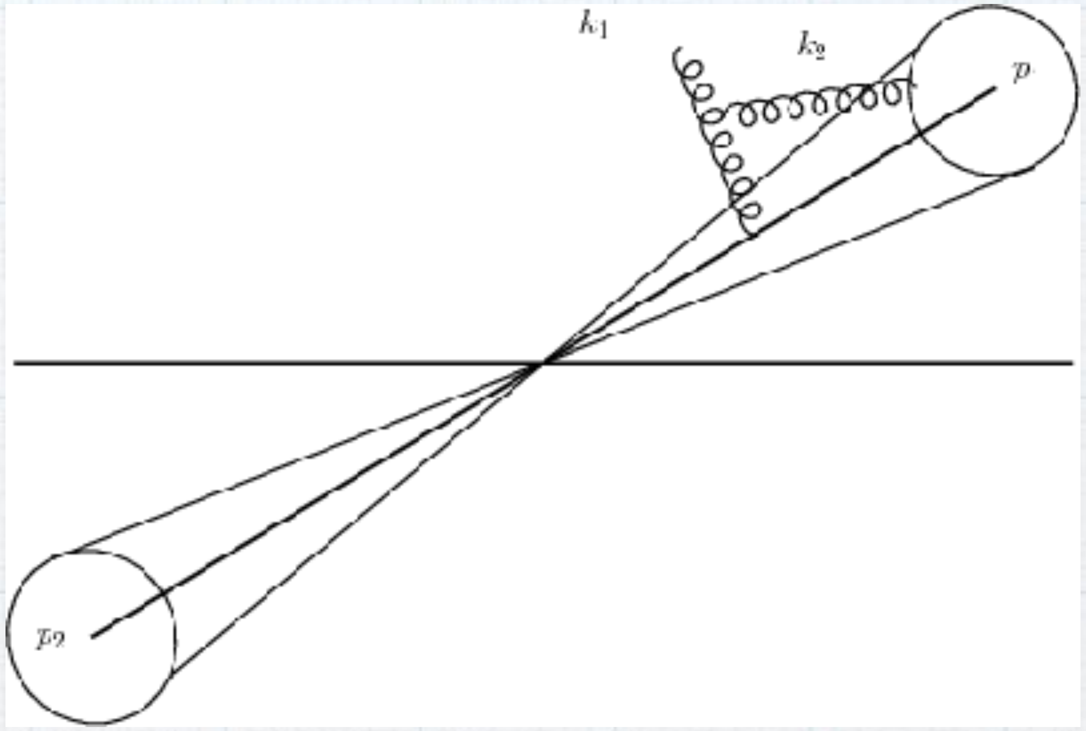
Lecture 2: calculating jet properties

* QCD in the soft/collinear limit
* inclusive and exclusive observables
* the jet mass as a case study



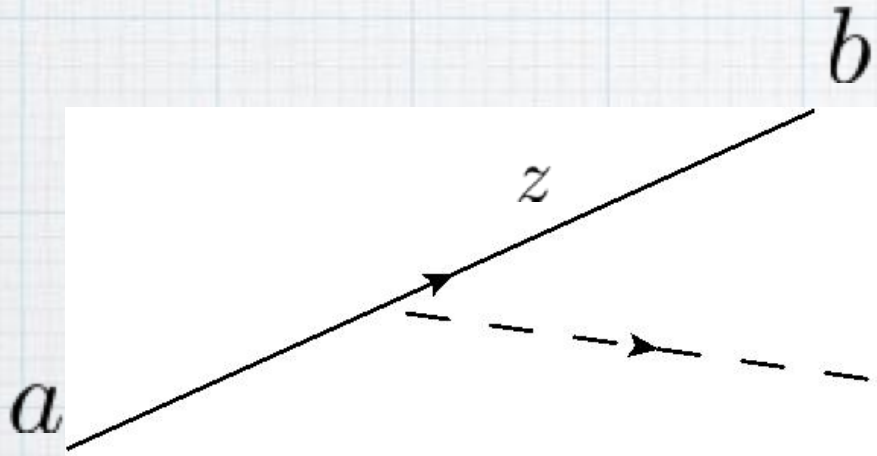
perturbative QCD

* asymptotic freedom allows us to apply perturbation theory for processes with high momentum transfer
* in this context, jets allows us to use the language of partons: we're inclusive over the details of the hadronic final state (pions, kaons, etc.)
* we will consider IRC safe observables and jet algorithms
* apply perturbative QCD to describe jet physics

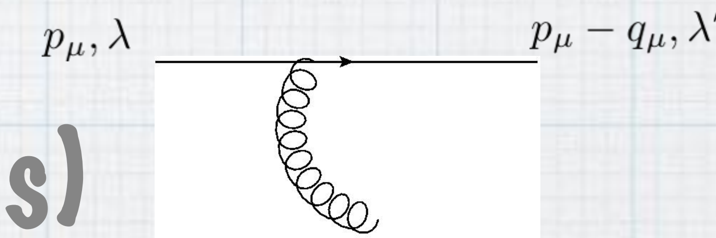
QCD in the soft/collinear limit

* factorization properties in both limits

* collinear limit (semi-classical)

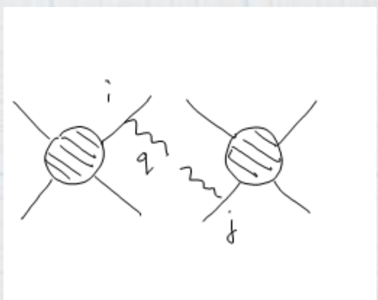


$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2} dz \; P_{ba}(z)$$



$$= 2 g p_\mu \delta_{\lambda\lambda'} T^a_{ij}$$

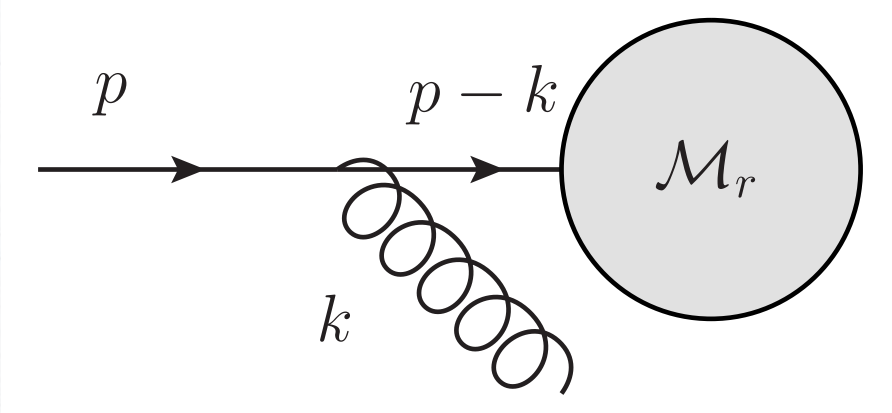
* soft limit (eikonal rules)



$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{2\pi}\frac{dE}{E}\frac{d\Omega}{2\pi}\sum_{ij} C_{ij} E^2 \, \frac{p_i \cdot p_j}{p_i \cdot q \; p_j \cdot q}$$

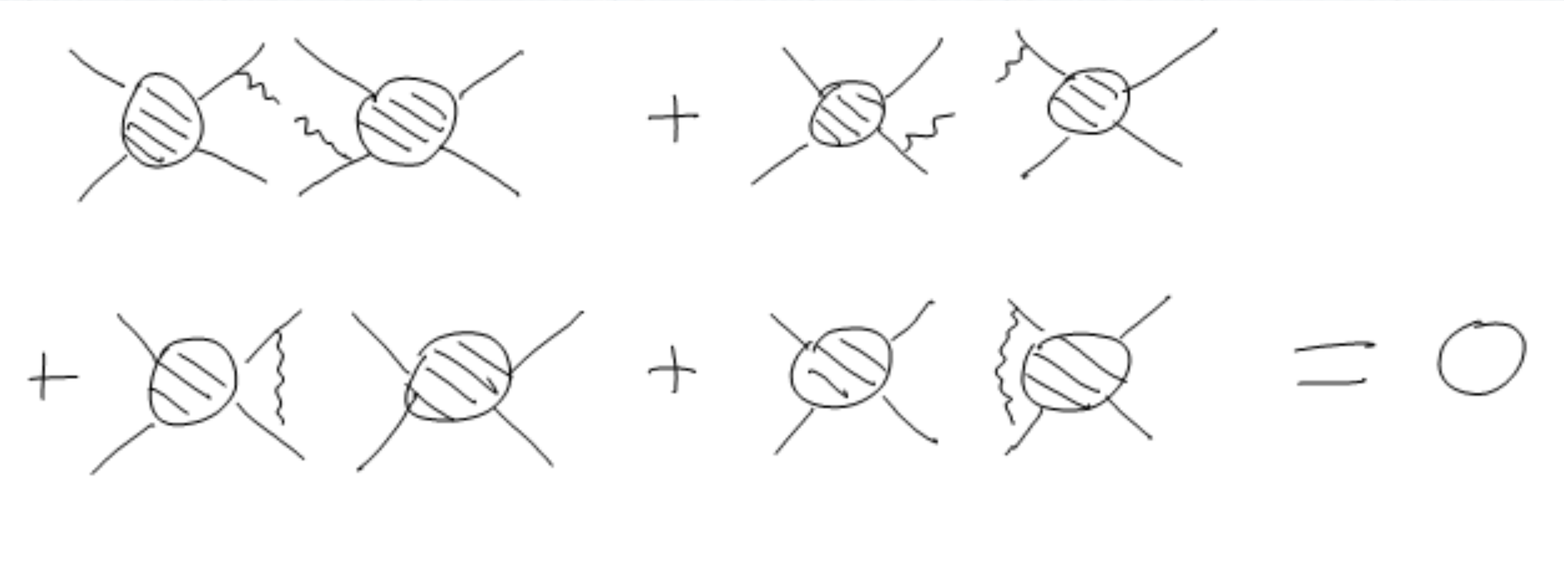
homework 3

* Considering the emission of a soft gluon as shown in the diagram below, derive the eikonal Feynman rule for a quark lin



IRC singularities

* QCD matrix elements are singular in the IRC limits
* rather general theorems ensure cancellation of singularities when real and virtual are added together



a model NLO calculation

* we want to calculate the cross-section for a jet observable at NLO (e^+e^- for simplicity)

$$\sigma(v) = \int d\Phi_n |M_{\rm LO}|^2 J_n + \int d\Phi_{n+1} |M_{\rm real}|^2 J_{n+1} + \int d\Phi_n |M_{\rm virt}|^2 J_n$$

* in the soft/collinear limit real and virtual are the same up to a sign

$$\sigma(v) \simeq \int d\Phi_n |M_{\rm LO}|^2 J_n + \int d\Phi_n |M_{\rm LO}|^2 \frac{d^3q}{(2\pi)^3} \frac{1}{2|q|} 2 g_s^3 C_F \frac{k_1 \cdot k_2}{k_1 \cdot q k_2 \cdot q} \left[J_{n+1} - J_n\right]$$

* IRC safety of the observable J guarantees cancellation of singularities

inclusive vs exclusive

* for observables that sum inclusively over IRC radiation this cancellation is perfect
* these observables can be safely computed order by order in the strong coupling, e.g. p_t distributions
* situation is different if the observable J restrict real radiation to a cornel of phase-space
* R/V cancellation is incomplete and large logarithms are left behind

$$\alpha_s \int_0^{Q_0} \frac{dk_t}{k_t}\bigg|_{\text{real}} - \alpha_s \int_0^{Q} \frac{dk_t}{k_t}\bigg|_{\text{virtual}} = -\alpha_s \int_{Q_0}^{Q} \frac{dk_t}{k_t}\bigg|_{\text{virtual}} = \alpha_s \ln \frac{Q_0}{Q}$$

jet properties

* we want to studies the properties of jets
* hence, we resolve a (high p_t) jet down to a smaller scale, e.g. its mass
* large logarithms appear invalidating the fixed-order expansion
* we need to reorganise the calculation so that we can consider any number of soft/collinear partons: resummation
* vast field with many approaches: dQCD, SCET, etc.

how do we model it ?

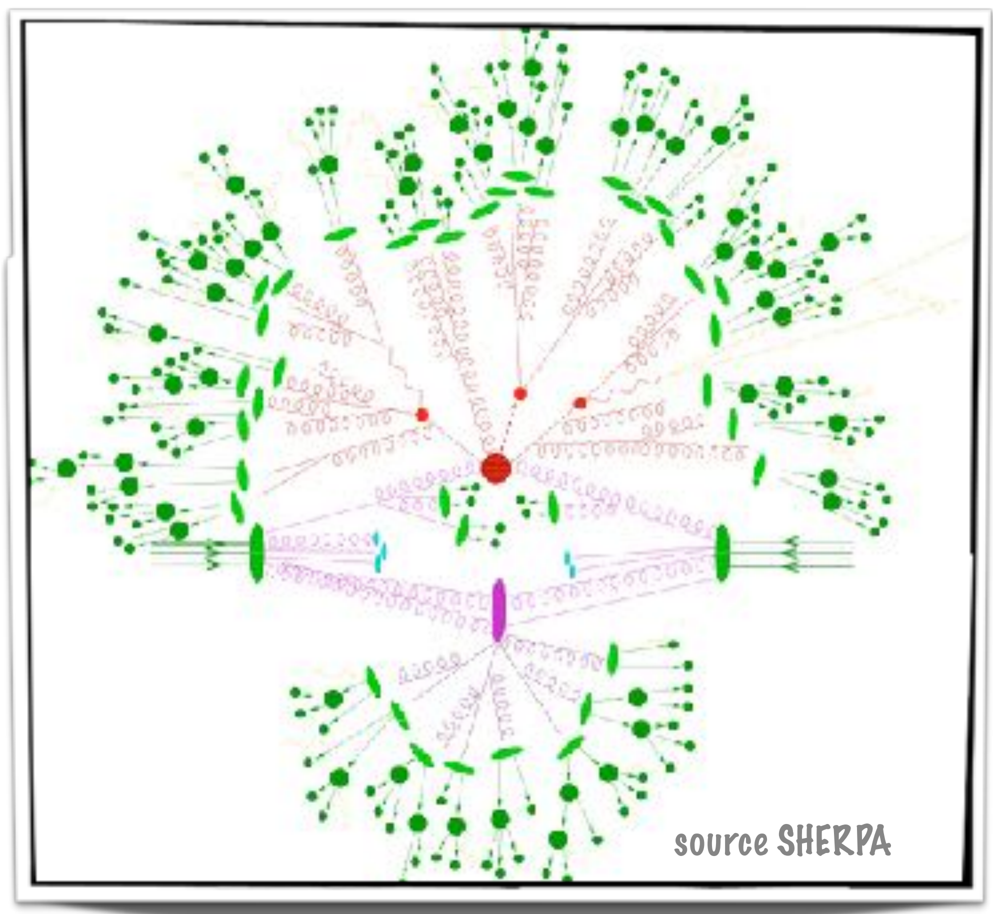
* jet properties: we want to compute x-sections and distributions with many particles in the final state
* fixed-order perturbation theory seems inadequate
* interesting physics happens at small angular separation and small energies
* all-order (resummed) calculations are possible and necessary !

Monte Carlo Parton Showers

emissions at small angles factorize

$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2} dz d\phi P(z,\phi)$$

we can write a computer program that simulates these classical branchings



how do we model it ?

* jet properties: we want to compute x-sections and distributions with many particles in the final state
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Analytic Resummation

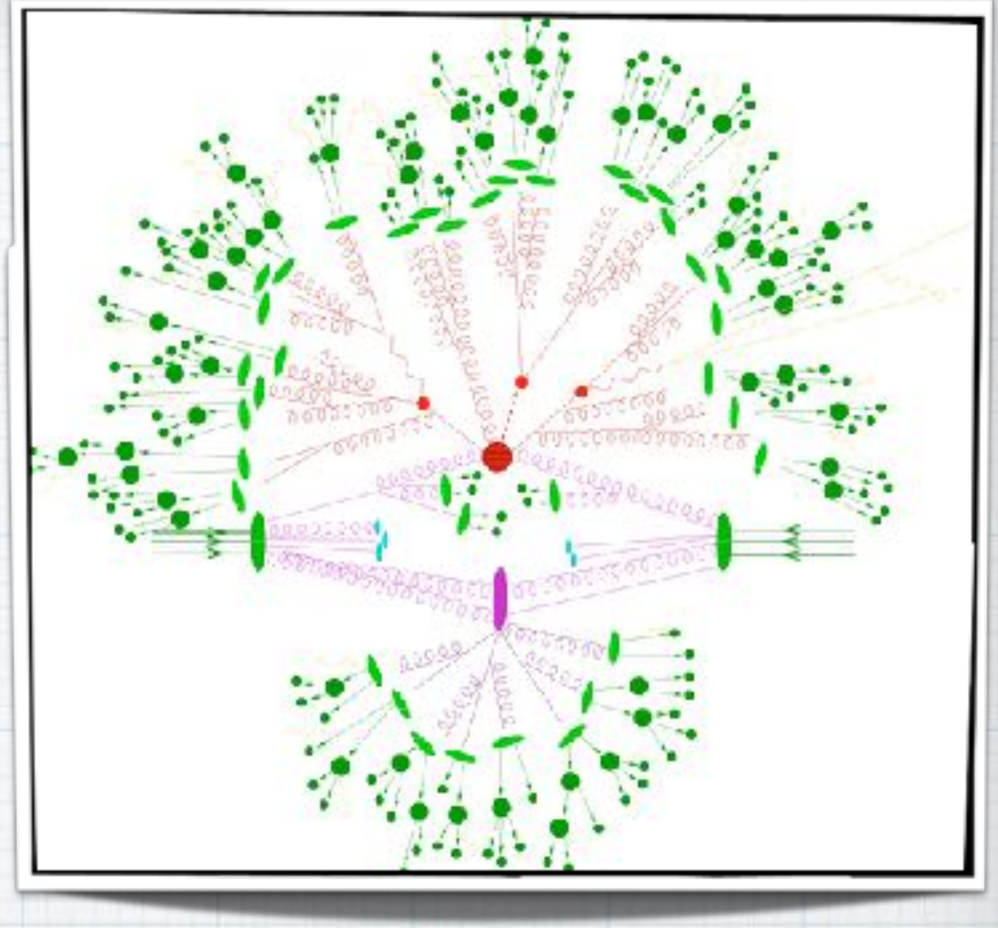
emissions at small angles factorize

$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} dz d\phi P(z,\phi)$$

soft emissions factorize in a subtle way

$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s}{2\pi} d\theta dz d\phi \sum_{i<j} C_{ij} D_{ij}(z,\theta,\phi)$$

$$\sigma_{res} = g_0 \exp[g_1(\alpha_s L)/\alpha_s + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots]$$



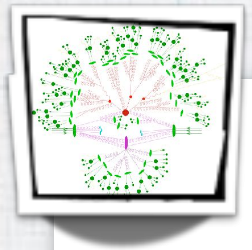
- powerful general-purpose tools
- provide fully differential events on which any observable can be measured
- interfaced with non-perturbative models to give a realistic description
- theoretical accuracy difficult to assess (often low)

VS

$$\sigma_{res} = g_0 \exp[g_1(\alpha_s L)/\alpha_s + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots]$$

- feasible for a limited number of observables
- well defined and improvable accuracy
- state-of-the art (resummation + fixed order)
- provide insights and understanding

the jet mass



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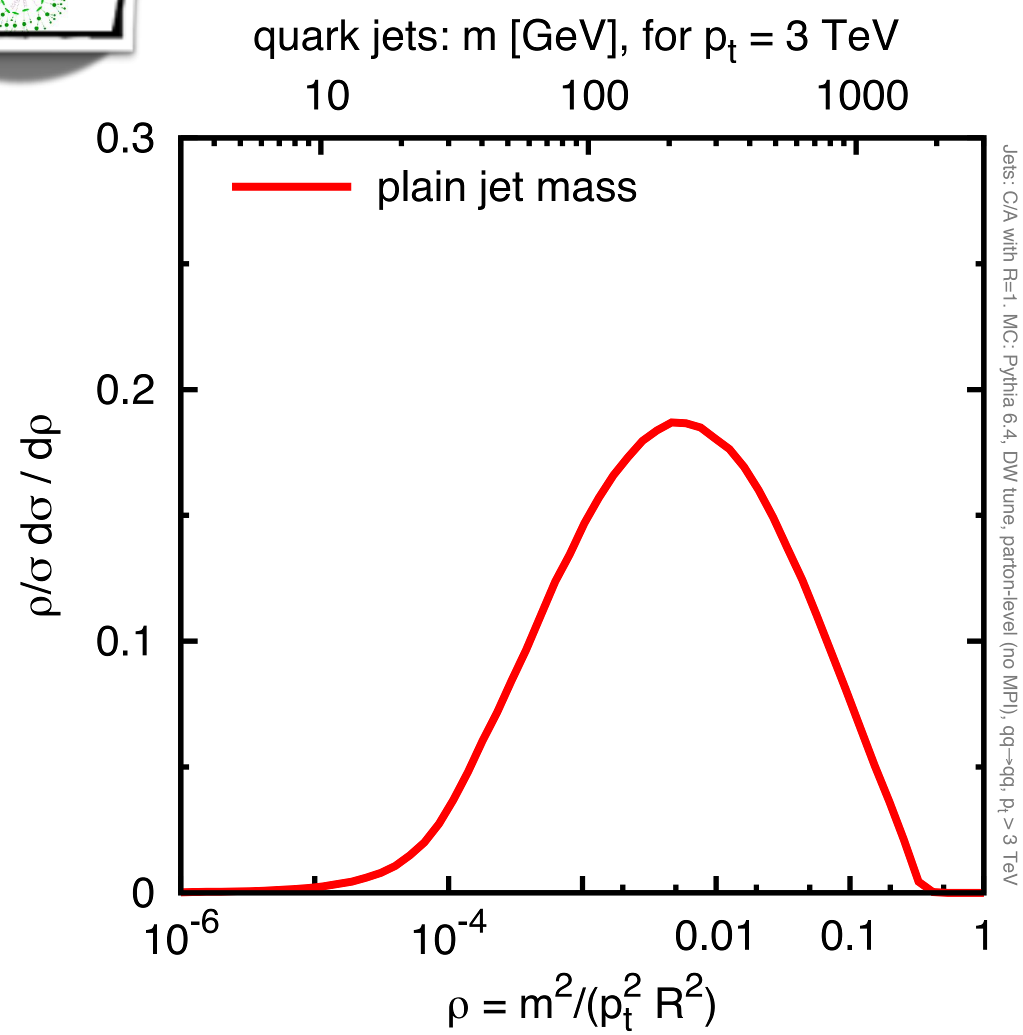
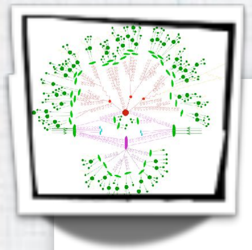


* plain jet mass: Sudakov peak, where does it come from ?
* let's do an easy calculation: one gluon emission in the collinear limit

the jet mass



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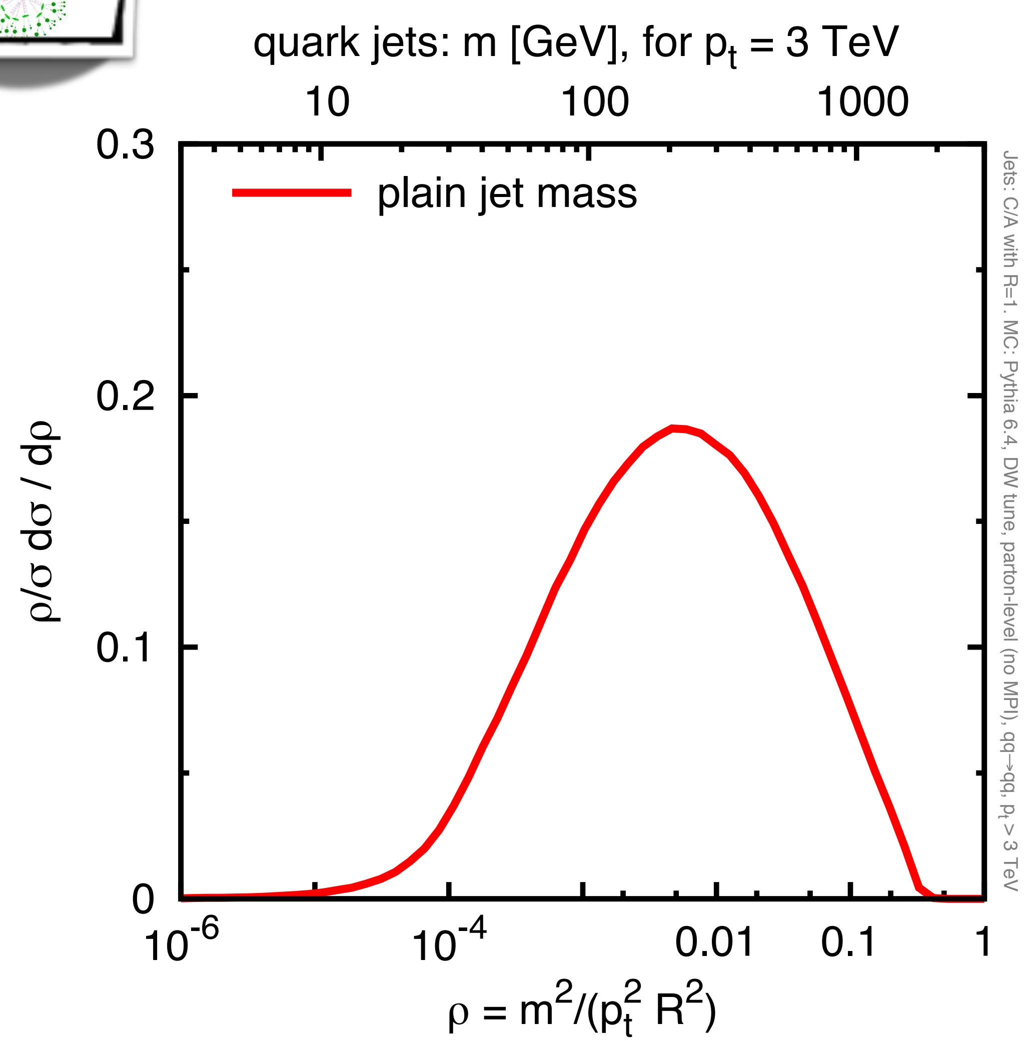

$$\frac{1}{\sigma}\frac{d\sigma^{\mathrm{LO}}}{dm^2} = \frac{\alpha_s}{2\pi}\int_0^{R^2}\frac{d\theta^2}{\theta^2}\int_0^1 dz P_{gq}(z) \times \delta\left(m^2 - z(1-z)\theta^2 p_T^2\right)$$

we demand a mass m

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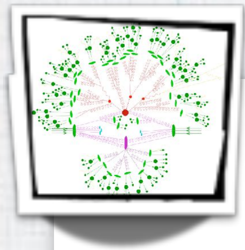
the jet mass



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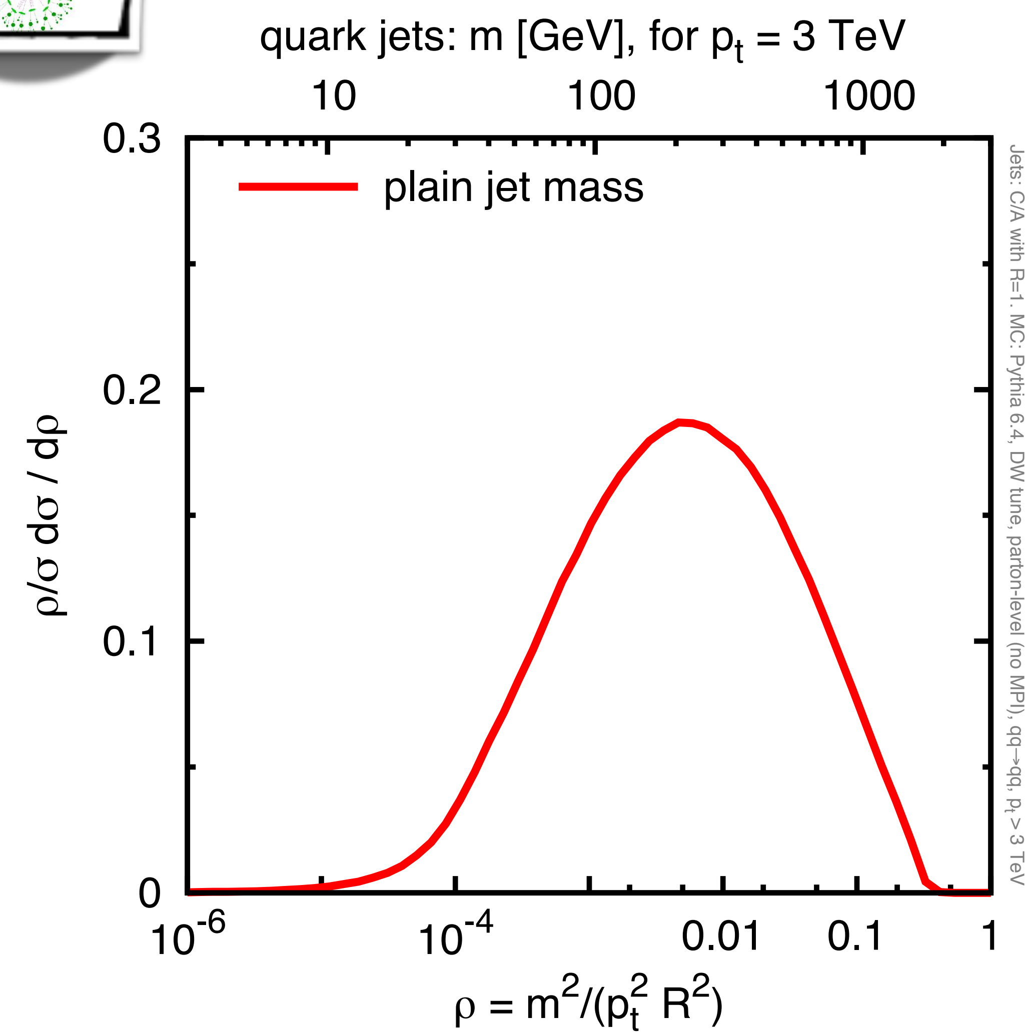

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we do the angular integral with the delta function

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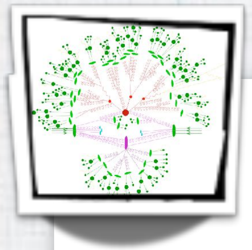
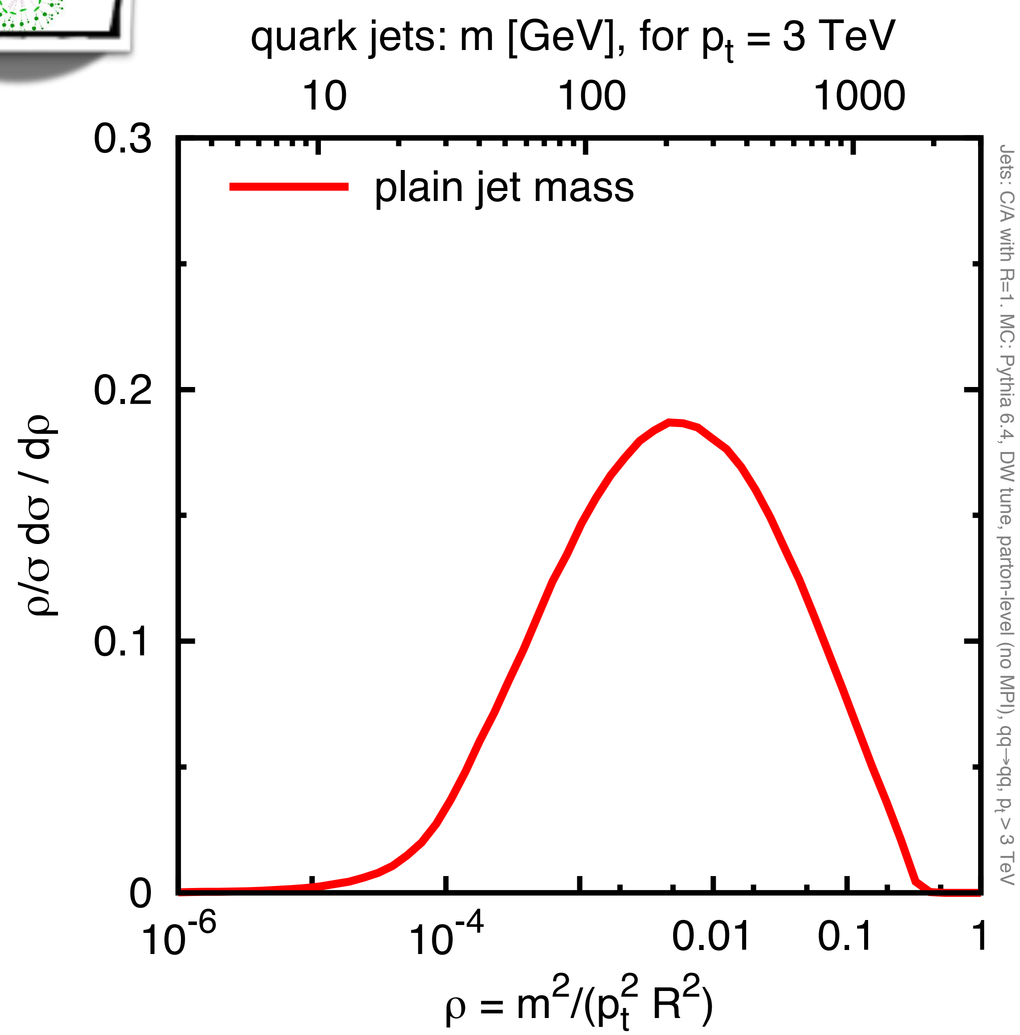
the jet mass

$\sigma_{res} = g_0 \exp[g_1(\alpha_s L)/\alpha_s + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots]$





$$
\begin{aligned}
\frac{1}{\sigma}\frac{d\sigma^{\mathrm{LO}}}{dm^2}
&= \frac{\alpha_s}{2\pi}\int_0^{R^2}\frac{d\theta^2}{\theta^2}\int_0^1 dz\,P_{gq}(z) \\
&\times \delta\left(m^2 - z(1-z)\theta^2 p_T^2\right) \\
&= \frac{\alpha_s C_F}{\pi} m^{-2}\int_{m^2/(p_T^2R^2)}^{1} dz \qquad P_{gq}(z) \\
&\simeq \frac{\alpha_s C_F}{\pi} m^{-2}\left[\ln\frac{p_T^2 R^2}{m^2} - \frac{3}{4}\right].
\end{aligned}
$$

* plain jet mass: Sudakov peak, where does it come from ?
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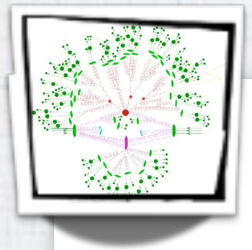

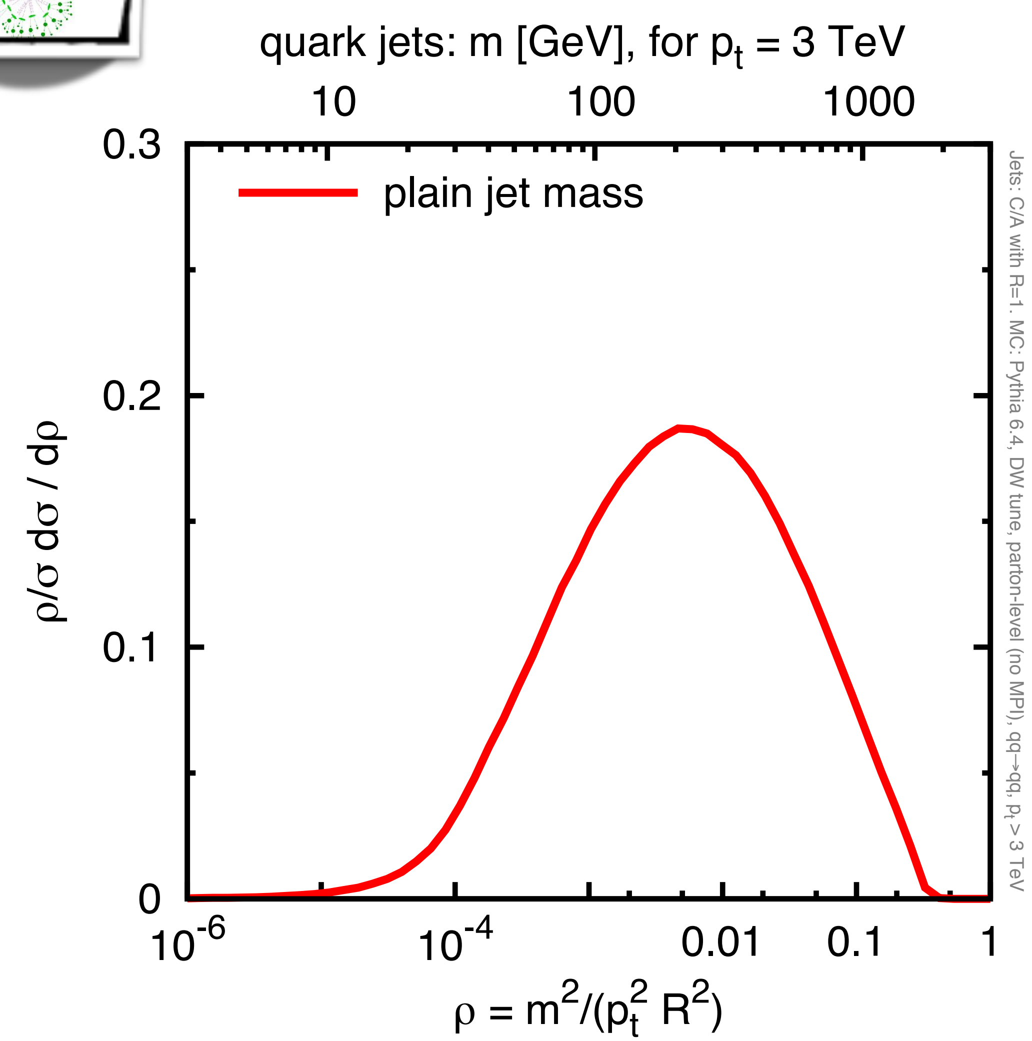

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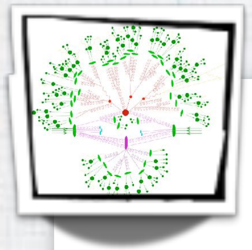
&\simeq \frac{\alpha_s C_F}{\pi} m^{-2}\left[\ln\frac{p_T^2 R^2}{m^2} - \frac{3}{4}\right]
\end{aligned}
$$

double log: soft & coll.

single log: hard coll.

- plain jet mass: Sudakov peak, where does it come from ?
- let's do an easy calculation: one gluon emission in the collinear limit

the jet mass



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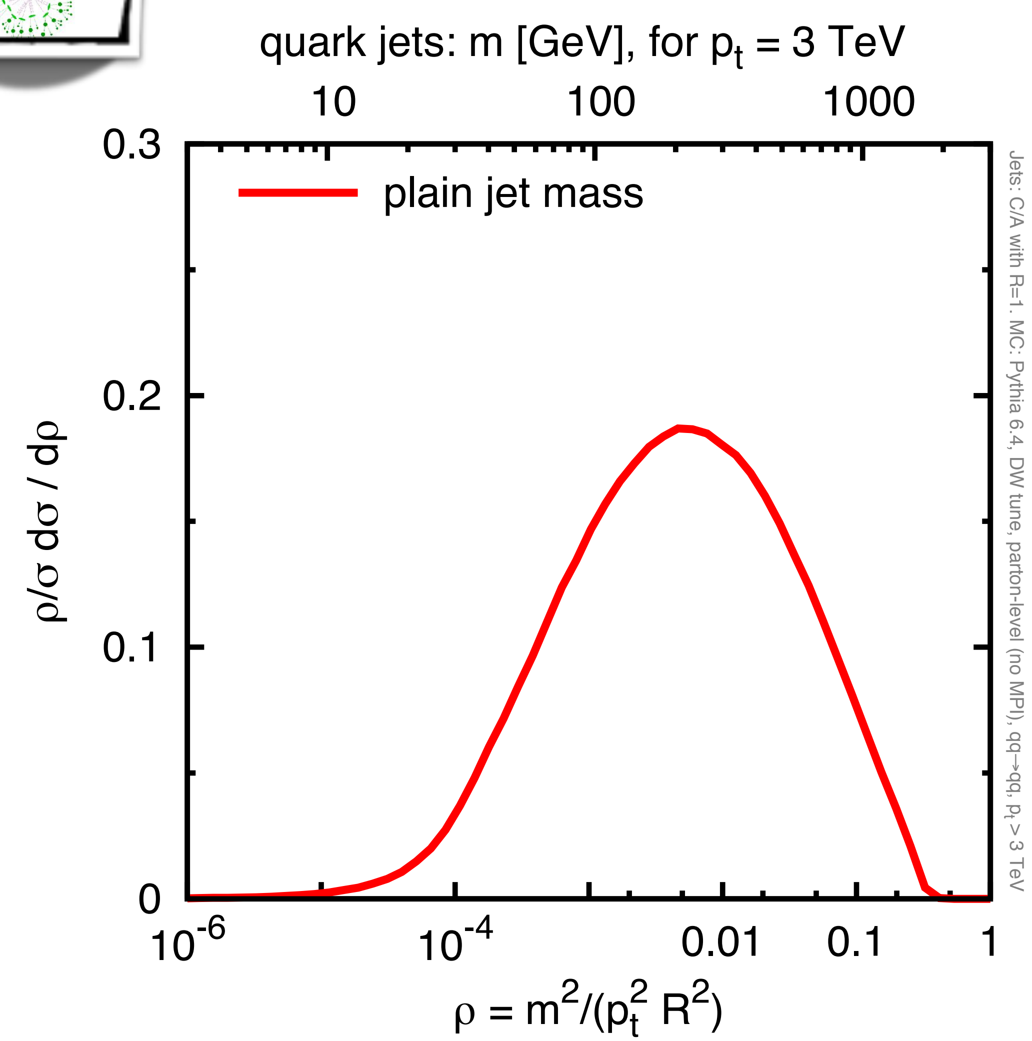
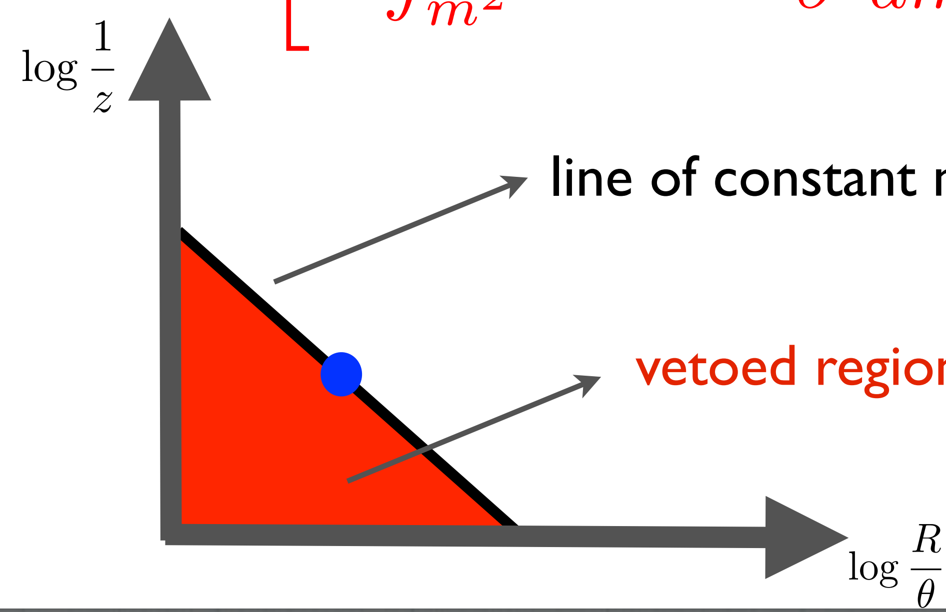
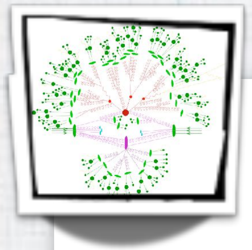


$$\frac{1}{\sigma}\frac{d\sigma^{\mathrm{LL}}}{dm^2} = \frac{1}{\sigma}\frac{d\sigma^{\mathrm{LO}}}{dm^2} \times \exp\left[-\int_{m^2}^{p_T^2 R^2} dm'^2 \frac{1}{\sigma}\frac{d\sigma^{\mathrm{LO}}}{dm'^2}\right]$$



* all-order leading logs: veto emissions which would give too big a mass
* exponential that gives the no-emission probability

the jet mass



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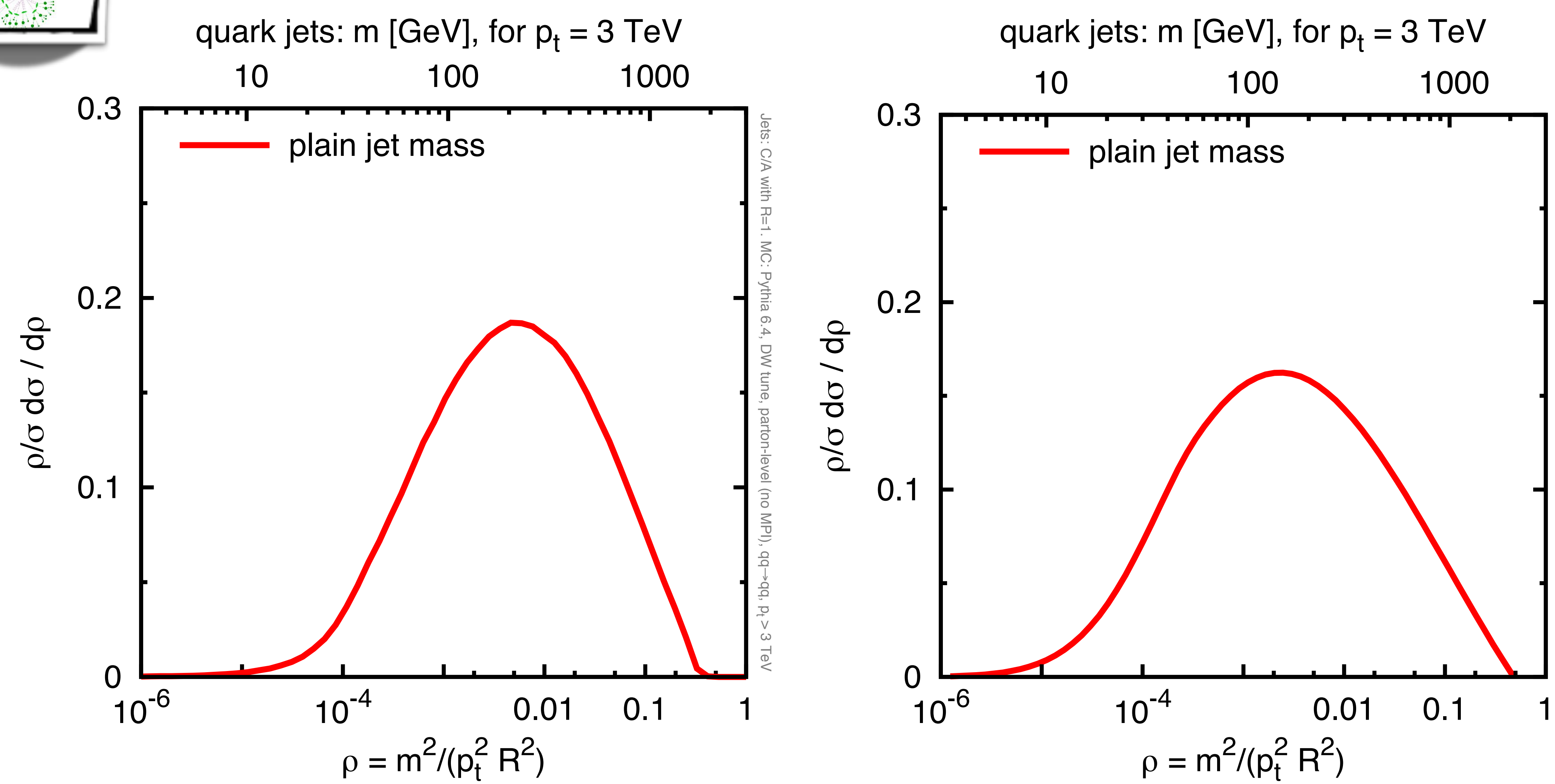


* all-order leading logs: veto emissions which would give too big a mass
* exponential that gives the no-emission probability

resummation

* the all-order calculation can be systemically improved

$$\sigma_{res} = g_0 \exp[g_1(\alpha_s L)/\alpha_s + g_2(\alpha_s L) + \alpha_s\, g_3(\alpha_s L) + \ldots]$$

* $g_1 \rightarrow$ leading logarithmic accuracy
* $g_2 \rightarrow$ next-to leading logarithmic accuracy
* $g_3 \rightarrow$ next-to next-to leading logarithmic accuracy
* $g_0 = 1 + \alpha_s C_1 + \ldots$

* g_1 is fairly simple: soft and collinear physics

* the structure of g_2 is already highly non trivial

collinear emissions

* collinear emissions are easy to deal with
* soft/collinear contributes to g_1 while hard collinear to g_2
* same calculation as before but with running coupling

$$g_{\text{coll}} = \int_{\rho}^{1} \frac{d\rho'}{\rho'} \int dz P_{gq}(z) \frac{\alpha_s(k_t)}{2\pi}$$

* very soft/collinear emissions are sensitive to the Landau pole: peak region receives important non-pert. contributions

soft emissions

* soft emissions at large angle are tough for (at least) 3 reasons
    * colour correlations
    * jet algorithm dependence
    * non-global logarithms
* all these effects result into NLL corrections, i.e. they enter the resummation function g_2

colour correlations

* remember that soft-factorization happens at the amplitude level

* in the soft limit, the soft gluon emission probability off an ensemble of hard partons can be written as

$$|\mathcal{M}|^2 = \mathcal{M}_0^\dagger \exp\left[\frac{\alpha_s}{2\pi}\left(-2\sum_{i<j} W_{ij}\, t_i \cdot t_j\right)\right]\mathcal{M}_0$$

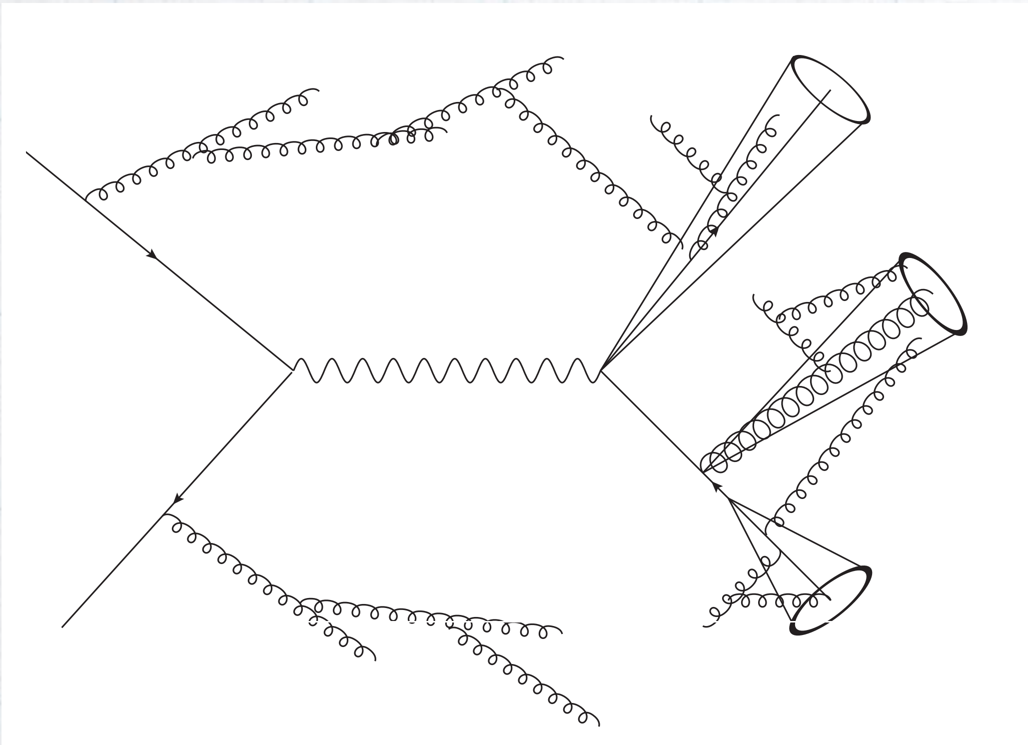
Kidonakis, Oderda, Sterman (1998)

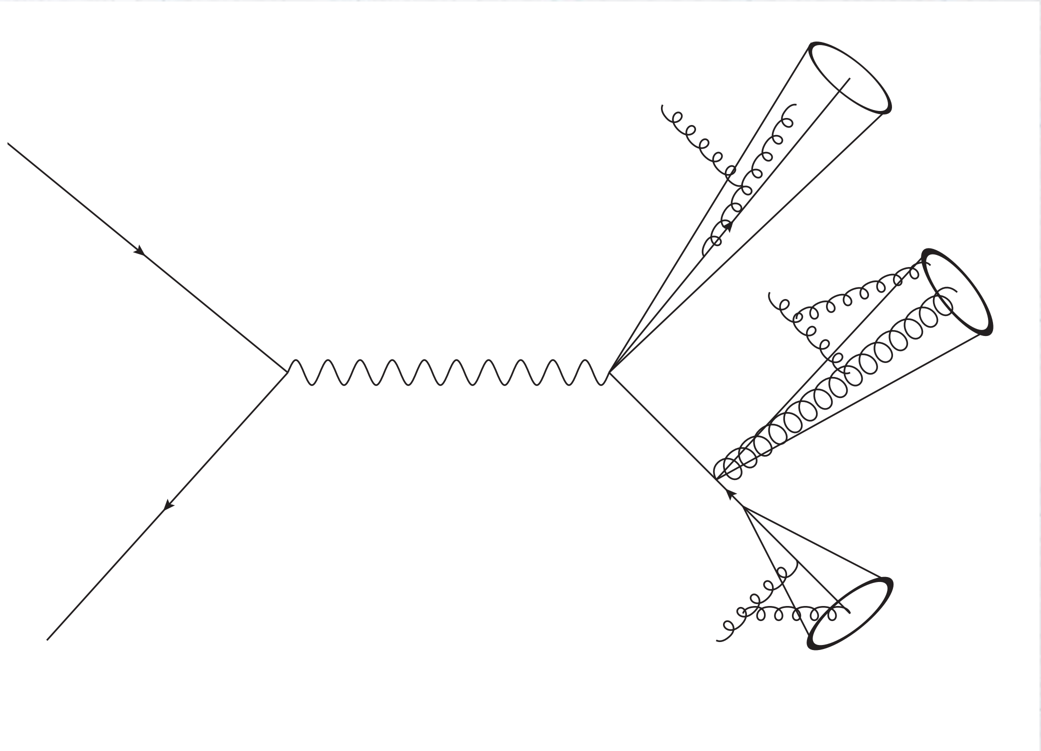
* the sum runs over all the possible dipoles in the hard scattering

* this gives rise to a matrix structure in colour space

* dimension of these matrices quickly increases with the number of hard directions

the small-R limit





* in the small-R limit a simpler picture emerges
* large-angle emissions are R-suppressed and each jet evolves independently
* finite R-corrections are at the percent level if R < 0.4, but they reach O(40%) for R=1.0
* they are dominated by $O(R^2)$ terms, while $O(R^4)$ are below 1% (even at R=1.0)

homework 4

* the bulk of the $O(R^2)$ contribution to the jet mass spectrum arises from the initial-state radiation . Calculate the contribution to the jet mass from the dipole which is formed by the initial-state partons [hint: it's easier to work with rapidity and azimuth]

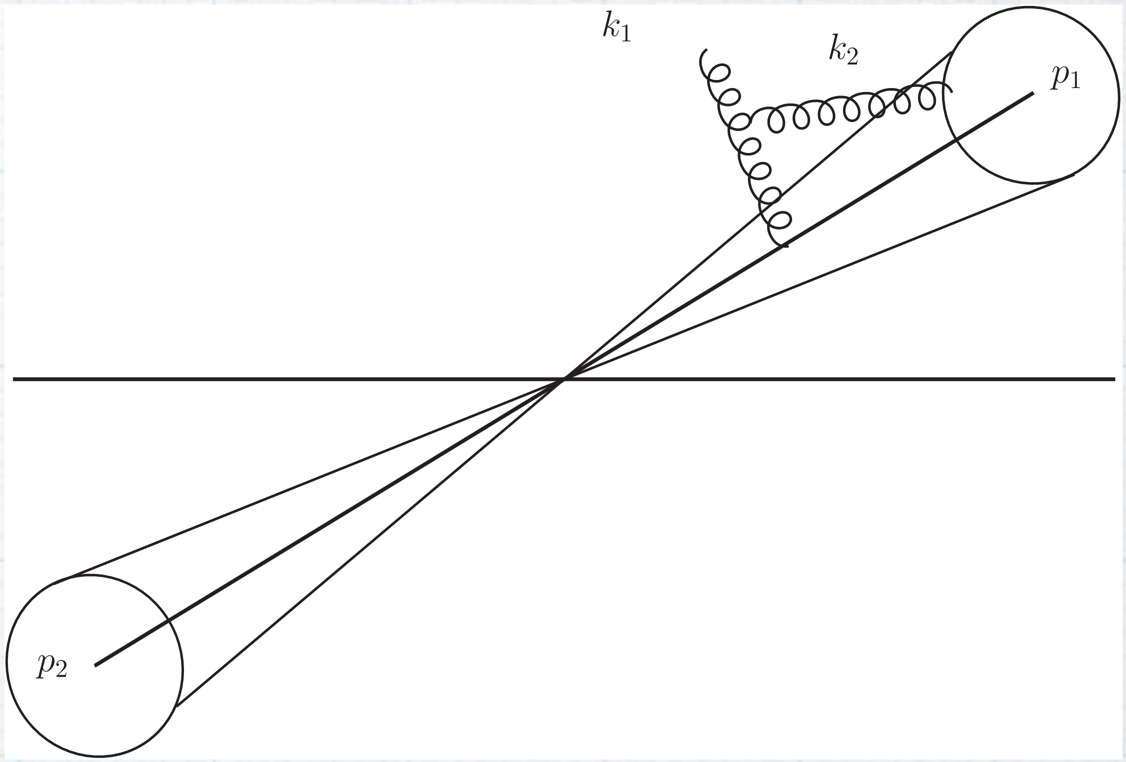
jet-algorithm dependence

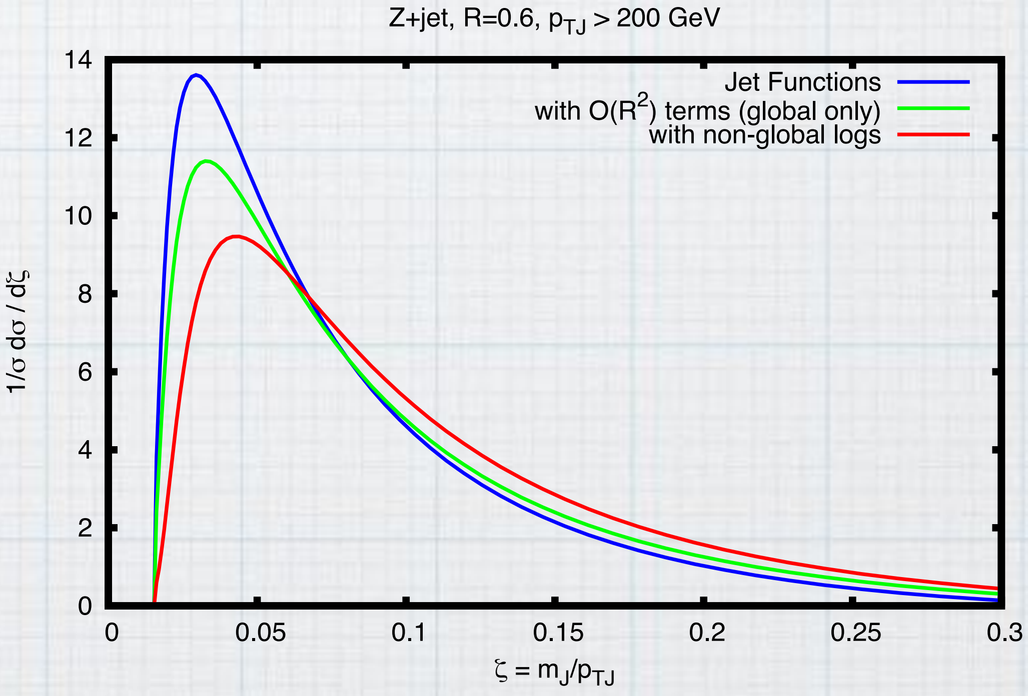
* does the resummed expression depend on the jet algorithm?
* two gluons are recombined with each other is their distance is smaller than R and the recombined momentum essentially lies along the harder one
* as a result a hard gluon can pull a softer one out of the jet
    * this does not happen if we use anti-k_t algorithm: two soft gluons are always far apart with this measure
    * with C/A and k_t a soft gluon outside the jet can recombine with a softer one inside, effectively pulling out the latter from the jet
* while the jet mass for anti-k_t does exponentiate, the corresponding distributions for C/A and k_t differ by single logs

calculation 2

non-global logarithms

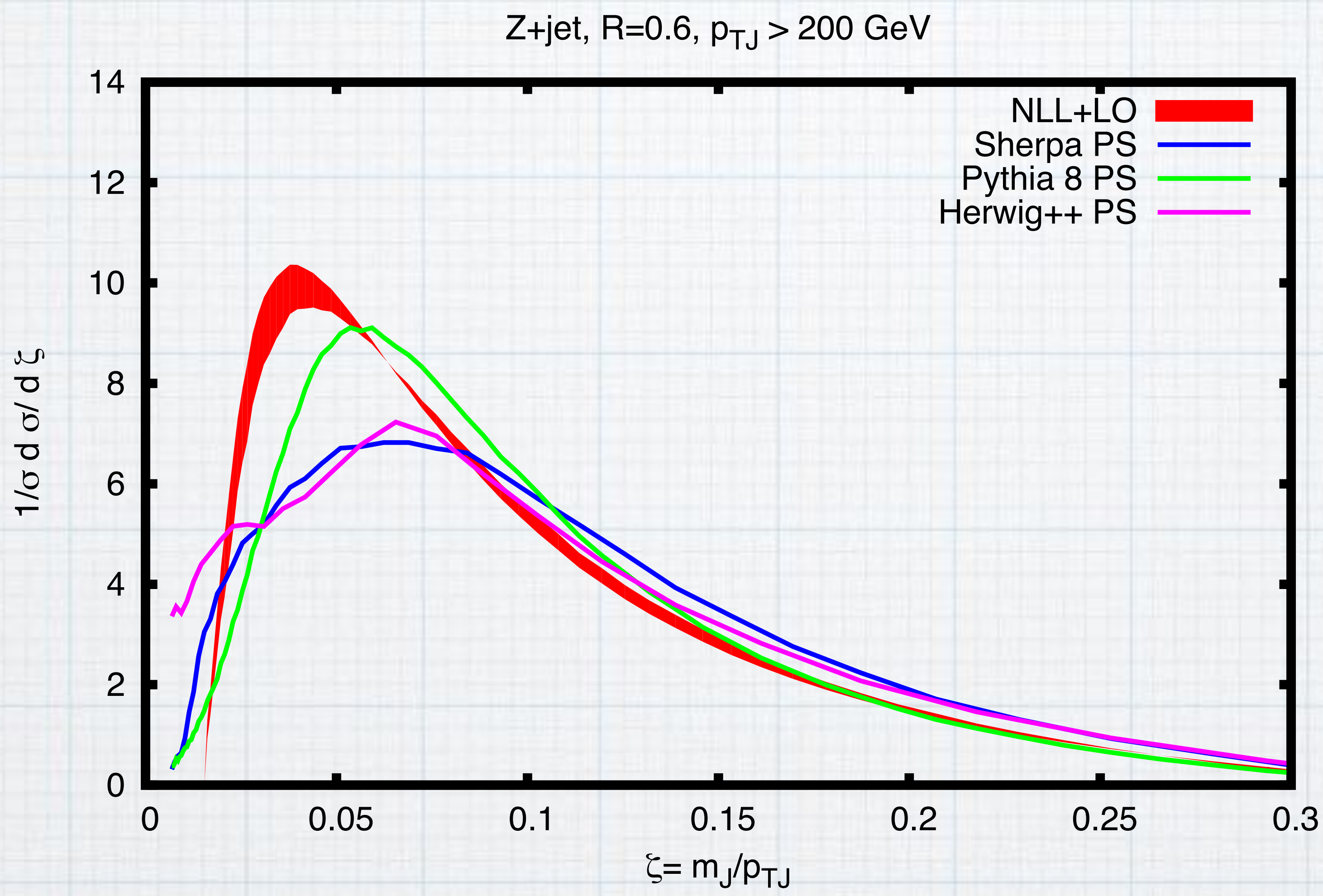
* **but**, even if we use anti-k_t, exponentiation of the independent emission is not the whole story
* the jet-mass is a non-global observable: it receives single log corrections from correlated emission
* this is a $C_F C_A$ term and it's missed by single gluon exponentiation
* in principle we need to consider any number of gluons outside the jet
* colour structure becomes intractable, so the resummation is performed in the large Nc limit





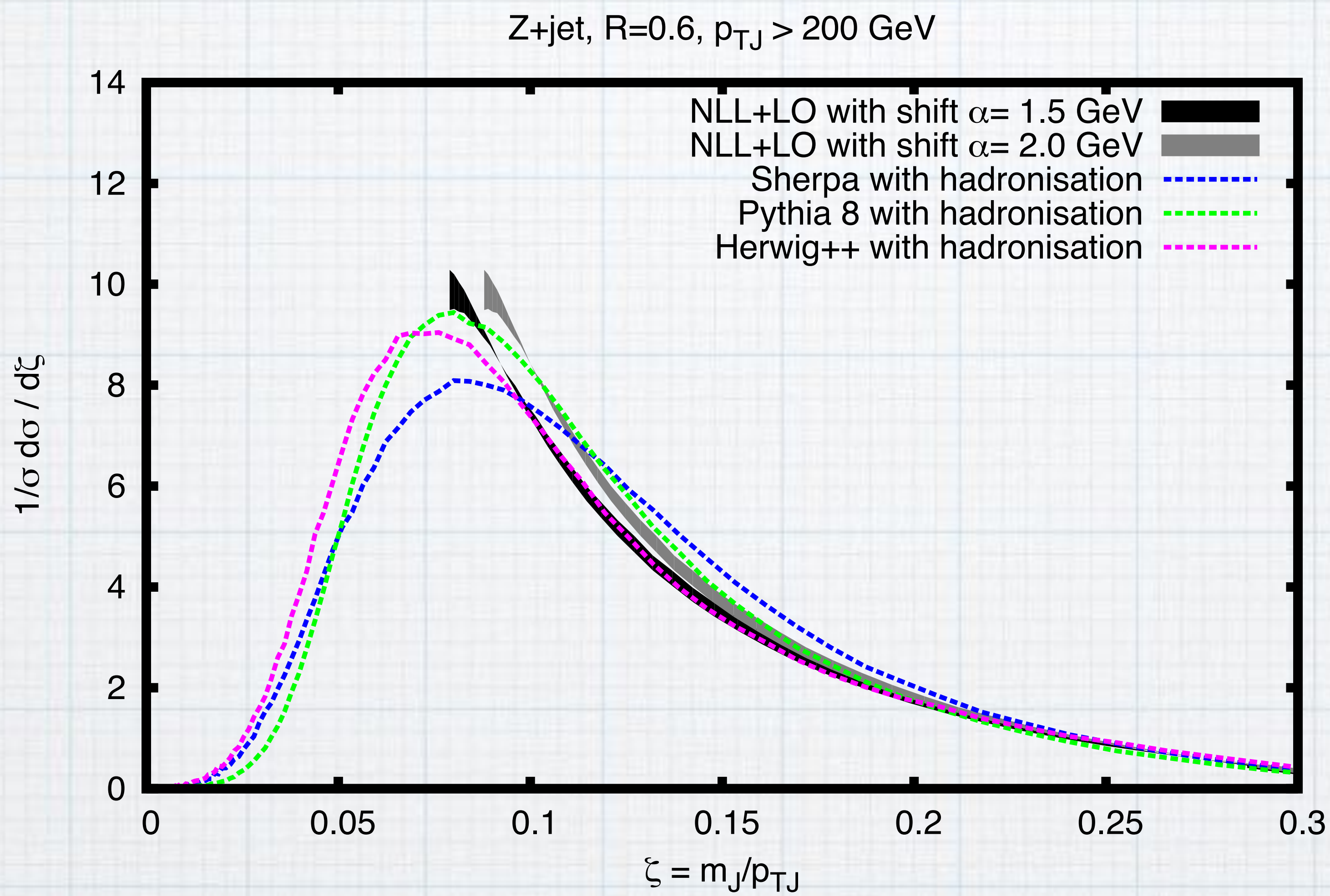
calculation 3

putting things together



- comparison between NLL and MCs
- general agreement but details depend on the shower

putting things together



* better agreement between at the hadron level
* can capture most of it by a shift of the perturbative result
* things get much worse with UE and pile-up!

summary of lecture 2

* jet properties can be computed using perturbative QCD provides
* precision: NLL is well-established and NNLL is also possible (although non-global logs are problematic)
* jet mass suffers from non-pert. corrections hadronisation, UE and pile-up