Electroweak Symmetry Breaking & Vacuum Stability

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Recap: Lecture 1

- The SM Higgs sector at tree level ensures *unitarity* and renormalisability of the theory of massive weak gauge boson
- Unitarity conditions necessitates the introduction of a physical Higgs field into the spectrum leading to the *Gauge Hierarchy Problem*
- This is a consequence of absence of symmetry to protect the Higgs mass

Reference: Contino, TASI lecture; 1005.4269

Lecture 1: SM Higgs sector

$$\mathcal{L}_{Higgs} \supset (D_\mu H(x))^\dagger (D^\mu H(x)) + V_0(h(x)) - \frac{y_u}{\sqrt{2}} \bar{Q}_l \tilde{H} u_R - \frac{y_d}{\sqrt{2}} \bar{Q}_l H(x) d_R$$

$$V_0(h) = -\frac{1}{2} m_h^2 h^2 + \frac{\lambda}{4} h^4$$

- The tree level Lagrangian is Unitary & Renormalizable
- The tree level relation between the W mass and Z mass reproduced
- Gives a gauge invariant mass to the massive gauge boson and fermions

Lecture 2: Radiative corrections at 1-loop

$$V_{1-loop}(h) = \sum_{n=0}^{\infty} \frac{h^n}{n!} \Gamma^{(n)}(P_i = 0)$$

C-W effective potential: DR+MS

$$V_{1-loop}(h) = \frac{1}{64\pi^2} \sum_i n_i m_i(h)^4 \left(\log\left[\frac{m_i(h)^2}{\mu^2}\right] - c_i \right)$$

$$n_{h,\chi,W,Z,t} = 1, 3, 6, 3, -12$$

$$c_{h,\chi,t} = 2/3$$

$$c_{W,Z} = 5/6$$

$m_h \rightarrow$ Higgs dependent mass

$\mu \rightarrow$ regularisation scale

Reference: Quiros; hep-ph/990131

Lecture 2: RG improved C-W potential

$$V(h,\lambda_i,\mu) = V_0(h,\lambda_i,\mu) + V_{1-loop}(h,\lambda_i,\mu)$$

Regularization scale invariance of Green's Function imply:

$$\mu\frac{dV}{d\mu} = \left(\mu\frac{\partial V}{\partial\mu} + \beta_{\lambda_i}\frac{\partial V}{\partial\lambda_i} - \gamma h\frac{\delta}{\delta h}\right)V = 0$$

Solution to RGE:

$$\mu \to \mu(t) \equiv \mu_0 e^t$$

$$\lambda(t) \to \lambda_i(t)$$

where $\frac{d\lambda_i}{dt} = \beta_{\lambda_i}$

$$h(x) \to h(t,x) \equiv exp(-\int_0^t \gamma(t')dt')h(x)$$

- Identify $\mu(t) \to h(x)$
- Perturbative expansion of CW-pot
- Can compute the beta functions directly

RG improved 1-loop effective Higgs potential

$$V_{SM}(h) \sim \frac{\lambda_{eff}(h)}{4} h^4$$

C-W Potential:

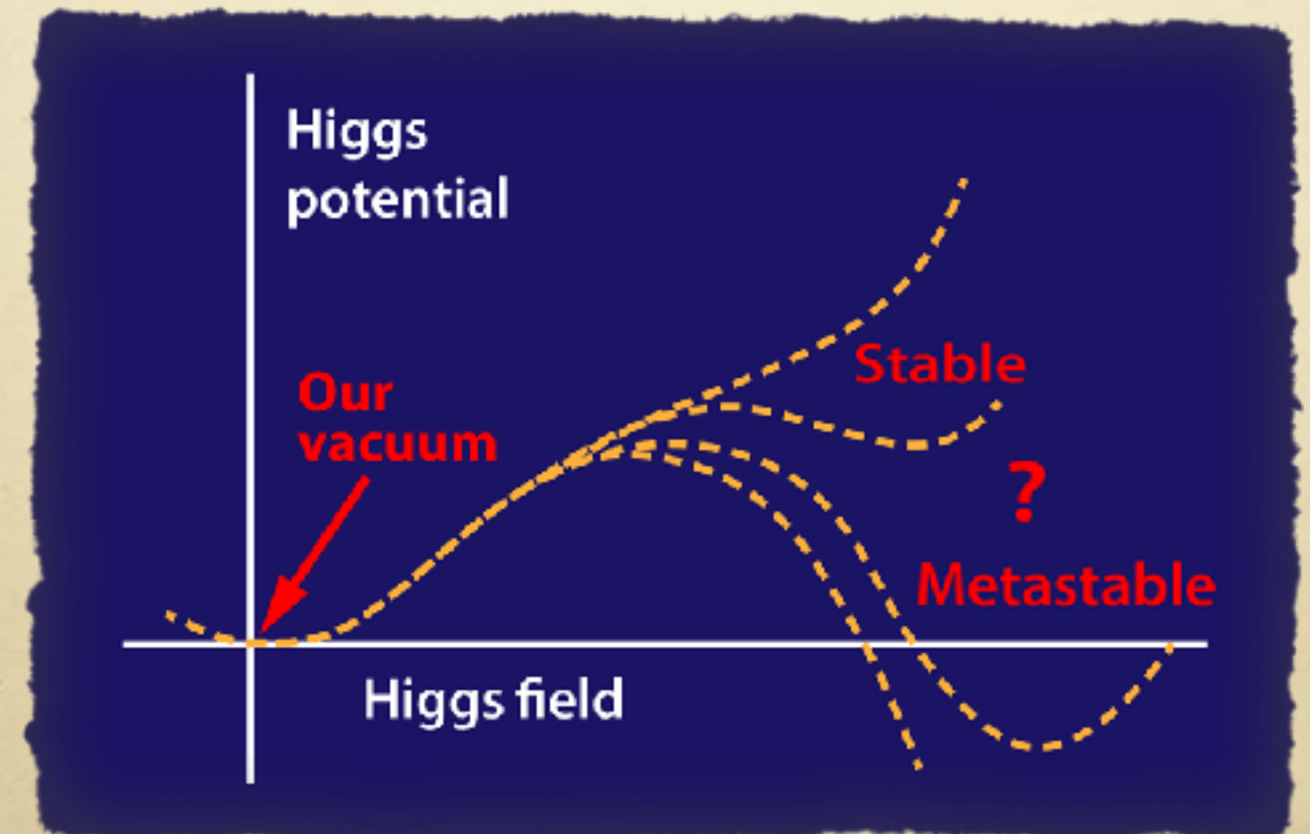
$$\begin{aligned}
\lambda_{eff}(h) =& e^{4\Gamma}\left[\frac{\lambda(h)}{4}\right. && \text{-RG improved tree} \\
&+ 3\left(\frac{g_1^2+g_2^2}{4}\right)^2\left(\log\left[\frac{g_1^2+g_2^2}{4}\right]-5/6\right) && \text{-Z boson} \\
&+ 6\left(\frac{g_2^2}{2}\right)^2\left(\log\left[\frac{g_2^2}{2}\right]-5/6\right) && \text{-W boson} \\
&- 12\left(\frac{y_t^2}{2}\right)^2\left(\log\left[\frac{y_t^2}{2}\right]-3/2\right) && \text{-t quark} \\
&+ \left(\frac{3\lambda}{2}\right)^2\left(\log\left[\frac{3\lambda}{2}\right]-3/2\right) && \text{-Higgs} \\
&+ 3\left(\frac{\lambda}{2}\right)^2\left.\left(\log\left[\frac{\lambda}{2}\right]-3/2\right)\right] && \text{-Goldstone}
\end{aligned}$$

Beta Function:

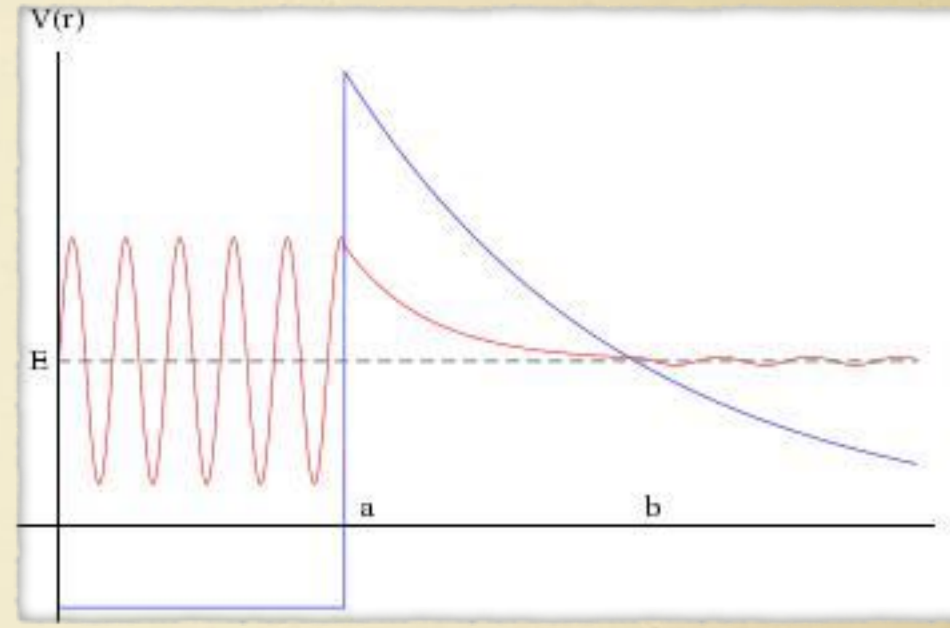
$$\begin{aligned}
\frac{d\lambda(h)}{d\log h} = \beta_\lambda = \frac{1}{16\pi^2}&\left(12\lambda^2 - 3y_t^4\right. \\
&+ 6y_t^2\lambda + \frac{9}{16}\left[g_2^2 + \frac{2}{3}g_2^2 g_1^2 + \frac{3}{25}g_1^4\right] \\
&\left.+\frac{9}{2}\lambda\left[g_2^2 + \frac{g_1^2}{5}\right]\right)
\end{aligned}$$

SM Vacuum: Stability

- The behaviour of the potential at large field related to Higgs quartic coupling behaviour at high scale
- Depending on the running of the quartic the potential may take various forms:



Metastable Vacuum: Lifetime calculations



WKB Approximation:

$$P \sim A e^{-B}$$

Bounce:

$$B = S_E = \int_{\infty}^{\infty} \mathcal{L}_E$$

Bounce calculation for SM:

$$\mathcal{L}_E = -\frac{1}{2}\left(\frac{\partial h}{\partial \tau}\right)^2 - \frac{1}{2}\left(\frac{\partial h}{\partial x_i}\right)^2 - V(h)$$

E-L Eqn.

$$\frac{\partial^2 h}{\partial \tau^2} + \frac{\partial^2 h}{\partial x_i^2} = \frac{\partial V(h)}{\partial h}$$

SO(4) symmetry

$$\rho \equiv \sqrt{\tau^2 + x_i x_i}$$

$$\frac{\partial^2 h(\rho)}{\partial \rho^2} + \frac{3}{\rho}\frac{\partial h(\rho)}{\partial \rho} = \lambda h^3(\rho)$$

Solution

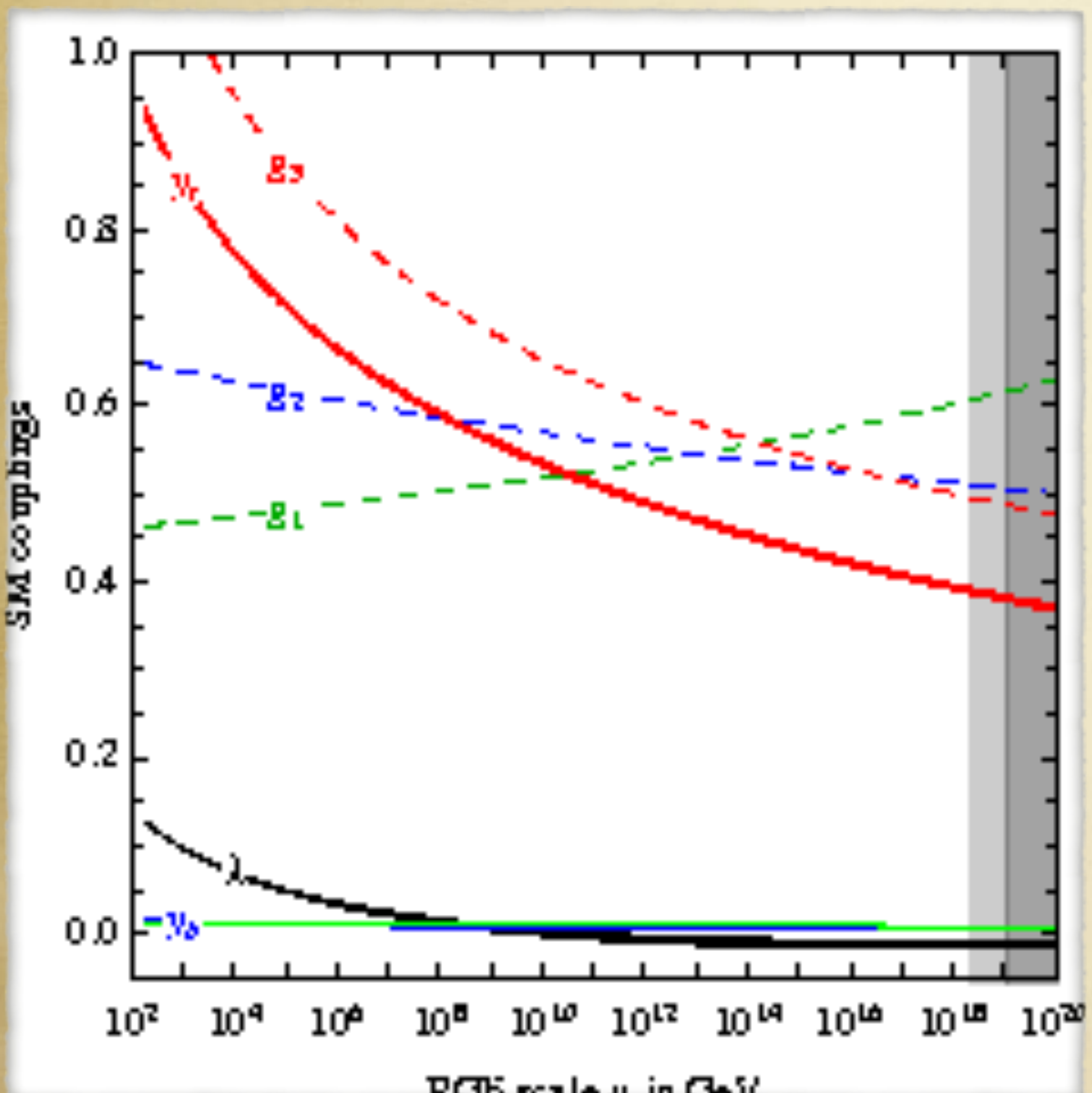
$$h(\rho) = \sqrt{\frac{2}{\lambda}}\frac{2\rho_0}{\rho^2 + \rho_0^2}$$

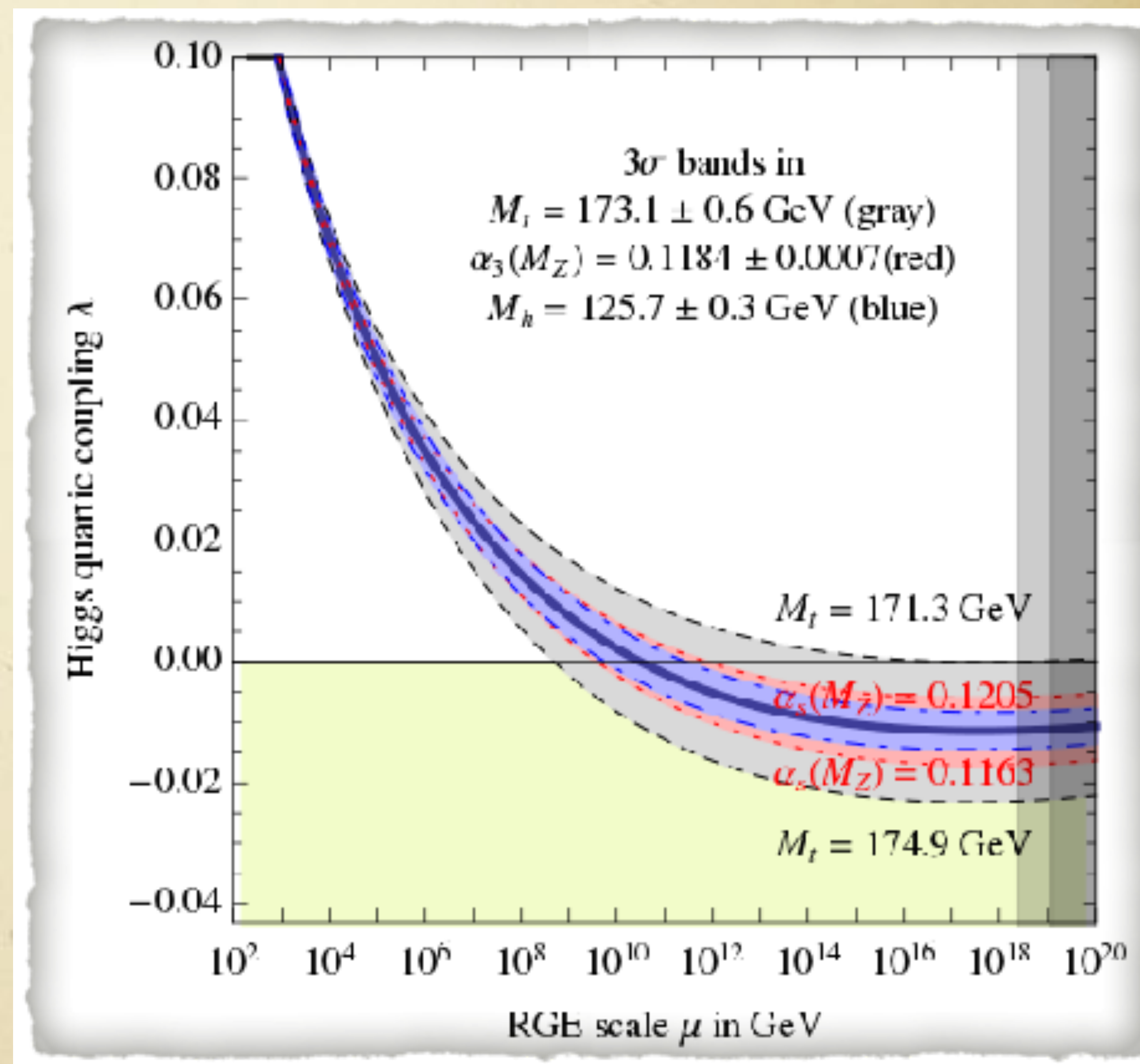
Plug back

$$B = S_E = \frac{8\pi^2}{3\lambda}$$

Reference: Coleman; PRD Vol 10 No15, 2929-2936

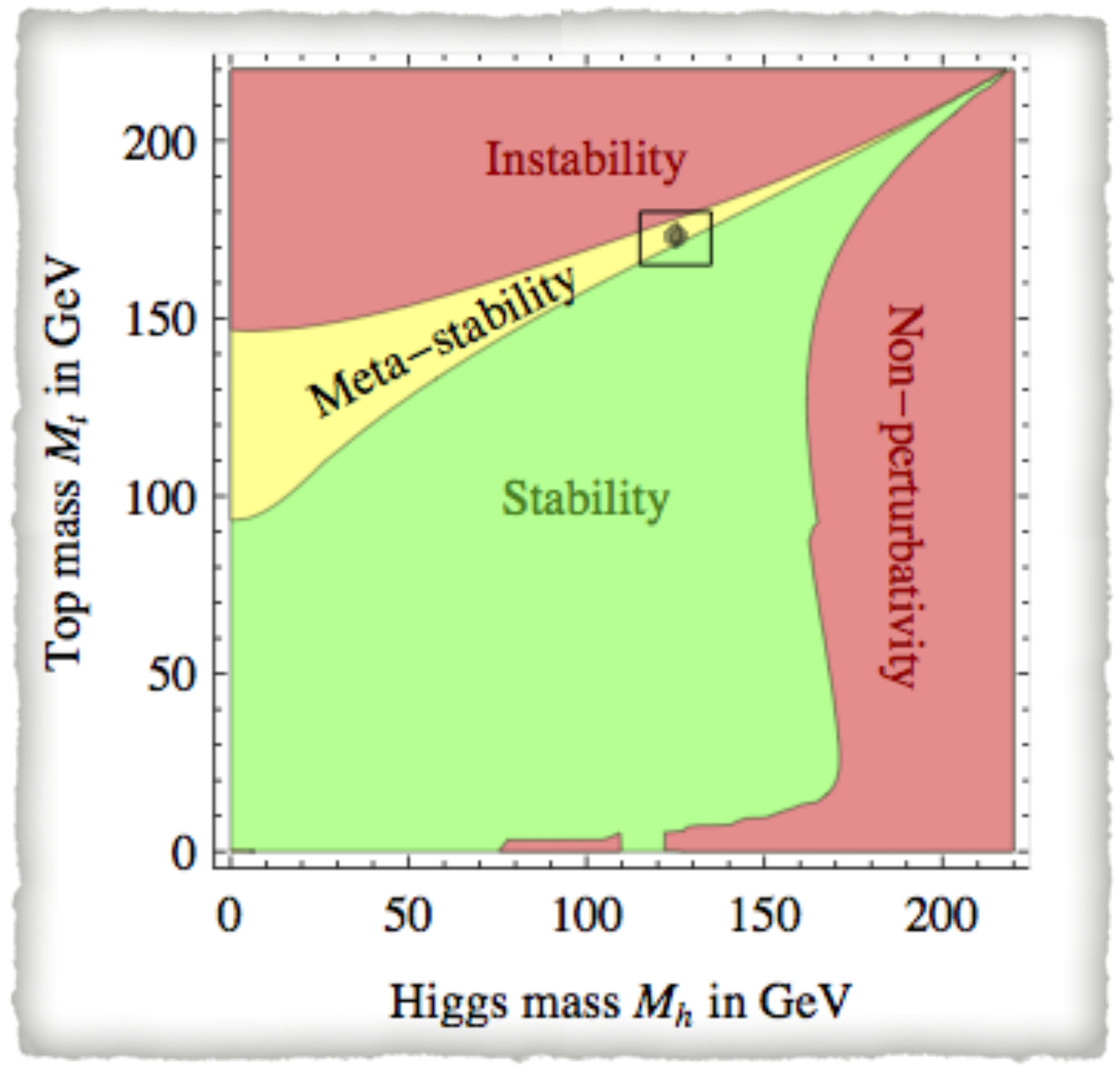
SM: RG Running of Higgs quartic

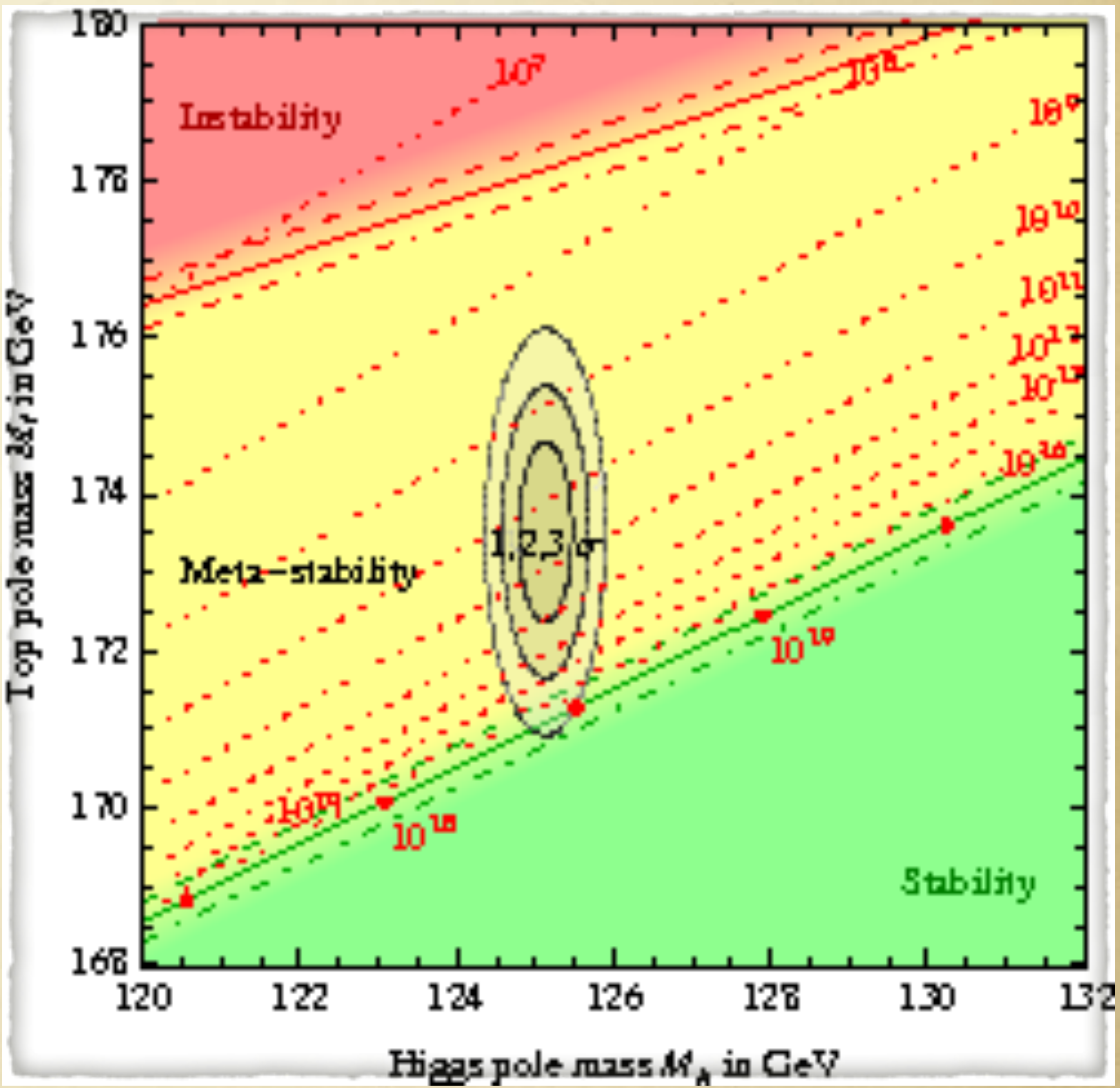




Reference: Di Vita et al; [arXiv:1205.6497]

Stability of SM Vacuum





Near Criticality of Higgs Mass

Reference: Di Vita et al; arXiv:1205.6497

What have we learned from the Higgs discovery: **Criticality**

- Why is the Higgs vev << M_{Planck} ?: Criticality of the EWSB
- Why is M_h close to129 GeV? : Criticality of the Higgs mass

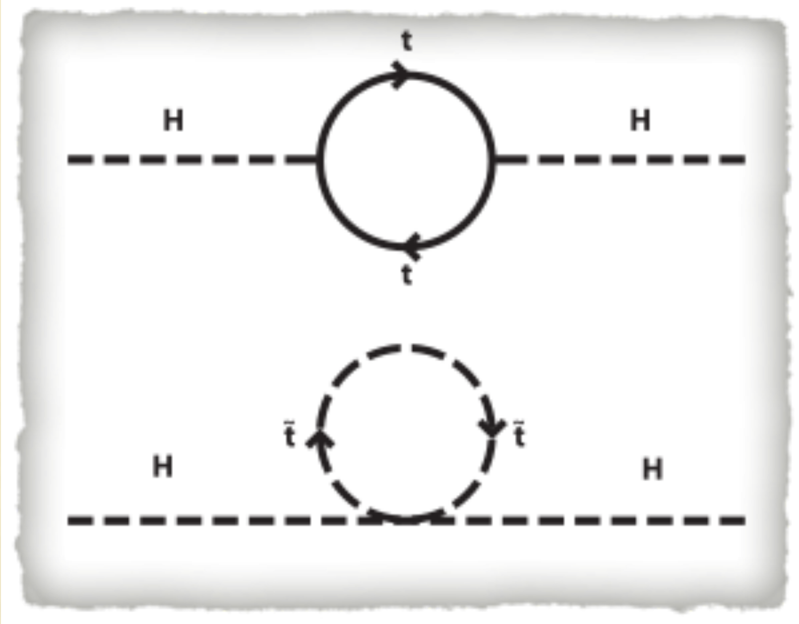
Why are we sitting on the edge?

BSM: Can do away with Criticality: SUSY, Composite Higgs

It can explain criticality dynamically: Relaxion

SUSY

- Imbibed chiral symmetry from super partner protects the Higgs mass



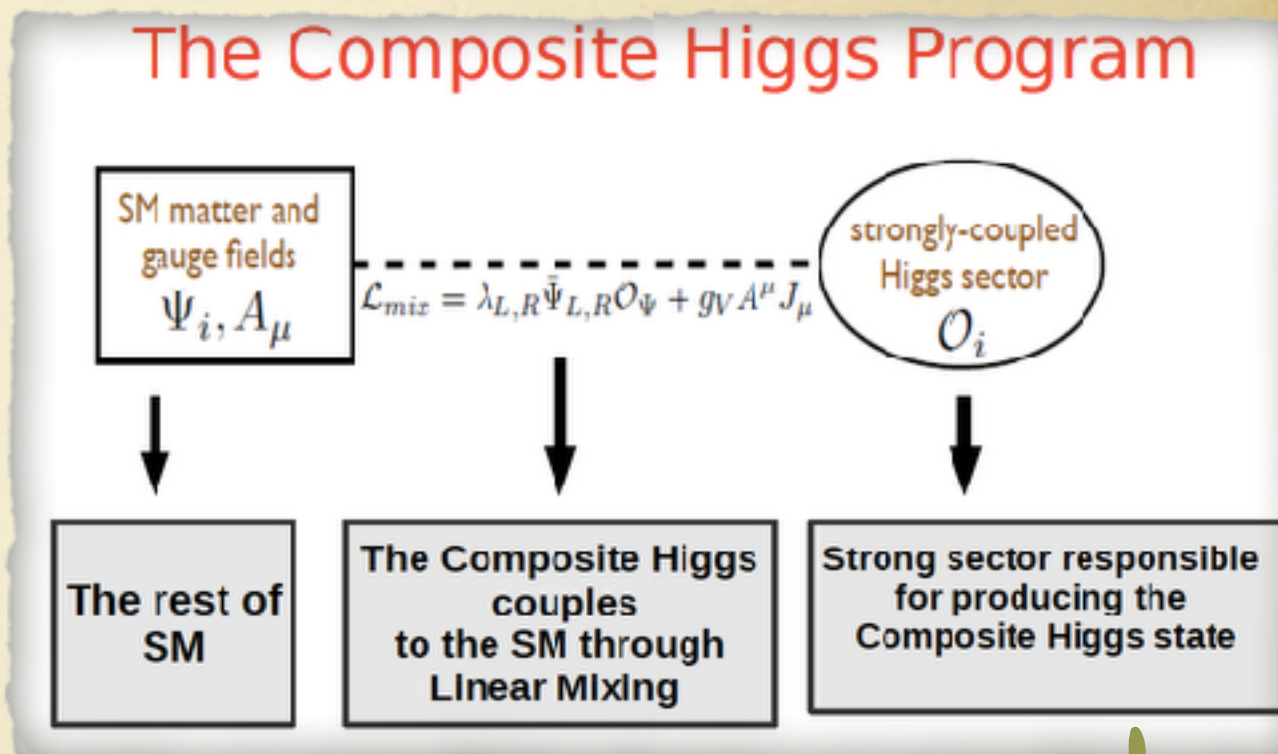
- The potential is positive definite. But one can have deeper charge/color breaking minima making the correct EW vacuum metastable

$$V(h) = F^\dagger F + D^\dagger D$$

$$\lambda_h \propto g_1^2 + g_2^2$$

Composite Higgs

- The inexact shift symmetry of the pNGB Higgs insulates it from the high scale



- The Higgs "melts " away into constituent state beyond the compositeness scale f where the potential becomes non-existent

Dynamics in strong sector produces pNGB mesons out of preons

Above f → ψ row; Below f → M row

	$Sp(2N_c)$	$SU(4)$
ψ	$\square$	**4**
M	**1**	**6**

Relaxion: Criticality

Extend SM with QCD Axion a(x): pNGB of PQ symmetry (protected from UV instability)

$$\mathcal{L}_{relaxion} \supset (-M^2 + qa(x))h^2 + V(a(x)) + \frac{1}{32\pi^2}\frac{a(x)}{f}\tilde{G}_{\mu\nu}G^{\mu\nu}$$

- At beginning $qa(x) >> M^2$: No Higgs vev
- As $q\,a(x) < M^2$: EWSB; chiral symmetry broken
- QCD Axion a(x) gets potential due to feedback:

$$V(a(x))_{QCD} = yvf_\pi^3 \cos(a(x)/f)$$