

30/8/07

$$ds^2 = -dt^2 + (\lambda(t))^2 \left\{ \frac{dr^2}{1-k r^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right\}$$

where $k = 0, -1, +1$

Einstein eqn. :- $\left(\frac{\dot{\lambda}}{\lambda}\right)^2 + \frac{k}{\lambda^2} = \frac{8\pi G}{3} \rho$

Energy-mom. conserv. :- $\frac{d}{dt}(\rho \lambda^3) = -3\dot{\lambda} \rho \lambda^2$

ρ and p are defined through :-

$T_{00} = -\rho g_{00}$, $T_{ij} = p g_{ij}$ [where $\rho = \rho(t)$ and $p = p(t)$]

Eqn. of state :-

$$p = f(\rho)$$

functional relationship between p & ρ

Non-relativistic matter :-

$$p = 0$$

Cosmological constant :-

$$p = -\rho$$

Radiation :- $p = \rho/3$

We haven't proven that p corr. to pressure

For an arbitrary mix. of the 3 ingredients, ρ is the total density & p is the total pressure — then we have too many unknowns & need some additional info.

One assumption (can't be applied always) \rightarrow they don't interact

Another assumption can be matter & radiation are in thermal eqn., though their individual $T_{\mu\nu}$ may not be consid.

Now, $T_{\alpha\mu} = g^{\alpha\beta} g_{\mu\nu} g^{\kappa\tau} \langle F_{\beta\kappa} F_{\nu\tau} \rangle$

Energy-mom. tensor for radiation

$$- \frac{1}{4} g^{\alpha\mu} g^{\beta\sigma} g^{\kappa\tau} \langle F_{\beta\kappa} F_{\sigma\tau} \rangle$$

(The avg. means if there are multiple em radiations going in diff. dirns, we just over all such systems)

$$T^{00} = -\rho g^{00}$$

$$\Rightarrow g^{00} g^{00} g^{\kappa\tau} \langle F_{0\kappa} F_{0\tau} \rangle - \frac{1}{4} g^{00} g^{\beta\sigma} g^{\kappa\tau} \langle F_{\beta\kappa} F_{\sigma\tau} \rangle = -\rho g^{00} \quad [i, j = 1, 2, 3]$$

For our present universe, we can take the 3 comp. to be non-interacting

$$\Rightarrow g^{00} g^{00} g^{i'j'} \langle F_{0i'} F_{0j'} \rangle - \frac{1}{4} g^{00} g^{00} g^{i'j'} \langle F_{0i'} F_{0j'} \rangle \times 2 - \frac{1}{4} g^{00} g^{i'j'} g^{kl} \langle F_{ik} F_{jl} \rangle = -\rho g^{00}$$

$$\Rightarrow \frac{1}{2} g^{00} g^{00} g^{i'j'} \langle F_{0i'} F_{0j'} \rangle - \frac{1}{4} g^{00} g^{i'j'} g^{kl} \langle F_{ik} F_{jl} \rangle = -\rho g^{00}$$

→ this gives ρ

$$T_{ij} = p(x) g_{ij} \Rightarrow g^{i'j'} T_{ij} = 3p$$

$$\Rightarrow p = \frac{1}{3} g^{i'j'} T_{ij} = \frac{1}{3} g_{ij} T^{ij}$$

ex. check that $p = \frac{1}{3} \rho$

