D-term potential in $\mathcal{N}=1$ supergravity:

→ Requires two more ingredients:

$f_{ab}(\Phi)$: A ~~ko~~ holomorphic function of chiral fields controlling the gauge kinetic term

$$-\frac{1}{4}\int d^4x \sqrt{-\det g}\left(f_{ab}(\Phi)\, F^{+a}_{\mu\nu} F^{+b\,\mu\nu} + h.c. \right).$$

X^i_a : A Killing vector field describing the action of the gauge generator T_a on the ~~[illegible]~~ space spanned by $\{\Phi^i, \bar{\Phi}^i\}$.

Define:
$$D_a = i\left(\partial_i K + \frac{\partial_i W}{W}\right) X^i_a$$

$$V_D = \frac{1}{2}\left((\mathrm{Re}\, f)^{-1}\right)^{ab} D_a D_b$$

arXiv: 1110.2174

$$\cancel{\left(\partial_i K\, \bar{\partial}_j K\, G^{i\bar{k}} - 3\right)}$$

$$\cancel{\partial_i G = \left(\partial_i K + \frac{\partial_i W}{W}\right)\left(\bar{\partial}_j K + \frac{\bar{\partial}_j W}{W}\right) G^{i\bar{j}}}$$

$W \neq 0 \Rightarrow e^{G}$ vanishes $e^{+K + \ln W + \ln \bar{W}}$

Massless 4-d fields from gauge fields:

Background gauge field $\in SU(3)$.

Surviving group $\subset E_8 \times E_8$ or $SO(32)$

that commutes with ~~SO~~ $SU(3)$.

$E_8 \subset SU(3) \times E_6$, $SO(32) \supset SU(3) \times U(1) \times SO(26)$

$\Rightarrow$ Unbroken gauge group: (at tree level)

$E_8 \times E_8$, $U(1) \times SO(26)$

$$A_\mu^a(x,y) = A_\mu^a(x) + \sum A_\mu^{a(n)}(x)\, f^{(n)}(y)$$

complex set of functions

$$D^\mu\left(\partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + f^{abc} A_\mu^a A_\nu^b\right) = 0.$$

$$\Rightarrow D^\mu\left(\partial_\mu A_\nu^a(x,y) - \partial_\nu A_\mu^a(x,y) + f^{abc} A_\mu^a A_\nu^b\right) = 0$$

$$\partial_\mu A_y^a(x,y) - \partial_y A_\mu^a(x,y) + f^{abc} A_\mu^a A_y^b = 0.$$

$$\partial_{y_i} A_{y_j}^a(x,y) - \partial_{y_j} A_{y_i}^a(x,y) + f^{abc} A_{y_i}^a A_{y_j}^b = 0.$$

$\Rightarrow$ Soln. to 4-d YM eq. $\Rightarrow$ Soln to 10-d YM eq.

$\to A_\mu^a(x) \to$ 4-d YM gauge fields.

$\subset \cancel{SU(3)}\,E_6 \times E_8$ or $U(1) \times SO(26)$

E_8 adjoint under $SU(3) \times E_6$.

$- (3, 27) + (\bar{3}, \overline{27})$.

$A_m^{(3,27)}(x,y)$, $A_m^{(\bar{3},\overline{27})}(x,y)$

$\to$ Could there be massless scalars among them?

$\to$ If so they will be part of chiral multiplet.

$m = i, \bar{i}$ — $\bar{i}$: chiral, i: anti-chiral. = focus on these.

m: internal real coordinate
$i, \bar{i}$: internal complex coordinates
$a, \bar{a}$: tangent space indices (internal)

$A_{\bar{i}}^{(3,27)}$ $\to$ global index does not couple to background SU(3) gauge fields.

$\downarrow$

(0,1) form.

$\in \mathcal{H}_1$ valued in 3-rep. of SU(3)

SU(3) gauge connection = SU(3) spin connection.

$3 \to$ Holomorphic tangent space index.

$A_m^{(3,27)} \in H_1(T)$ $\qquad A_{\bar{i}}^{\overset{1,\cdots 3}{(a,}\overset{1,\cdots 27}{\alpha)}}(x,y) = \phi^{(\alpha)}(x)\, f_{\bar{i}}^{\,a}(y)$

Non-trivial results for massless $\phi^{(\alpha)}(x)$

$F_{mn}^{(a)} = 0.$

$= D_m A_n^{(3,27)} - D_n A_n^{(3,27)} = 0$

$\Downarrow$

Covariant derivative.

(Background gauge field = spin connection)

For constant $\phi^{(\alpha)}(x)$, $A_{\bar{i}}^{(a,\alpha)}(x,y)$ should satisfy field eq. & appropriate gauge condition (so that it is not pure gauge).

$D^m \underset{\sim}{F}_{mn} = 0. \qquad D^m \underset{\sim}{A}_m = 0.$

$\to$ linear eqs. on $f_{\bar{i}}^{\,a}$

$\Rightarrow f_{\bar{i},jk} = f_{\bar{i}}^{\,a} E_a^{\,i}\, \Omega_{ijk}$

regarded as (2,1) form is harmonic

$(\bar{\partial} * \bar{\partial} * + * \bar{\partial} * \bar{\partial})\, f = 0$

Conclusion: # of chiral multiplet in 27 rep. of E_6 $= h_{2,1}$.

Similarly chiral multiplets in $\overline{27}$ rep: $\Leftrightarrow$ $f_i^{\bar{a}} \in H^1(T)$

$f_i^{\bar{a}} E_{\bar{a}}^{\bar{j}} g_{\bar{j}k} = f_{\bar{i},k}$

Gauge condition + Eq. of motion

$\Rightarrow$ $f_{\bar{i},k}$ regarded as (1,1) form is harmonic.

$\Rightarrow$ # of chiral multiplet in $\overline{27}$ representation is $h_{1,1}$.

What are the significance of 27 representations.

$E_6 \supset SU(5) \times U(1) \times U(1)$

$27 = 10 + \bar{5} + 5 + \bar{5} + 1$

$SU(5) \supset SU(3) \times SU(2) \times U(1)$
→ GUT group Quarks + Leptons $\in 10 + \bar{5}$

In SUSY SU(5) GUT: Higgs $\in 5 + \bar{5}$

The SU(5) singlet $\to$ candidate for right handed neutrino needed to give ~~it~~ the neutrino a mass.

$h_{2,1} - h_{1,1}$ = # of generations (3 in our world)

Superpotential: Unlike the moduli fields, there can be superpotential involving these new fields.

~~Cons~~: E_6 singlet.

$27 \times 27 \times 27$ has a singlet

$\overline{27} \times \overline{27} \times \overline{27}$ has a singlet.

$\Rightarrow$ Cubic coupling of 27's & independently $\overline{27}$ is possible.

Consider the $h_{1,1}$ $\overline{27}$'s. ϕ^A

ω_A: basis of 2 forms.

Consider the 2-form:

$\Phi = \sum_A \phi^A \omega_A \quad (= f_{\bar{i}}{}^{\bar{a}} E_{\bar{a}}{}^{\bar{j}} g_{\bar{j}k} \, dy^{\bar{i}} \wedge dy^k$

Then $W = \int \Phi \wedge \Phi \wedge \Phi$

$= \sum_{A,B,C} \phi^A \phi^B \phi^C \underbrace{\int \omega_A \wedge \omega_B \wedge \omega_C}_{\text{Topological}}$.

$\Rightarrow$ The cubic superpotential of $\overline{27}$'s.

is topological.

Similarly consider the 27's.

$\in H_1(T)$.

(0,1) form valued in holomorphic tangent space.

$A_{\bar{i}}{}^a$

$A^a = A_{\bar{i}}{}^a \, dy^{\bar{i}}$

$W = \int \Omega \wedge A^a \wedge A^b \wedge A^c \underbrace{\epsilon_{abc}}_{\text{3-index object}}$

The result depends on ~~complex~~ the cohomology class of A & on the complex structure via Ω.

Caveat: The kinetic terms of 27 and $\overline{27}$'s do not have standard normalization

$\Downarrow$

Requires knowledge of the metric.

What is the Kahler potential for these scalars?

$$K = \cdots + \underbrace{f^{(1)}_{ij} \, C_i^* C_j}_{27} + \underbrace{f^{(2)}_{ij} \, \tilde{C}_i^* \tilde{C}_j}_{\overline{27}} + \cdots$$

Computation of $f^{(1)}, f^{(2)}$ requires knowledge of metric in general.

$\Rightarrow$ Physical Yukawa couplings require knowledge of metric.