Fermions: 16-component Majorana in $D=10$

Under $SO(6) \times SO(3,1) \subset SO(9,1)$

\[ (4, \overline{2}) + (\overline{4}, 2) \rightarrow 16 \]

Under $SU(3) \subset SO(6)$

\[ 4 \rightarrow 3 + 1 \quad \overline{4} \rightarrow \overline{3} + 1 \]

\[ \Rightarrow \text{Fermion in } 4 \text{ rep. } + SU(4) \]

\[ \Psi_i \rightarrow \Psi^m_\rho \text{ Ex}_m \text{ } g_{m\bar{n}} \]

(Anti-holomorphic vector + scalar), as far as complexing to background metric is concerned.

\[ \Psi_i \rightarrow \text{left handed from 4-d viewpoint} \]

\[ \Psi^i_\rho \rightarrow \text{right handed from 4-d viewpoint} \]

Same as $A^\mu_\rho$, $A^\mu_\rho$, etc.

Explains why $A_\rho^\mu$ is part of (left) Chiral multiplet and $A^\mu_\rho$ is part of anti-Chiral multiplet.
Breaking E6

Suppose we have a discrete \( \mathbb{Z}_n \) group under which \( \text{Re}_3 \) is invariant.

Take quotient of \( \text{Re}_3 \) by the group.

Keep only those fields which are invariant under \( \mathbb{Z}_n \).

\[ x \rightarrow x/n, \]

- reduce \# of generations - anti-generations.

We can also quotient by \( \mathbb{Z}_n \) with a twist.

1. Identify a \( \mathbb{Z}_n \) subgroup of \( E_6 \times E_6 \).

2. Given an element of \( \mathbb{Z}_n \) acting on \( m \), keep only fields which are invariant under simultaneous action of \( \mathbb{Z}_n \) and \( \mathbb{Z}_n \).

= Switching on Wilson line along the cycle generated by \( \mathbb{Z}_n \) quotient.
Now only the subgroup $F_{\tilde{6}}$ invariant under $\mathbb{Z}_2$ will survive as a gauge group.

Example: Take $E_6$ gauge field $\omega^a$. Regarded as a 10-d field, it is constant along $M_{xy}^3$.

$\mathbb{Z}_2$ acts trivially.

Only $\mathbb{Z}_2$ invariant gauge fields will survive.

Thus, this way we can break $E_6$ to a lower $SU(3) \times SU(2) \times U(1)$ group.

Note: Since $\mathbb{Z}_2$ acts on electric charges, it can be embedded in the Carter subgroup.

The Carter generators survive.

Rank remains 6.

(can be reduced by giving up spin connection = gauge connection)
e) \( E_6 \rightarrow SU(3) \times SU(2) \times U(1) \times U(1) \times U(1) \)

\[ c = \frac{(1 + J^5)}{1 + 5J} = \frac{(1 + 5J + 10J^2 + 10J^3)}{1 - 5J} \]

Coefficient of \( J^3 \):
- \( 64 + 80 - 40 + 10 = 6 \)
- \( -125 + 125 - 50 + 10 = -40 \)

\[ X = 5 \times (-40) = -200 \]

Degree (product of degrees):
\[ h_{1,1} - h_{2,1} = -200 \]
\[ h_{1,1} = 1 \]
\[ h_{2,1} = \frac{-201}{200} \]