Precision QCD at the LHC

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Outline

Motivation

- Essential ingredients of pQCD
- NNLO top pair production
- NNNLO inclusive Higgs production
- Summary

Higgs production mechanism @ LHC

• Gluon fusion (loop induced), dominant Higgs production mechanism at the LHC



• Compute in pQCD in an EFT, top quark is infinitely heavy and has been integrated out. Higgs boson is coupled directly to gluons via a 5-dim effective operator

$$\mathcal{L}_{eff} = \mathcal{L}_{SM,5} - \frac{C}{4} H G^a_{\mu\nu} G^{a\mu\nu}$$

other $n_f = 5$ flavours treated massless



 \bullet Corrections due to finite top quark mass known up to NNLO, at most of the order of 1%

Higgs production mechanism @ LHC (NNLO)

 \bullet NLO QCD corrections amount to 80-100% of the LO contributions at LHC energies





R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. 88 (2002) 201801

V. Ravindran, J. Smith, and W. L. van Neerven, Nucl. Phys. B665 (2003) 325

• Stability of perturbative expansion of Higgs boson production x-sec at LHC?

Exclusion Plot for Higgs mass



• Discovery of the Higgs boson, the SM is a fully predictive theory, with all its parameters determined experimentally

Stability of EW vacuum

• At tree-level the SM Higgs potential has an absolute minimum corresponding to the EW vacuum

• In the absence of any new physics, the running of the Higgs quartic coupling λ from $M_W \rightarrow M_P$ is governed by loop corrections, which changes the picture drastically



• Precise evolution of λ strongly depends on the values of M_h , M_t , α_s • For current best fit values of the SM parameters, λ changes sign at large RG scale $\sim 10^{10}$ GeV and reaches a *-ive* min at $10^{16} - 10^{18}$ GeV

(M_h, M_t) parameter space, EW vacuum stability

• Metastable ($\tau_{EW} > \tau_U$, yellow) or unstable ($\tau_{EW} < \tau_U$, red); τ_{EW} is the EW vacuum life time and τ_U is the age of the universe



- ullet Ellipses give the experimental values at 1, 2 and 3 σ
- Demonstrate the need of a very precise calculation of the stability bound to determine the properties of the EW vacuum

Implications on EW vacuum (NNLO)

• Condition of absolute stability of EW vacuum, $\lambda(M_P) \ge 0$ when SM extrapolated upto M_P puts lower bound on

$$m_h \; [\text{GeV}] > 129.4 + 1.4 \left(rac{m_t \; [\text{GeV}] - 173.1}{0.7}
ight) - 0.5 \left(rac{lpha_s(m_Z) - 0.1184}{0.0007}
ight) \pm 1.0 \, GeV$$

State of the art NNLO stability analysis in the SM involves the RGE of all couplings constants upto 3-loop level



 $M_h > 129.4 \pm 1.8 \; \mathrm{GeV}$

• EW vacuum is stable or not up to the largest possible high-energy scale, relies on a precise determination of m_h , m_t and α_s

Degrassi, Vita et. al. JHEP08(2012)098

Bezrukov, Kalmykov, Kniehl, Shaposhnikov JHEP10(2012)140

Factorisation of the hard scattering process

- Perturbative evaluation of the factorisation formula is based on a power series expansion in $\alpha_s(\mu)$
- An inclusive hard scattering process at the LHC

$$P(p_1) + P(p_2) \rightarrow t\overline{t}(Q, \{\cdots\}) + X$$

• At short distances, asymptotic freedom in QCD guarantees that, the partons in hadron are almost free, and are sampled essentially one at a time in hard collisions— QCD improved parton model



• Hadronic cross section for production of $t\bar{t}$ factorises, into a convolution of $\hat{\sigma}^{ij \to t\bar{t}}(x_1p_1, x_2p_2; \alpha_s, \mu_F, \mu_R)$ and $f_{i/P}(x_1, \alpha_s, \mu_F)$

$$\sigma(p_1, p_2) = \sum_{ij} \int_0^1 \int_0^1 dx_1 \ dx_2 \ f_{i/P}(x_1) \ f_{j/P}(x_2) \ \hat{\sigma}^{ij}(x_1, x_2) + \mathcal{O}(\Lambda/Q)$$

i, j are partons carrying a fraction $x_{1,2}$ of the proton momentum

- Strong coupling $\alpha_s(\mu_R^2)$, depends on the UV renormalisation scale
- Parton Distribution Functions (PDF) $f_{a/P}(x, \mu_F^2)$
- Stability of the perturbative expansion $\hat{\sigma}(x_1, x_2, \alpha_s, \mu_R, \mu_F)$
- Missing higher terms as result of truncation of the perturbative expansion

 $\alpha_s(m_z^2) = 0.1181 \pm 0.0011$

 $[\delta \alpha_s(m_z^2)/\alpha_s(m_z^2) \approx 1\%]$

Strong coupling $\alpha_s(\mu)$

• Of all fundamental constants in nature α_s is the least precisely known $\circ \ \delta G/G \approx 10^{-5} \qquad \circ \ \delta G_F/G_F \approx 10^{-8} \qquad \circ \ \delta \alpha/\alpha \approx 10^{-10}$

• Free parameter of the QCD Lagrangian. Its evolution governed by renormalisation group equation. However its value at a given reference scale must be determined from experimental data

• Determination of the sign of the leading 1-loop β_0 and shortly later 2-loop corrections β_1 , lead to the discovery of asymptotic freedom of non-Abelian gauge theories and paved the way for establishing QCD as the theory of strong interaction

Scale dependence of strong coupling $\alpha_s(\mu)$

 β function controls, evolution of $\alpha_s(\mu)$, known to 5-loops in $\overline{\mathrm{MS}}$ scheme

$$\frac{\partial a_s}{\partial \ln \mu^2} = \beta(a_s) = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 - \beta_4 a_s^6 \cdots$$

For physical case $N_c = 3$ and n_f is number of active quark flavors

$$\begin{array}{rcl} \beta_{0}=&11-\frac{2}{3}n_{f} & \beta_{1}=102-\frac{38}{3}n_{f} & \beta_{2}=\frac{2857}{2}-\frac{5038}{20}n_{f}+\frac{325}{4}n_{f}^{2} \\ \beta_{3}=&\left(\frac{149753}{6}+3564\zeta_{3}\right)-\left(\frac{1078361}{162}+\frac{6503}{27}\zeta_{3}\right)n_{f}+\left(\frac{50065}{162}+\frac{6472}{81}\zeta_{3}\right)n_{f}^{2}+\frac{1093}{729}n_{f}^{3} \\ \text{4-loop} & Ritbergen, Vermaseren, Larin Phys. Lett. B400 (1997) 379 \\ & Czakon, Nucl. Phys. B710 (2005) 485 \end{array}$$

$$\beta_{4} = \frac{1}{4^{5}} \left(\frac{8157455}{16} + \frac{621885}{2} \zeta_{3} - \frac{88209}{2} \zeta_{4} - 288090 \zeta_{5} \right. \\ \left. + \left[-\frac{336460813}{1944} - \frac{4811164}{81} \zeta_{3} + \frac{33935}{6} \zeta_{4} + \frac{1358995}{27} \zeta_{5} \right] n_{f} \right. \\ \left. + \left[\frac{25960913}{1944} + \frac{698531}{81} \zeta_{3} - \frac{10526}{9} \zeta_{4} - \frac{381760}{81} \zeta_{5} \right] n_{f}^{2} \right. \\ \left. + \left[-\frac{630559}{5832} - \frac{48722}{243} \zeta_{3} + \frac{1618}{27} \zeta_{4} + \frac{460}{9} \zeta_{5} \right] n_{f}^{3} + \left[\frac{1205}{2916} - \frac{152}{81} \zeta_{3} \right] n_{f}^{4} \right] \right] \right]$$

5-loop Baikov, Chetyrkin, K\"uhn PRL 118 (2017) 082002 [1606.08659] \\ Herzog, Ruiil, Ueda, Vermaseren, Vogt JHEP 02 (2017) 090 [1701.01404]

Renormalization constants (five-loop order)

- ghost-ghost-gluon vertex; ghost propagator; gluon propagator
- Typical 5-loop diagrams contributing to different color structures • n_f^3 term



 $\circ n_f^4$ term



 n_f is the number of active flavours, quark is active if $m_f \ll \mu$

Numerical value of the coefficients for $n_f = 3 - 6$

$$\overline{\beta}(n_f = 3) = 1 + 1.78 a_s + 4.47 a_s^2 + 20.99 a_s^3 + 56.59 a_s^4$$

$$\overline{\beta}(n_f = 4) = 1 + 1.54 a_s + 3.05 a_s^2 + 15.07 a_s^3 + 27.33 a_s^4$$

$$\overline{\beta}(n_f = 5) = 1 + 1.26 a_s + 1.47 a_s^2 + 9.83 a_s^3 + 7.88 a_s^4$$

$$\overline{\beta}(n_f = 6) = 1 + 0.93 a_s - 0.29 a_s^2 + 5.52 a_s^3 + 0.15 a_s^4$$

- N⁴LO correction much smaller than N³LO contribution, perturbative convergence of QCD β -function is pretty good
- Strong coupling is characterized by two important features:
 - asymptotic freedom $\alpha_s
 ightarrow 0$ UV
 - confinement $\alpha_s \to \infty$ IR

Extractions based on at least NNLO QCD predictions

• Top-pair x-sec (NNLO+NNLL) first hadronic collider measurements at LHC and Tevatron that constraint $\alpha_s(m_z)$

EPJ C 77 (2017) 778

• Precise determination of α_s is essential to reduce theoretical uncertainties for any high-precision pQCD observables that depends on higher powers of α_s



Measurements of α_s as a function of the respective energy scale Q

• Decoupling of heavy quarks at $\mu < m_F$ would need an effective QCD with $(n_f - 1)$ flavours as compared to a full QCD with n_f flavours at $\mu > m_F$

• Curves are QCD predictions for the combined world average of $\alpha_s(M_Z)$, is (n+1)-loop running with n-loop threshold matching at the heavy quark mass threshold



• From M_{τ} to M_Z the number of active flavor matching is need at $n_f = 3 \rightarrow 4$ at $\mu = 3$ GeV and $n_f = 4 \rightarrow 5$ at $\mu = 10$ GeV

• Starting with the $\alpha_s(M_{\tau})$ one arrives at $\alpha_s(M_Z)$, after running and matching at the charm and bottom threshold

Number of loops	$\alpha_s^{(3)}(M_\tau)$	$\alpha_{s}^{(5)}(M_{Z})$	$\alpha_{s}^{(5)}(M_{H})$
3	0.33 ± 0.014	0.1200 ± 0.0016	0.1145 ± 0.0014
4	0.33 ± 0.014	0.1199 ± 0.0016	0.1143 ± 0.0014
5	0.33 ± 0.014	0.1198 ± 0.0016	0.1143 ± 0.0014

• Excellent agreement between α_s values from vastly different energy scales, persists in higher orders for fit to electroweak precision data (collected in Z-boson decays)

 $\alpha_s^{(5)}(M_Z) = 0.1196 \pm 0.0030$

Asymptotic freedom ensures application of perturbative QCD to study the dynamics of quarks and gluons at high energies— but in nature we have to deal with hadrons

Parton Distribution Functions (PDF) $f(x, \mu^2)$

- Domain of the PDFs is the initial state of hadronic collisions
- PDFs are universal and their scale dependence is determined by pQCD via the DGLAP evolution equations, but being non-perturbative objects they have to be extracted from global fits to hard scattering data
- Accurate PDFs are an essential ingredient for LHC phenomenology. Uncertainties in PDF, limits:
 - accuracy of extraction of Higgs coupling at the LHC
 - reaches of massive BSM particles in the TeV scale
 - accuracy of the determination of fundamental parameters
 - • •

Evolution of PDFs

• $f_i(x, \mu^2)$ are not calculable in pQCD, but its scale dependence is perturbatively controlled

• Universality allows for the determination of PDFs in global fits to experimental data

• Independence of any physical observable on scale μ gives rise to evolution equation for PDFs, which is a system of coupled integro-differential equations corresponding to different possible parton splittings

$$\frac{d}{d\ln\mu^2} \begin{pmatrix} f_{q_i}(x,\mu^2) \\ f_g(x,\mu^2) \end{pmatrix} = \frac{\alpha_s}{2\pi} \sum_j \int_x^1 \frac{dz}{z} \begin{pmatrix} P_{q_iq_j}(z,\alpha_s) & P_{q_ig}(z,\alpha_s) \\ P_{gq_j}(z,\alpha_s) & P_{gg}(z,\alpha_s) \end{pmatrix} \begin{pmatrix} f_{q_j}(\frac{x}{z},\mu^2) \\ f_g(\frac{x}{z},\mu^2) \end{pmatrix}$$

 \bullet Splitting functions P_{ij} are universal quantities, calculable in pQCD to an order in α_s

$$P_{ij}(z, \alpha_s) = P_{ij}^{(0)} + \alpha_s P_{ij}^{(1)} + \alpha_s^2 P_{ij}^{(2)} + \cdots$$

 Calculational tools, necessary for a consistent NNLO pQCD treatment of Tevatron & LHC hard scattering cross sections, available since 2004 Moch, Vermaseren, Vogt NPB 688 (2004) 101; 691 (2004) 129
 First results on N3LO splitting function moments are now available Ruijl, Ueda, Vermaseren, Davies, Vogt 1605.08408 Moch, Ruijl, Ueda, Vermaseren, Vogt 1707.08315

Data set (x, Q^2) coverage included in global analysis



• low-x, low- Q^2 : inclusive HERA st fn, • high-x, low- Q^2 : fixed tgt DIS st fn,

• high-x, large- Q^2 : collider jet, DY, $t\bar{t}$, • high-x, very high Q^2 (few TeV): inclusive jet production ATLAS, CMS

Hard-scattering processes used to constrain PDFs

 \bullet Hadron–level process, dominant parton–level process, partons which are constrained in each case and the corresponding \times range

	Process	Subprocess	Partons	x range
Fixed Target	$\ell^{\pm}\left\{p,n\right\}\to\ell^{\pm}+X$	$\gamma^* q \rightarrow q$	q, \bar{q}, g	$x \gtrsim 0.01$
	$\ell^{\pm} n/p \rightarrow \ell^{\pm} + X$	$\gamma^* d/u \rightarrow d/u$	d/u	$x \gtrsim 0.01$
	$pp \rightarrow \mu^+ \mu^- + X$	$u\bar{u}, d\bar{d} \rightarrow \gamma^*$	\tilde{q}	$0.015 \leq x \leq 0.35$
	$pn/pp \rightarrow \mu^+\mu^- + X$	$(u\bar{d})/(u\bar{u}) \rightarrow \gamma^*$	d/ū	$0.015 \lesssim x \lesssim 0.35$
	$\nu(\bar{\nu}) N \rightarrow \mu^{-}(\mu^{+}) + X$	$W^*q \rightarrow q'$	q, \bar{q}	$0.01 \lesssim x \lesssim 0.5$
	$v N \rightarrow \mu^- \mu^+ + X$	$W^*s \rightarrow c$	5	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \rightarrow \mu^+ \mu^- + X$	$W^*\bar{s} \rightarrow \bar{c}$	ŝ	$0.01 \lesssim x \lesssim 0.2$
	$e^{\pm} p \rightarrow e^{\pm} + X$	$\gamma^* q \rightarrow q$	g, q, \bar{q}	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \rightarrow \bar{v} + X$	$W^+ \{d, s\} \rightarrow \{u, c\}$	d, s	$x \gtrsim 0.01$
Collider DIS	$e^{\pm}p \rightarrow e^{\pm}c\bar{c} + X$	$\gamma^* c \rightarrow c, \gamma^* g \rightarrow c \bar{c}$	c. g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^{\pm}p \rightarrow e^{\pm}b\bar{b} + X$	$\gamma^* b \rightarrow b, \gamma^* g \rightarrow b \bar{b}$	b, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^*p \rightarrow \text{jet} + X$	$\gamma^* g \rightarrow q \bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
Tevatron	$p\bar{p} \rightarrow jet + X$	$gg, qg, qq \rightarrow 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
	$p\bar{p} \rightarrow (W^{\pm} \rightarrow \ell^{\pm}\nu) + X$	$ud \rightarrow W^+, \bar{u}\bar{d} \rightarrow W^-$	u, d, \bar{u}, \bar{d}	$x \gtrsim 0.05$
	$p\bar{p} \rightarrow (Z \rightarrow \ell^+ \ell^-) + X$	$uu, dd \rightarrow Z$	u, d	$x \gtrsim 0.05$
	$p\bar{p} \rightarrow t\bar{t} + X$	$qq \rightarrow t\bar{t}$	q	$x \gtrsim 0.1$
LHC	$pp \rightarrow jet + X$	$gg, qg, q\ddot{q} \rightarrow 2j$	g, q	$0.001 \lesssim x \lesssim 0.5$
	$pp \rightarrow (W^{\pm} \rightarrow \ell^{\pm} v) + X$	$u\bar{d} \rightarrow W^+, d\bar{u} \rightarrow W^-$	$u, d, \bar{u}, \bar{d}, g$	$x\gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+ \ell^-) + X$	$q\bar{q} \rightarrow Z$	q, \bar{q}, g	$x\gtrsim 10^{-3}$
	$pp \rightarrow (Z \rightarrow \ell^+ \ell^-) + X, p_\perp$	$gq(\bar{q}) \rightarrow Zq(\bar{q})$	g, q, \bar{q}	$x \gtrsim 0.01$
	$pp \rightarrow (\gamma^* \rightarrow \ell^+ \ell^-) + X$, Low mass	$q\bar{q} \rightarrow \gamma^*$	q, \bar{q}, g	$x\gtrsim 10^{-4}$
	$pp \rightarrow (\gamma^* \rightarrow \ell^+ \ell^-) + X$, High mass	$q\bar{q} \rightarrow \gamma^*$	\bar{q}	$x \gtrsim 0.1$
	$pp \rightarrow W^{*}\bar{c}, W^{-}c$	$sg \rightarrow W^{+}c, \bar{s}g \rightarrow W^{-}\tilde{c}$	5,5	$x \sim 0.01$
	$pp \rightarrow t\bar{t} + X$	$gg \rightarrow t\bar{t}$	g	$x \gtrsim 0.01$
	$pp \rightarrow D, B + X$	$gg \rightarrow c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \rightarrow J/\psi, \Upsilon + pp$	$\gamma^*(gg) \rightarrow c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \rightarrow \gamma + X$	$gq(\bar{q}) \rightarrow \gamma q(\bar{q})$	g	$x \ge 0.005$

• Medium to small-x region ($x \le 0.01$), HERA and now some LHC data

 \bullet In the high energy limit $m\ll Q,$ we can set $m\rightarrow 0,$ as it is much smaller than the relevant scale Q

• As a result if the x-sec diverges due to collinear singularities, we need to define collinear safe observable, like jets, or introduce PDF/ fragmentation functions that can absorb the collinear singularities which are universal

 $\label{eq:absorbing} \begin{array}{|c|c|c|c|c|} \hline & \mbox{Absorbing the divergences into redefinitions:} \\ & \mbox{Renormalisability} & \mbox{UV} & \alpha_s \rightarrow \alpha_s(\mu_R^2) & \beta\mbox{-function evolution} \\ & \mbox{Factorisation} & \mbox{IR} & f_i(x) \rightarrow f_i(x,\mu_F^2) & \mbox{DGLAP evolution} \end{array}$

- μ_F : Factorisation scale separates short distance and long distance physics
- For $t\bar{t}$ production we set the scale to the relevant hard scale:
- \circ Total x-sec, the only scale available is m_t
- Differential distributions better choices viz. $m_T = \sqrt{p_T^2 + m_t^2}, \cdots$

• μ_R : Renormalisation scale is the scale at which the strong coupling constant is evaluated

• μ_F and μ_R are varied around a judiciously chosen, default scale called the central scale, natural choice $\mu_F = \mu_R = Q \equiv m_t$

Estimate Theoretical Uncertainties

• To estimate the theoretical uncertainties, it is usual to vary either μ_R or μ_F about a central value Q: $1/2 < \xi < 2$ Together Independently Inversely μ_F , $\mu_R = \xi Q$ $\mu_R = Q$; $\mu_F = \xi Q$ $\mu_R = \xi Q$; $\mu_F = \xi^{-1} Q$ $\mu_F = Q$; $\mu_R = \xi Q$

• On physical grounds these scales have to be of the same order as Q, but their value can not be unambiguously fixed

 $\begin{aligned} f_{a/h_1}(x_1, \mu_F^2) \ f_{b/h_2}(x_2, \mu_F^2) & \hat{\sigma}_{ab}(x_1p_1, x_2p_2; Q, \{\dots\}; \mu_R, \mu_F; \alpha_s(\mu_R)) \\ \mu_F &= Q \quad \downarrow \quad \mu_R = Q \\ f_{a/h_1}(x_1, Q^2) \ f_{b/h_2}(x_2, Q^2) & \hat{\sigma}_{ab}(x_1p_1, x_2p_2; Q, \{\dots\}; \alpha_s(Q)) \end{aligned}$

• Theoretical uncertainty as a result of truncation, can ONLY be reduced by actually computing more terms in perturbation theory

• In the absence of direct evidence of any news physics one need to look for deviations in precision measurements

• Important for new physics searches & backgrounds to have better control over the theoretical uncertainties

QCD precision frontiers

NNLO de facto benchmark for LHC phenomenology:



Rely on 5 different types of 2-loop amplitudes

 $\bullet\ 2 \to 2$ process with some off-shell legs



 $\bullet~2~\rightarrow~1$ process with off-shell final state





- tt pair production
- only known numerically

Top-quark pair production to NNLO

First NNLO computation with two colored partons and massive fermions at a hadron collider which is exact and complete

Partonic cross section

 X_1 one additional parton

• Partonic cross sections are perturbatively calculable order by order in α_s

$$\hat{\sigma}^{i \ j \to t\bar{t}}(\alpha_s, \mu_F, \mu_R) = \alpha_s(\mu_R)^2 \Big\{ \hat{\sigma}^{LO} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{NLO}(\mu_F, \mu_R) \\ + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{NNLO}(\mu_F, \mu_R) + \mathcal{O}(\alpha_s^3) \Big\}$$

full NNLO

$$d\hat{\sigma}^{NNLO} = d\hat{\sigma}^{VV} + d\hat{\sigma}^{RV} + d\hat{\sigma}^{RR} \qquad \qquad \mathcal{O}(\alpha_s^4)$$

 NNLO cross sections beyond the known threshold expansions was essential and the missing ingredients involved

double-real
 real-virtual

- LO $i j \rightarrow t\bar{t}$ $ij \equiv q\bar{q}; gg$ $\mathcal{O}(\alpha_s^2)$
- NLO $i j \rightarrow t\bar{t} + X_1$ $ij \equiv q\bar{q}; gg; q(\bar{q})g$ $\mathcal{O}(\alpha_s^3)$ $\mathcal{O}(\alpha_s^4)$
- NNLO $i j \rightarrow t\bar{t} + X_2$ $ij \equiv q\bar{q}; qq'(\bar{q}'); gg; q(\bar{q})g$

 X_2 **two** additional parton

- New channels open up, as one goes higher up in the perturbative order
- Important development is the development of sector-improved residue subtraction scheme (STRIPPER) to handle the NNLO computations. Czakon (2010); (2011); Czakon, Heymes (2015)

Partonic cross sections $t\bar{t} + X$ to NNLO

 NLO $\mathcal{O}(\alpha_s^3)$ $\circ |i | i \rightarrow (n+1)|^2$ \circ *i j* \rightarrow *n* (LO) \times *i j* \rightarrow *n* (1-loop amplitude) Nason, Dawson, Ellis 1988 Beenakker, Kuijf, van Neerven, Smith 1989 Czakon, Mitov 2010 $\mathcal{O}(\alpha_{\epsilon}^{4})$ NNLO $\circ |i | i \rightarrow (n+2)|^2$ (RR) $\circ i j \rightarrow (n+1)$ (NLO) \times 1-loop amplitudes (RV) \circ n-parton (LO) \times 2-loop amplitudes (VV) \circ |n-parton 1-loop|² (VV)• guark initiated: $\circ a\bar{a} \rightarrow t\bar{t}X_2$ Bärnreuther, Czakon, Mitov 2012 $\circ qq \rightarrow t\bar{t}X_2, \circ qq' \rightarrow t\bar{t}X_2, \circ q\bar{q}' \rightarrow t\bar{t}X_2$ Czakon, Mitov 2012 • quark-gluon initiated: $\circ q(\bar{q}) g \to t\bar{t}X_2$ Czakon, Mitov 2013 • gluon initiated: \circ gg $\rightarrow t\bar{t}X_2$ Czakon, Fiedler, Mitov 2013

• First NNLO computation with two colored partons and massive fermions at a hadron collider which is exact and complete

Total inclusive top pair production

•

$$\sigma_{\text{tot}} = \sum_{i,j} \int_{0}^{\beta_{\text{max}}} d\beta \ \Phi_{ij}(\beta, \mu_F^2) \ \hat{\sigma}_{ij}(\alpha_s(\mu_R^2), \beta, m^2, \mu_F^2, \mu_R^2)$$

$$g_{\text{max}} \equiv \sqrt{1 - 4m^2/S} \qquad \beta = \sqrt{1 - \rho} \qquad \rho \equiv 4m^2/s \qquad 0 < \rho < 1$$

$$\beta \to 0 \quad \text{Threshold limit} \qquad \rho \to 1 \qquad 4m^2 \approx s \text{ (soft radiation)}$$

$$\beta \to 1 \quad \text{High energy limit} \qquad \rho \to 0 \qquad 4m^2 \ll s \text{ (massless limit)}$$

m: top quark mass, a scheme dependent quantity. Usually the pole mass \bullet Partonic flux

$$\Phi_{ij}(eta,\mu_F^2) = rac{2eta}{1-eta^2} \; \mathcal{L}_{ij}\left(rac{1-eta_{ ext{max}}^2}{1-eta^2},\mu_F^2
ight)$$

• Partonic luminosity

$$\mathcal{L}_{ij}(x,\mu_F^2) = x(f_i \otimes f_j)(x,\mu_F^2) = x \int_0^1 dy \int_0^1 dz \, \delta(x-yz) f_i(y,\mu_F^2) f_j(z,\mu_F^2) \, .$$

• Partonic cross section upto NNLO ($\mu_F = \mu_R = \mu$)

$$\hat{\sigma}_{ij} = \frac{\alpha_s^2}{m^2} \left\{ \sigma_{ij}^{(0)} + \alpha_s \left[\sigma_{ij}^{(1)} + L \sigma_{ij}^{(1,1)} \right] + \alpha_s^2 \left[\sigma_{ij}^{(2)} + L \sigma_{ij}^{(2,1)} + L^2 \sigma_{ij}^{(2,2)} \right] \right\}$$
$$L = \ln(\mu^2/m^2)$$

Partonic cross section \times flux



• Exact NNLO (black), in comparison with approximate NNLO (blues, red); approximates the exact result close to the threshold

• Outside of the threshold region, they do not agree, difference in between is pure NNLO, which the threshold resummation could not predict

• Integrating over β , one gets the contribution to total cross section and the exact gg contribution is a factor of 2 larger then the approximate results, in the $q\bar{q}$ case the ration is smaller

qq: Bärnreuther, Czakon, Mitov 2012gg: Czakon, Fiedler, Mitov 2013

Relative contribution of partonic sub channels

• Tevatron and LHC collider energies to NNLO+NNLL

	Tevatron	LHC@7 TeV	LHC@8 TeV	LHC@14 TeV
gg	15.4%	84.8%	86.2%	90.2%
$qg+ar{q}g$	-1.7%	-1.6%	-1.1%	0.5%
qq	86.3%	16.8%	14.9%	9.3%

Czakon, Mangano, Mitov, Rojo 2013

• Top-quark pair production, sensitive to gluon (LHC)/quark (Tevatron) and can be consistently included in a NNLO PDF fit without any approximations

Scale variation Fixed Order NNLO



- Perturbative expansion well convergent
- \bullet Scale variation of NNLO \subset NLO \subset LO implies the scale variation approximates the missing higher order terms well
- PDF sets used, match the accuracy of the Fixed order



Soft-gluon resumation



- Scale dependence for various Fixed Order and Logarithmic accuracy
- Impact of soft-gluon resummation on the size of the scale dependence and the theoretical central value
- PDF sets match the accuracy of the Fixed order
- Perturbative convergence preserved after the inclusion of soft-gluon resummation
- Inclusion of resummation with logarithmic accuracy decreases the scale dependence

Czakon, Fiedler, Mitov, Rojo 2013 35/62

Total $t\bar{t}$ cross section LHC & Tevatron

LHC & Tevatron measurements of top-pair production x-sec



As a function of √s compared to NNLO QCD + NNLL resummation
 Theory band reflects uncertainties due to • renormalisation μ_R and factorisation scale μ_F, • PDFs and • strong coupling α_s(μ_R²)
 Input m_t = 172.5 GeV

Differential top-quark pair production

Dynamical scales

• Variation of μ_F , μ_R is a proxy for missing higher order terms

 \bullet Functional form of default central scale μ_0 is a prerequisite to scale variation. Possible options:

• Note the proportionality constant, also need to be fixed Czakon, Heymes, Mitov 2016

Total cross section

• Fixed scale $\mu_F = \mu_R = m_t$ for LO, NLO, NNLO and NNLO+NNLL. Plots normalised to NNLO+NNLL cross section evaluated with corresponding PDF sets at scale $\mu_0 = m_t$



- Scale at which perturbative convergence is maximised is slightly above $m_t/2$, significantly lower than the std choice $\mu_0 = m_t$
- Numerical agreement between the fixed order result evaluated at a lower scale and the usual resumed result is significant
- \bullet Difference between the two PDF sets decreases fast with higher order and almost negligible at NNLO and NNLO+NNLL
- For both inclusive top-pair and Higgs production x-sec exhibit faster perturbative convergence at scales lower than usual, m_t and $m_h/2$

Total cross section

- Two criteria that is used:
- \circ principle of Fastest convergence and \circ minimum sensitivity
- \bullet Min sensitivity for which NLO curve plateaus is particularly low $\sim m_t/4$
- Significant shift when going from NLO to NNLO:
 min sensitivity scale
 value of x-sec at these two scales
- Both inclusive top-pair and Higgs production x-sec exhibit fastest perturbative convergence at scales lower than the usual ones
- Fixed NNLO x-sec at the scale of fastest convergence is only about 0.5% higher than the NNLO+NNLL resummed result evaluated at the usual scale $\mu_0 = m_t$
- Fast rise of resummed x-sec at larger values of μ , indicates that the resummed perturbative series is not converging well at high scales— large scales should be avoided

Comparison of different dynamic scales at NNLO



- Predictions are rather stable w.r.t choice of PDF sets at this level of perturbation theory
- \bullet Typical scale μ_0 used in past studies reduced by a factor 2 is a better option
- Such functional forms of μ_0 leads to faster perturbative convergence, small to moderate scale error and NNLO total x-sec not too different from NNLO+NNLL $\sigma_{tot}(m_t)$

Average t, \bar{t} , p_T differential x-sec at NNLO

• Ratio of $\mu_0 = H_T/4$; $H_{T,int}/2$; m_T ; $m_{t\bar{t}}/4$ w.r.t default scale $m_T/2$. Bands describe scale variation



• Scale $m_T/2$ leads to K-factors close to unity— best fits requirement for fastest perturbative convergence in the full kinematical range

• Scale $m_T/2$ has smallest scale variation

Comparison $m_{t\bar{t}}$ differential x-sec at NNLO

• Ratio of $\mu_0 = H_{T,int}/2$; $H_T/2$; $m_{t\bar{t}}/4$; $m_{t\bar{t}}/2$ w.r.t default scale $H_T/4$



- Dynamical scale $\mu_0 = H_T/4$ best fits the requirement for fastest perturbative convergence
- Note $m_{t\bar{t}}$ based scale lead to poor perturbative convergence

- Scale choice, based on the principle of fastest perturbative convergence
- Require the scale be such that the K-factor at NLO and NNLO introduces the smallest K-factor across the full kinematical range
- $\mu_0 = m_T/2$ for p_T distribution
- $\mu_0 = H_T/4$ for all other distribution

Normalised p_T^t and $m_{t\bar{t}}$ distribution to NNLO



• ATLAS and CMS results are compared to NNLO calculations. The shaded bands show the total uncertainty on the data measurements in each bin. The lower panel shows the ratio of the data measurements to the NNLO calculation

LHC*top*WG

Phenomenological implications of NNLO computation

Allow to undertake accurate phenomenological analysis of LHC data

- Extract NNLO PDFs from LHC data
- Improved determination of top-quark mass
- Validation of different implementation of higher order effect in MC event generators
- Direct measurement of the running of α_s at high scales

Inclusion of Tevatron and LHC inclusive top-quark pair measurements into a NNLO global PDF fit



Correlation between PDFs and Cross-Section

• Comparison of default fit v/s fits with inclusion of Tevatron and LHC top quark data

• Gluon in the large x range, where the gluon PDFs uncertainties are large can be constrained by top cross section at the LHC

• NNPDF 3.0 and MMHT14 have included *t*t̄ total cross section data *Czakon, Mangano, Mitov, Rojo 2013*

PDF fits: Differential distributions to NNLO

• Study the impact on the large-x gluon of top-quark pair differential distributions measured by ATLAS and CMS at $\sqrt{S} = 8$ TeV. • Transverse momentum p_T^t , rapidity y_t of top quark or antiquark, rapidity $y_{t\bar{t}}$ and invariant mass $m_{t\bar{t}}$ of the top-quark pair system



• Global with $t\bar{t}$ total x-sec, • Global with differential $t\bar{t}$ distributions and • Global with total x-sec and differential distributions

• Analysis, performed in the NNPDF3.0 framework at NNLO accuracy, allows to identify the optimal combination of LHC top-quark pair measurements that maximize the constraints on the gluon

Czakon, Hartland, Mitov, Nocera, Rojo- JHEP 04 (2017) 044

- \bullet Top-pair data from the LHC used to constrain gluon PDF within NNLO global analysis
- Improved gluon PDFs is used to provide updated prediction for other top-quark observables or gluon driven processes
- Achieve significant reduction of theory uncertainties, improve prospects of both SM measurements and BSM searches

Higgs Boson Production at Hadron Colliders to N3LO in QCD



top quark assumed to be infinitely heavy and all other quarks massless

Gluon fusion Higgs boson production cross-section

$$\sigma_{PP \to H+X} = \tau \sum_{ij} \int_{\tau}^{1} \frac{dz}{z} \int_{\tau/z}^{1} \frac{dx_1}{x_1} f_i(x_1) f_j\left(\frac{\tau}{x_1 z}\right) \frac{\hat{\sigma}_{ij}(z, m_h^2)}{z}$$



• Expand cross section as a series expansion about the threshold $z = \frac{M_h^2}{s} \rightarrow 1$, compute as many terms so that the series stabilises

Contributions @ N3LO

• Perturbative calculation at N3LO contains many pieces: • virtual corrections through 3-loops $gg \rightarrow H$

Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser, PRL 102 (2009) 212002

Gehrmann, Glover, Huber, Ikizlerli, Studerus, JHEP 06 (2010) 094

o single-real-emission corrections through 2-loops

 $gg
ightarrow Hg; \qquad qg
ightarrow Hq; \qquad q\bar{q}
ightarrow Hg$

Duhr, Gehrmann, PLB 727 (2013) 452; Li, H.X. Zhu, JHEP 11 (2013) 080 Dulat, Mistlberger, arXiv:1411.3586; Duhr, Gehrmann, Jaquier, JHEP 02 (2015) 077

Anastasiou, Duhr, Dulat, Herzog, Mistlberger, JHEP 12 (2013) 088; Kilgore, PRD 89 (2014) 073008

double-real-emission corrections through 1-loop

 $gg
ightarrow Hgg; \quad gg
ightarrow Hq ar q; \quad qar q
ightarrow Hgg; \quad qg
ightarrow Hqg; \quad qq'
ightarrow Hqq'$

Anastasiou et al., PLB 737 (2014) 325; Li, von Manteuffel, Schabinger, Zhu, PRD 90 (2014) 053006

Anastasiou, Duhr, Dulat, Furlan, Herzog, Mistlberger, JHEP 08 (2015) 051

Anastasiou, Duhr, Dulat, Herzog, Mistlberger, PRL 114 (2015) 212001

triple-real emission at tree level

 $gg \rightarrow Hggg; \quad gg \rightarrow Hgq\bar{q}; \quad q\bar{q} \rightarrow Hggg; \quad qg \rightarrow Hqq\bar{q}; \quad qq' \rightarrow Hqq'g$

Anastasiou, Duhr, Dulat, Mistlberger, JHEP 07 (2013) 003; Anzai, Hasselhuhn, Höschele, Hoff, Kilgore, Steinhausera, Uedad, JHEP 07 (2015) 140

• Both VR^2 and R^3 were evaluated as an expansion in $z \to 1$ (about 30 terms) which is sufficient for phenomenological applications, (but for qq' channel, were the computation is exact in m_H , s),

Appropriate UV and IR counter terms

• Effective H-g-g coupling has been computed to 4-loop accuracy

Chetyrkin, Kniehl, Steinhauser, NPB510 (1998) 61; Schröder, Steinhauser, JHEP 01 (2006) 051; Chetyrkin, Kühn, Sturm, NPB 744 (2006) 121

• $\mathcal{O}(\epsilon)$ contributions to NNLO MI

Pak, Rogal, Steinhauser, JHEP 09 (2011) 088; Anastasiou, Buehler, Duhr, Herzog, JHEP 11 (2012) 062

• LO,NLO,NNLO partonic x-sec expanded to $\mathcal{O}(\epsilon^i)$, i = 3, 2, 1 resptly

Höschele, Hoff, Pak, Steinhauser, Ueda, PLB 721 (2013) 244; Buehler, Lazopoulos, JHEP 10 (2013) 096

ullet 3-loop UV counterterms of strong coupling α_{s}

Tarasov, Vladimirov, Zharkov, PLB 93 (1980) 429; Larin, Vermaseren, PLB 303 (1993) 334

• 3-loop UV counterterms the operator in the effective Lagrangian

Spiridonov, IYal-P-0378, Academy of Sciences of the U.S.S.R., Moscow Russia (1984)

Contributions @ N3LO

• 3-loop correction $(VV^2 \text{ and } V^3)$



virtual corrections only contribute at threshold, known completely as an expansion in the dimensional parameter ϵ

• Real emmision of one parton to 1-loop and 2-loop contributions



• Real emmision corrections of two partons to 1-loop and three partons to tree level



Threshold expansion @ N3LO

$$rac{\hat{\sigma}_{ij}(z)}{z} = \hat{\sigma}^{SV} \ \delta_{ig} \ \delta_{jg} + \sum_{N=0}^{\infty} \hat{\sigma}_{ij}^{(N)} \ (1-z)^N$$

 $\bullet~1^{\rm st}$ term, Soft-Virtual (SV), kinematic configurations where any parton produced in conjunction with Higgs boson, is soft

$$\hat{\sigma}^{SV} = a \ \delta(1-z) + \sum_{k=0}^{5} b_k \ \left[\frac{\log^k(1-z)}{1-z} \right]_+$$

contains linear combinations of a δ -fn and + distributions that act on PDFs. $\delta(1-z)$ terms have have been computed recently and reconfirmend independently

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger, PLB737 (2014) 325

Li, von Manteuffel, Schabinger, Zhu, PRD91 (2015) 036008

• + distributions were know much earlier

Moch, Vermaseren, Vogt, NPB726 (2005) 317; Ravindran, NPB746 (2006) 58; Idilbi, Ji, Yuan, NPB753 (2006) 42

N3LO_{SV} (**DY**)



• Dependence N3LO (Threshold) result on the renormalization scale at various orders, plotted $R^{(i)} = \sigma^{i(\mu_R)}/\sigma^{i(Q)}$ where i = NLO, NNLO, $N3LO_{SV}$ versus μ_R/Q and the reduction in the scale dependence evident

Ahmed, Mahakhud, Rana, Ravindran PRL 113 (2014) 112002

Regular contributions

 $\bullet~2^{\rm nd}$ term, subleading soft emissions

$$\sum_{\mathsf{N}=0}^{\infty} \hat{\sigma}_{ij}^{(\mathsf{N})} \; (1-z)^{\mathsf{N}}$$

• Coefficients are polynomials in log(1 - z) at N3LO

$$\hat{\sigma}_{ij}^{(N)} = \sum_{k=0}^{5} c_{ijk}^{(N)} \log^{k} (1-z)$$

coefficients in this polynomial are zeta values

• VRV and V^2R contributions known exactly. VR^2 and R^3 , until recently known as an approximation based on a power series around threshold z = 1 truncated at $\mathcal{O}((1-z)^{30})$. Exact results known for qq' channel.

 \bullet Very recently, first exact formula for a partonic hadron collider x-sec at N3LO in pQCD

Mistlberger, arXiv:1802.00833

Higgs boson, Gluon-fusion x-sec to N3LO in QCD

• First complete computation of a x-sec at N3LO at a hadron collider



- Inclusive gluon fusion Higgs boson production cross-section has a slowly convergent perturbative expansion in QCD, finally stabilises at N3LO
- N3LO scale variation well contained within the N2LO scale variation

Anastasiou, Duhr, Dulat, Herzog, and Mistlberger, PRL. 114 (2015) 212001

Inputs to gluon-fusion cross section in EFT to N3LO

$$\hat{\sigma}_{ij} \simeq R_{LO} \left(\hat{\sigma}_{ij,EFT} + \delta_t \hat{\sigma}_{ij,EFT}^{NNLO} + \delta \hat{\sigma}_{ij,EW} \right) + \delta \hat{\sigma}_{ij,ex;t,b,c}^{(N)LO}$$

$\circ \hat{\sigma}_{ij,EFT}$ contribution to N3LO in EFT

• $\delta_t \hat{\sigma}_{ij,EFT}^{NNLO}$ subleading corrections as an expansion in inverse top mass • $\delta \hat{\sigma}_{ij,EW}$ includes EW corrections to NLO in α , mixed QCD-electroweak corrections through $\alpha \alpha_s^3$ in EFT

 $\circ~\delta\hat{\sigma}^{(N)LO}_{ij,ex;t,b,c}$ denotes exact results upto NLO, including all mass effects from t,~b,~c quarks

• $R_{LO} \equiv \sigma_{ex;t}^{LO} / \sigma_{EFT}^{LO}$ validity of ET can be enhanced by rescaling

 $\sigma = 48.58 \text{ pb}_{-3.27 \text{ pb}}^{+2.22 \text{ pb}} (+4.56\%) \text{ (theory)} \pm 1.56 \text{ pb} (3.20\%) \text{ (PDF} + \alpha_s)$

$$\sqrt{S} = 13 \text{ TeV}$$
 $m_H = 125 \text{ GeV}$ $\mu_R = \mu_F = m_H/2$

$\delta(scale)$	$\delta(trunc)$	$\delta(PDF-TH)$	$\delta(EW)$	$\delta(t, b, c)$	$\delta(1/m_t)$
+0.10 pb -1.15 pb	$\pm 0.18~{ m pb}$	± 0.56 pb	$\pm 0.49 \ \text{pb}$	$\pm 0.40 \text{ pb}$	$\pm 0.49~{ m pb}$
+0.21% -2.37%	$\pm 0.37\%$	$\pm 1.16\%$	$\pm 1\%$	±0.83%	$\pm 1\%$

$48.58\mathrm{pb} =$	$16.00\mathrm{pb}$	(+32.9%)	(LO, rEFT)
	$+20.84\mathrm{pb}$	(+42.9%)	(NLO, rEFT)
	- 2.05 pb	(-4.2%)	((t, b, c), exact NLO)
	+ 9.56 pb	(+19.7%)	(NNLO, rEFT)
	+ 0.34 pb	(+0.7%)	(NNLO, $1/m_t$)
	+ 2.40 pb	(+4.9%)	(EW, QCD-EW)
	+ 1.49 pb	(+3.1%)	$(N^{3}LO, rEFT)$

rEFT: cross section in EFT approx rescaled by R_{LO}

CERN Yellow Report Vol. 2/2017, arXiv:1610.07922

Summary

• With the discovery of the Higgs boson, the SM is a fully predictive theory, with all its parameters determined experimentally

• Initiates an era of precision studies of the properties of the Higgs boson at the LHC, where precise theory prediction for the Higgs observables play an indispensable role

 \bullet Inclusive gluon fusion cross section at the LHC evaluated to N3LO in pQCD, the first ever complete computation of a cross section at N3LO at a hadron collider

• N3LO corrections moderately increase (~ 3%) the cross-section for $\mu_F = \mu_R = m_H/2$, but notably stabilises the scale variation, reducing it almost by a factor of five compared to NNLO

• Opens avenues for theoretical predictions for large class of inclusive processes to N3LO *viz.*, DY production, Higgs production via b-quark fusion etc

Summary

• Driven by the vast number of top quark pairs produced at the LHC, top physics is entering into a high precision phase

• New generation of high precision calculations available, NNLO+NNLL total inclusive top-pair production and NNLO differential distribution

• First NNLO computation with two colored partons and massive fermions at a hadron collider which is exact and complete

• These developments will allow one to undertake number of high caliber phenomenological studies at the LHC:

 \circ Direct measurement of running α_{s} at high scales

 \circ Extraction of NNLO PDF from LHC data. Improved determination of poorly known large-x gluon PDFs.

 $\circ\,$ Total cross-section data and differential distributions already included in several PDF sets

 $\circ\,$ Translates into more accurate predictions of BSM heavy particles and high mass tail of the top pair invariant mass distribution

 \circ Improved determination of top-quark mass