30/7/07

$$g^{\mu\nu} g^{\tau\rho} D_\mu F_{\nu\rho} + J^\tau = 0$$

Generalisation of Maxwell's eqn to the curved space.

Flat space version :-

$$\eta^{\mu\nu} \eta^{\tau\rho} \partial_\mu F_{\nu\rho} + J^\tau = 0$$

$$\Rightarrow \underbrace{\eta^{\mu\nu} \eta^{\tau\rho} \partial_\tau \partial_\mu F_{\nu\rho}}_{=0} + \partial_\tau J^\tau = 0$$

(due to antisymm.)

$$\Rightarrow \partial_\tau J^\tau = 0$$

[We ask is the above eqn satisfied or does it give some additional condn.? It would be strange if it gives some " " bcos Maxwell's eqn with source term should contain all info. - let's see what is $\partial_\tau J^\tau$]

$$J^\mu = \sum_n e_n \int du \, \delta^{(4)}(x^\nu - X^\nu_n(u)) \frac{dX^\mu_n}{du}$$

$$\therefore \partial_\mu J^\mu = \frac{\partial}{\partial x^\mu} J^\mu = \sum_{\mu=0}^{3} \sum_n e_n \int du \left(\prod_{\nu \neq \mu} \delta(x^\nu - X^\nu_n(u)) \right) \left\{ \delta'(x^\mu - X^\mu_n(u)) \frac{dX^\mu_n(u)}{du} \right.$$

Why does not ∂_μ act on $\frac{dX^\mu_n}{du}$?

→ bcos X^μ_n is just the particle trajectory & the delta fn. is ~~the~~ one which tells us that when we happen to be on the trajectory, we will have non-zero source action

$$-\frac{\partial}{X^\mu_n(u)} \delta(x^\mu - X^\mu_n) \frac{dX^\mu_n}{du} = -\frac{d}{du} \delta(x^\mu - X^\mu_{n(u)})$$

(All sums & products will be written explicitly — here we won't follow summation convention.)

$$= -\sum_n e_n \int du \underbrace{\sum_{\mu=0}^{3} \left(\prod_{\nu \neq \mu} \delta(x^\nu - X^\nu_n(u)) \right) \frac{d}{du} \delta(x^\mu - X^\mu_n(u))}_{\frac{d}{du}\left(\prod_{\nu=0}^{3} \delta(x^\nu - X^\nu_n(u)) \right)}$$

$$= -\sum_n e_n \left[\prod_{\nu=0}^{3} \delta(x^\nu - X_n^\nu(u)) \right]_{u=-\infty}^{u=\infty}$$

$$= 0$$

Consider $\delta(x^0 - X^0(u))$

Now, $X^0(-\infty) = -\infty$

$X^0(\infty) = \infty$

(bcos trajectory is there from infinite past to infinite future)

$\therefore \delta(x^0 - X^0(u)) = 0$ for any finite x^0

This is expected bcos

This shows/proves charge is conserved in a covariant way — This extra condn automatically follows even if e.o.m. isn't satisfied

So, we don't get any additional condns on charges — then that would have implied that the motion of the charges can be det. from this contraint → but this should not be the case; the condn must be satisfied automatically if eqns of motion are satisfied — consistency of the E.O.M of EM

as it is a classical th. & once we specify initial & final condns, trajectory is fixed

Claim

$$D_\tau \left(g^{\mu\nu} g^{\tau\rho} D_\mu F_{\nu\rho} \right) = 0$$

$$D_\tau J^\tau = 0$$

Both follow automatically with imposing additional condns on the charges or fields

To prove, go to locally inertial frame.

$g_{\mu\nu} = \eta_{\mu\nu}$, $\Gamma^\mu_{\nu\rho} = 0 \Leftrightarrow \partial_\rho g_{\mu\nu} = 0$