What kind of trajectory does it describe in 3d subspace?

→ Forget about time. The geodesic in 4d is also a ~~trajectory~~ geodesic in 3d.

That geodesic is a curve/line for which $\theta = 0$, $\phi = 0$ at every pt.

⇒ The light travels along the spacetime curve

$$-dt^2 + \lambda(t)^2 \frac{dr^2}{1-kr^2} = 0$$

$$\Rightarrow \int_{t_1}^{t_0} \frac{dt}{\lambda(t)} = \int_0^{r_0} \frac{dr}{\sqrt{1-kr^2}}$$

$t_1 \Rightarrow$ initial time (when the light was emitted from the source)

$$= \begin{cases} r_0 & \text{for } k=0 \\ \sin^{-1} r_0 & \text{for } k=1 \\ \sinh^{-1} r_0 & \text{for } k=-1 \end{cases}$$

① ✓ t_1 can be found in terms of r_0, H_0, Ω_m, Ω_Λ, Ω_r.

✓ Suppose the source emits a total energy E per unit time.

per unit cosmic time

Energy/unit area measured by the observer: $F = \dfrac{E}{4\pi r_0^2 \lambda_0^2} \dfrac{\lambda(t_1)}{\lambda_0}$

The energy of each photon decreases by $\dfrac{\lambda(t_1)}{\lambda_0}$

② ✓ F is observed.

F is known for a Standard Candle

$\Rightarrow$ relation between r_0, t_1, H_0, Ω_m, Ω_Λ, Ω_r

We do integral over θ & ϕ for a fixed r ($=r_0$) & so we get 4π for $k=0, 1, -1$:- $r_0^2 \int \sin^2\theta \, d\theta \int d\phi = 4\pi r_0^2$

using ① & ②

Using this, we can determine t_1 for a given source as a fn. of the Cosmological parameters.

using $F_1 = \frac{E}{4\pi r_0^2 \lambda_0^2} \frac{\lambda}{\lambda_0}$ & ① .....

$$\frac{\lambda(t_1)}{\lambda(t_0)} = 1 - H_0(t_1 - t_0) - \frac{1}{2} q_0^2 H_0^2 (t_0 - t_1)^2 + \ldots\ldots$$

observed for a given source

known as a fn. of the cosmological parameters & luminosity observation.

[But we don't know the cosmological parameters.
But every obsn. is same eqn. of cosmological parameters
$\longrightarrow$ can be det. from these obsns]

For H_0 the analysis can be simplified

$$\frac{\lambda(t_0)}{\lambda_0} = 1 - H_0(t_0 - t_1) + O\{(t_0 - t_1)^2\}$$

To calculate H_0, we need to study the relations for small $(t_0 - t_1)$.