$\therefore$ ① reduces to

$$f_1(t', F(t', \vec{x}')) = f_1(t'; \vec{x}')$$

For $dx'^k dx'^l$:—

③ $$f_{3ij}(t', \vec{F}) \frac{\partial F^i}{\partial x'^k} \frac{\partial F^j}{\partial x'^l} = f_{3kl}(t'; \vec{x}')$$

Homogeneity :—

Given any two points : $(t, \vec{x})$ & $(t, \vec{y})$, there is an isometry F such that

$$\vec{F}(t, \vec{x}) = \vec{y}$$

This implies $f_1(t', \vec{y}) = f_1(t', \vec{x}')$ for any pair of points $(\vec{x}, \vec{y})$ ~~[illegible]~~ in space

[follows from ①]

$$\Rightarrow \boxed{f_1(t'; \vec{x}') = f_1(t')} \quad (\text{indep. of } \vec{x})$$

(Isotropy is not going to give any further restriction on f_1 bcos anyway it is indep. of $\vec{x}$)

$$f_{3ij}(t', F) \frac{\partial F^i}{\partial x'^k} \frac{\partial F^j}{\partial x'^l} = f_{3kl}(t'; \vec{x}')$$

Isometries associated to the space we consider here are homogeneity & isometry

Fix t' (think of it as a parameter without thinking 4D spacetime) & think of f_{3ij} as a 3-dim. metric.

Then, $x^i = F^i(\vec{x}')$ is an isometry of the 3-dimensional metric $f_{3ij}(t', \vec{x}')$ for every t'.

$ds_3^2 \equiv f_{3ij}(t', \vec{x}')\, dx'^i dx'^j$

& make a coord. trs. $x'^i = F^i(\vec{x}'')$

then this is an isometry of the 3-d metric

if $\boxed{f_{ij}(t', \vec{F}) \dfrac{\partial F^i}{\partial x''^k} \dfrac{\partial F^j}{\partial x''^l} = f_{3kl}(t', x'')}$

is satisfied.

The statement ~~of~~ about the 4-d metric is converted to the statement about the 3-d metric — the condn. can be restated in the 3-d language without any reference to time.

→ Reinterpretation

This could be done bcos we have the notion of COSMIC TIME

→ any isometry trs. should never change that time

List of homogeneous and isotropic metrics in D = 3

① flat space :-

$$f_{3ij}\, dx^i dx^j = \hat{f}(t) \{ (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \}$$

$\hat{f}(t)$ → arbitrary fn. of time

$\vec{x} \longrightarrow \vec{x} + \vec{a}$: establishes homogeneity

SO(3) rotation around origin → ~~isometry~~ establishes isotropy

② Surface of a 3-dimensional sphere S^3 :-

$$\sum_{i=1}^{4} (z^i)^2 = R^2$$

$$ds_4^2 = dz^i dz^i$$

SO(4)

Given any 2 pts. on the 2D sphere S^2, there is a rot. about origin which takes first pt. to the other pt.

Ricci scalar is −ve & const. → +ve curvature metric