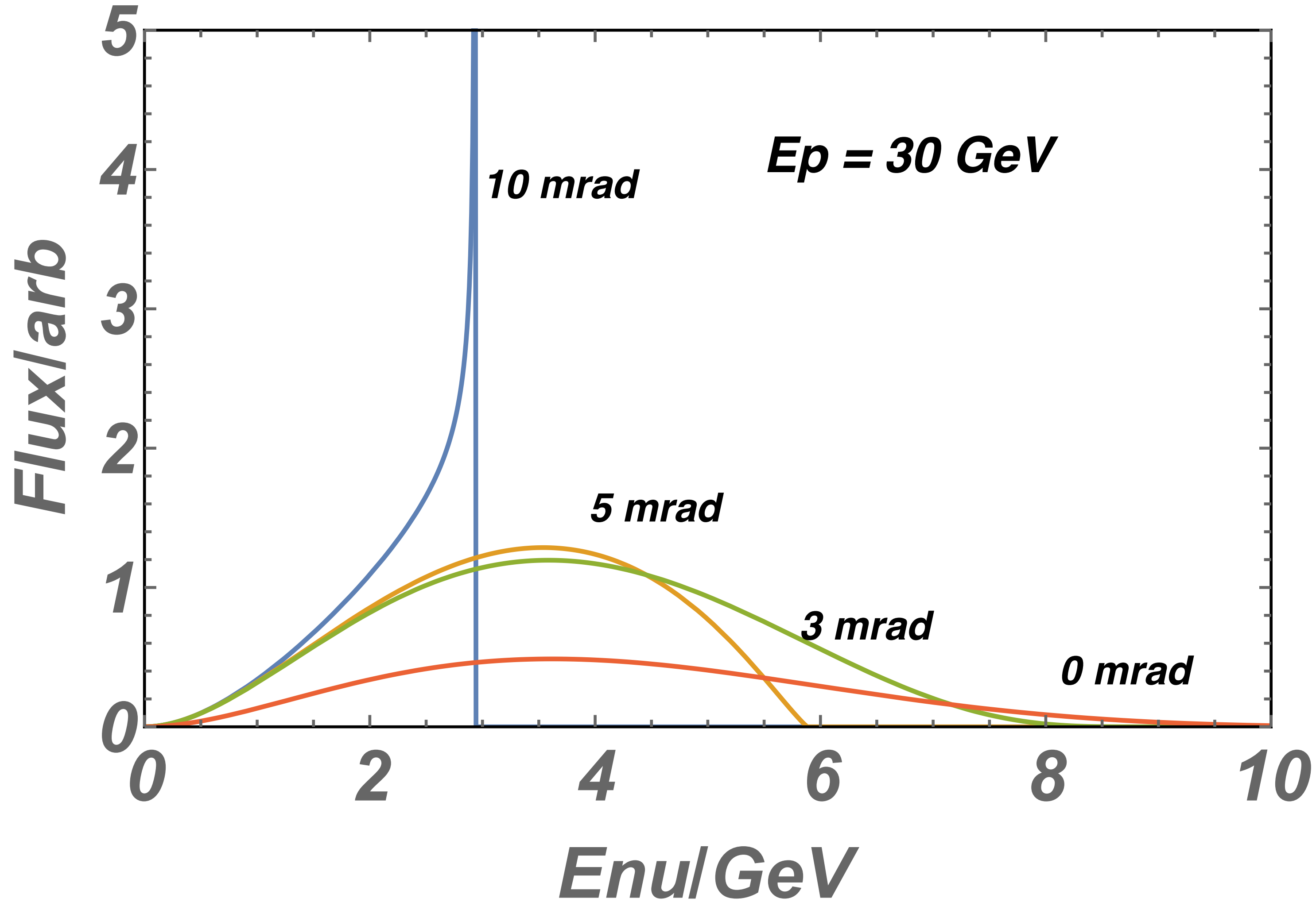
$$J = Det\begin{bmatrix} \frac{\partial E_\pi}{\partial E_\nu} & \frac{\partial \cos\theta^*}{\partial E_\nu} \\ \frac{\partial E_\pi}{\partial \cos\theta} & \frac{\partial \cos\theta^*}{\partial \cos\theta} \end{bmatrix}$$

Jacobian graphically



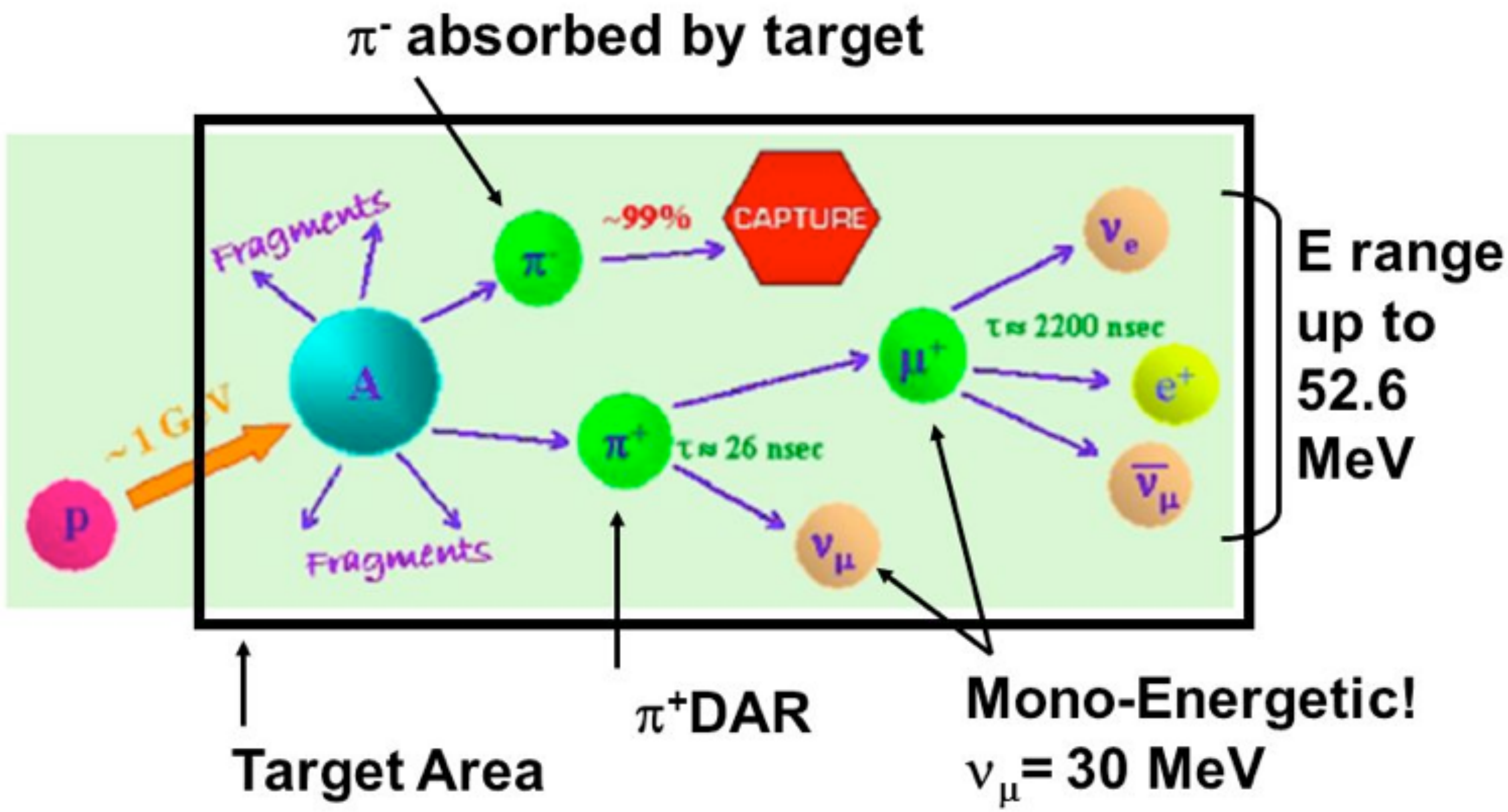
- From the left each rectangle has a probability which gets assigned to a parallelogram on the right.
- As the lab angle moves away from forward, the probability is getting concentrated in a narrow band which creates a sharp peak.
- Two rectangles from left get on top of each other on right. This is the quadratic ambiguity.

Simple calculation of accelerator flux



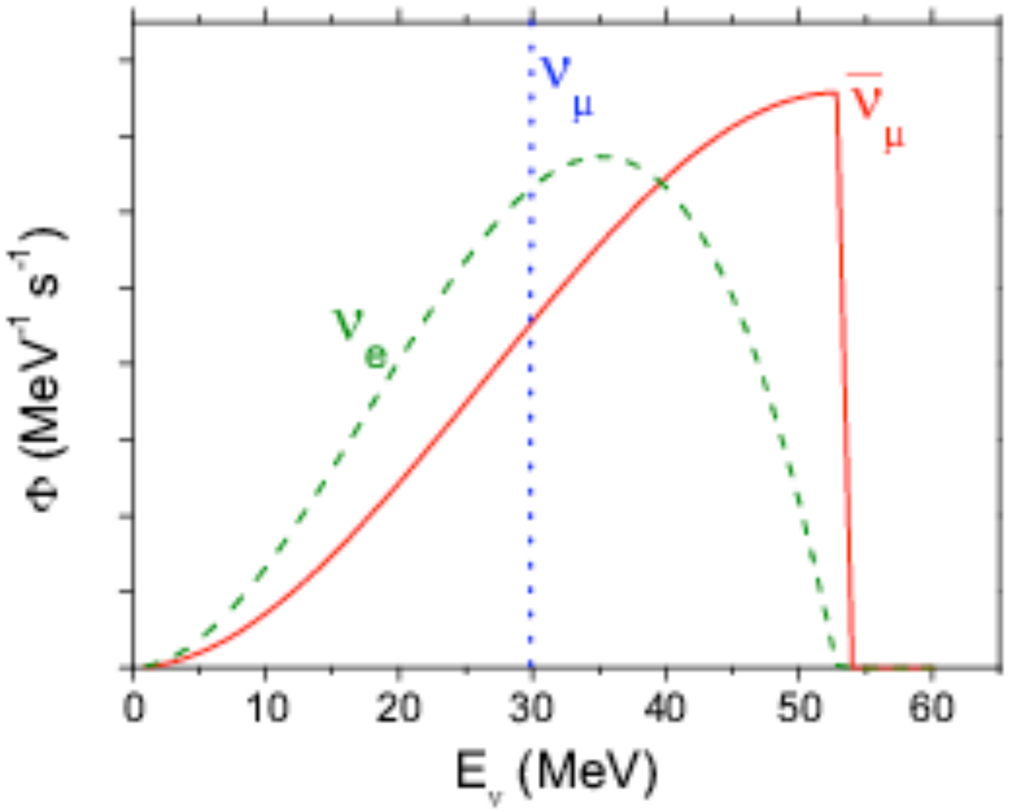
In practice this is modified by the initial transverse momentum distribution of the pions. It will broaden the sharp peak and lower the flux. I have not been careful about the units of flux here.

Decay at rest



Charged pion and muon stop in << 1 ns

DAR flux and event rate



$\pi^- \rightarrow$ Nuclear Absorption.

$\pi^+ \rightarrow \mu^+ + \nu_\mu$

$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$

- Coupled set of calculations
- Optimum proton energy is 600-1500 MeV. Above 1500 energy goes to make other particles.

Burman,Potter,Smith, NIM A291, 621 (1990) , etc.

Pion production has many channels. Must be calculated by Monte Carlo.

$p + N \rightarrow p + N$ (elastic)

$p + N \rightarrow p + N + \pi$

$p + N \rightarrow p + R \rightarrow p + N + \pi$ (resonance)

$p + N \rightarrow R + R \rightarrow N + \pi + N + \pi$

$R + N \rightarrow N + N$ (Absorption)

$\pi + N \rightarrow \pi + N$ (Elastic and chargexchange)

$$F_\nu = 1.3 \times 10^{15} \,/\sec/\, MW \;(for\ each\ flavor)$$

$$\phi_\nu = \frac{F_\nu}{4\pi R^2} = 2.6 \times 10^7\, cm^{-2} \sec^{-1} MW^{-1}$$

$$N_{\text{target}} = (2/3) \times 10^{29}\, ton^{-1}$$

$$\sigma(\bar{\nu}_e p) = 8.5 \times 10^{-41} cm^2 \;(for \sim 30\ MeV)$$

$$Events = \phi_\nu \bullet N_{\text{target}} \bullet \sigma = 4600\, ton^{-1} MW^{-1} yr^{-1}$$

Notice that the muon neutrinos are below threshold for charged current events.
Above assumes that all anti-muon-neutrinos are converted to anti-electron-neutrinos

If neutrinos have mass; the massive states need not be the same as the Weak interaction states. A neutrino could be in a classic superposition of states.

This will lead to interference effects

$$\begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Flavored neutrinos

Massive Neutrinos

$$\nu_a(t) = \cos(\theta)\nu_1(t) + \sin(\theta)\nu_2(t)$$

$$P(\nu_a \to \nu_b) = |<\nu_b|\nu_a(t)>|^2$$

$$= \sin^2(\theta)\cos^2(\theta)|e^{-iE_2 t} - e^{-iE_1 t}|^2$$

Sufficient to understand most of the physics:

$$P(\nu_a \to \nu_b) = \sin^2 2\theta \sin^2 \frac{1.27((m_2^2 - m_1^2)/eV^2)(L/km)}{(E/GeV)}$$

$$P(\nu_a \to \nu_a) = 1 - \sin^2 2\theta \sin^2 \frac{1.27(\Delta m^2/eV^2)(L/km)}{(E/GeV)}$$

Oscillation nodes at $\pi/2, 3\pi/2, 5\pi/2, \ldots$ $(\pi/2)$: $\Delta m^2 = 0.0025 eV^2$, $E = 1 GeV$, $L = 494 km$.

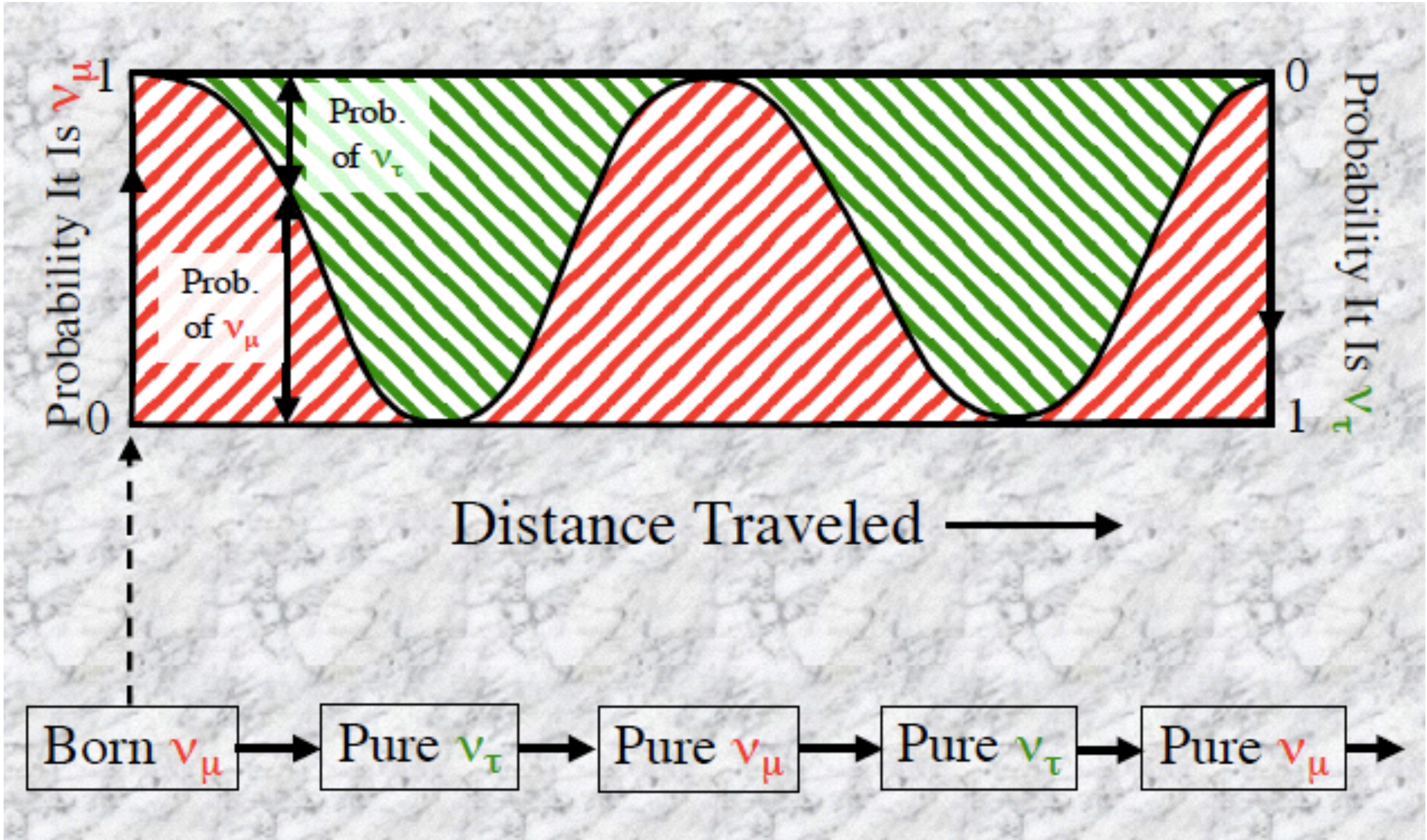
Definition

$Appearance: \nu_a \rightarrow \nu_b \quad \Rightarrow$ Make beam type (a) and Detect (b) after some distance to see if neutrino (a) tranformed to (b).

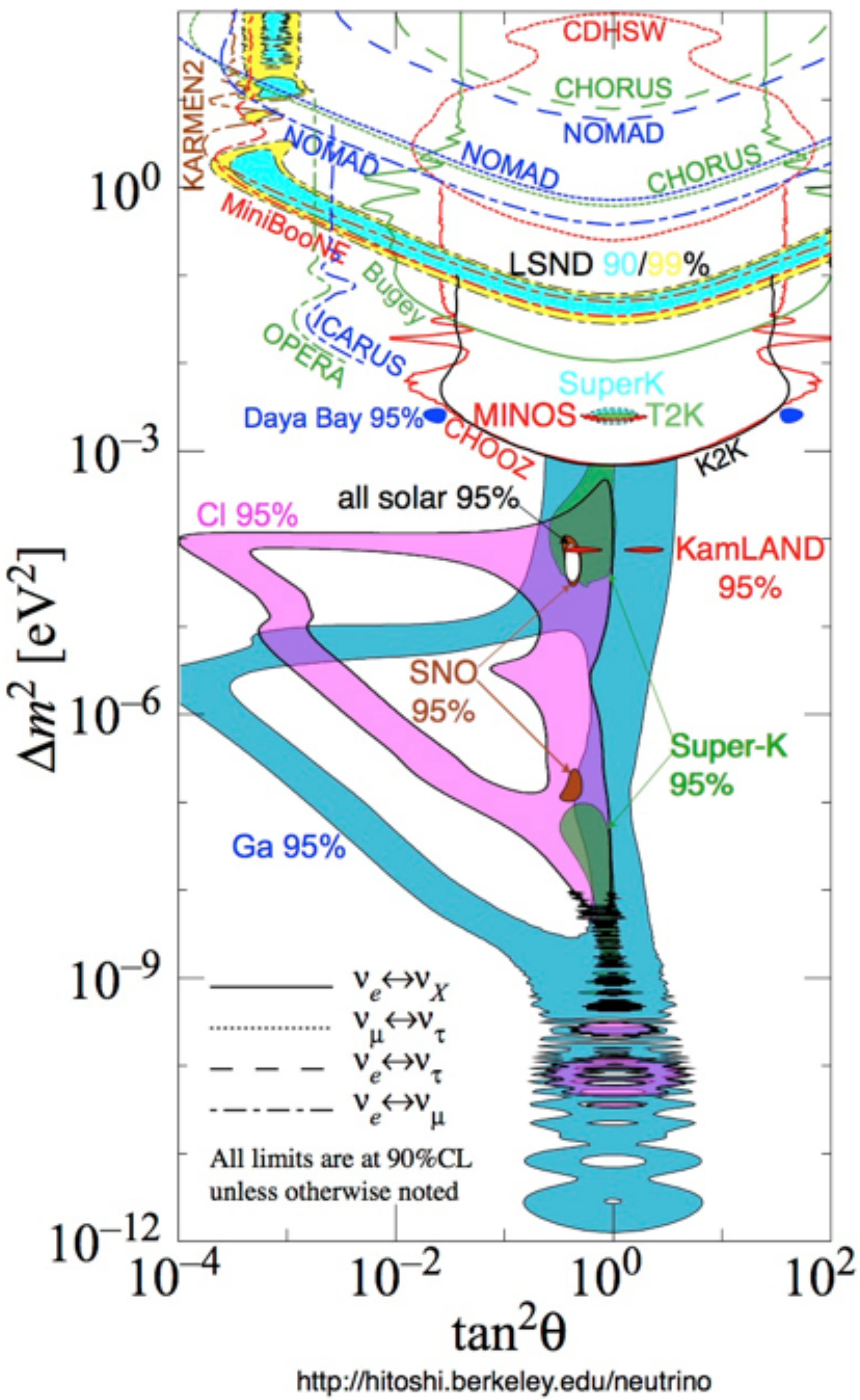
$Dis-appearance: \nu_a \rightarrow \nu_a \quad \Rightarrow$ Make beam type (a) and Detect (a) to see how many are left after traveling some distance

- **In both cases we must know how many (a) type were created and sent to the far detector. A precise prediction in the far detector is needed.**
- **In the first case, we must know if any events will fake the signature of (b) (these are called backgrounds.**
- **In the second we must know how many (a) to expect.**

Picture with θ = 45 deg



Everything we know about neutrino properties comes from this astonishing effect.



All oscillations data from PDG in two parameter plot

$$P(\nu_a \rightarrow \nu_b) = \mathrm{Sin}^2\theta\,\mathrm{Sin}^2(1.27\Delta m^2 \frac{L}{E})$$

Δm^2 *in* eV^2

L in km

E in GeV

Summary

- 3 types of neutrinos from many sources.
- Still many unknowns about these particles, but they play an important role in the early universe as well as current astrophysical processes. If they have mass there are fundamental consequences.
- We described the flux and spectra for most of the important known sources.
- Sources that we did not cover are:
  - Diffuse supernova or sum of all supernova from the past.
  - Extreme high energy sources from outer space.
  - Low energy sources from intense radioactive sources.
- In general, clever ideas about sources are always needed.