Type I on CY3

\[ \Rightarrow \text{similar to } SO(32) \text{ heterotic at least at the supergravity level.} \]

\[ \Rightarrow \text{same low energy phenomenology.} \]

We shall now return to type II theories and look for more general compactification with reduced supersymmetry.

To have control, need, at the leading order:

1. Unbroken SUSY
2. Minkowski 4-d space-time.

Later we have to break SUSY and look for de Sitter solns.
II B action

$$\frac{1}{2k^2} \int d^{10}x \sqrt{-g} \left[ R_g - \frac{1}{2k^2} \right]$$

$$\frac{1}{2\pi^2} \int \frac{1}{3!} \left( G^{(3)} - G^{(1)} \right) \epsilon^{\mu \nu \rho} g_{\mu \nu} g^{\rho \sigma} g_{\sigma \theta}$$

$$\frac{1}{2} \int \frac{1}{5!} \left( F^{(5)} \right)^2$$

$$\frac{1}{4k^2} \int C^{(4)} \wedge H^{(3)} \wedge F^{(3)}$$

$$\tau = \tau_1 + (\tau_2 - 2\Omega_0) + i e^{-\Phi}$$

$$G^{(3)} = F^{(3)} - \tau H^{(3)}$$

$$H^{(3)} = dB^{(2)} \quad F^{(3)} = dC^{(2)}$$

$$F^{(5)} = dC^{(4)} - \frac{1}{2} C^{(4)} \wedge H^{(3)} + \frac{1}{2} B^{(2)} \wedge F^{(3)}$$

We look for a general class of solutions with the ansatz:

$$ds^2 = e^{2A(y)} \eta_{\mu \nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn}(y) dy^m dy^n$$

$$\tau = \tau(y)$$

$$y^m: \text{coordinates along compact direction} \quad 0 \leq y \leq \pi$$
$x^k$: Coordinates along 4-d space-time $0 \leq k \leq 3$

$F^{(2)}$, $\tilde{F}^{(1)}$ can be non-zero but preserves 4-d Lorentz invariance.

$F^{(2)} \equiv F^{(2)}_{\mu
u}(y)$, $\tilde{F}^{(1)} \equiv \tilde{F}^{(1)}_{\mu
u}(y)$

$F^{(1)} = (1 + \ast) \delta \phi^8(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$

manifest: 1-form along compact direction
self-duality.

$dF^{(1)} = \delta F^{(2)} \wedge H^{(3)}$

has only 6 components along compact direction.

$dF^{(1)} = \delta \delta \phi^8(y) \wedge dx^0 \wedge \cdots \wedge dx^3$

$+ d \ast (A(y) \wedge dx^0 \wedge \cdots \wedge dx^3)$

Must 5-form in compact direction vanish.

$\delta \delta \phi^8(y) = 0 \Rightarrow \delta \phi^8(y) = dx^0 \wedge \cdots \wedge dx^4$

assuming $A_0^i = 0 \iff pS(y) = dx^i$ (assuming $\mathbf{X}(\mathbf{Y}) = 0$)
\[ F(\sigma) = (1 + \kappa^2) \alpha \wedge \beta \wedge \gamma \wedge \delta \wedge \epsilon \wedge \zeta \]

We shall also add localized sources of various fields along compact directions.

E.g. a 3-brane along a non-compact space-time, sitting at a given point on the compact space \( M \).

5-brane along \( x^0, \ldots, x^3 \) wrapping a 2-cycle on \( M \).

7-brane along \( x^0, \ldots, x^3 \) wrapping a 4-cycle on \( M \). etc.

\[ \Rightarrow \text{preserves} \ (3+1) \text{ dim. Poincaré invariance.} \]

Note: Most of the discussion also valid when \( (3+1) \)-d space is \( \text{AdS} \) or \( \text{dS} \).

Use isometries of \( \text{AdS} \) and \( \text{dS} \) instead of Poincaré invariance.
Metric equation:

$$\frac{\delta S}{\delta g_{MN}} = 0 \quad \text{if } M, N = 0, \ldots, 9$$

$$\Rightarrow R_{MN} - \frac{1}{2} R g_{MN} = k_{10} T_{MN}$$

$T_{MN}$: Stress tensor due to various fields other than gravity + due to localized source.

$$T_{MN} = -\frac{2}{\sqrt{-\det g}} \frac{\delta S'}{\delta g_{MN}} + T_{loc}$$


Taking trace:

$$R = -\frac{1}{2} k_{10} T^M_M$$

$$\Rightarrow R_{MN} = k_{10} \left( T_{MN} - \frac{1}{8} T_{PP} g_{MN} \right)$$
Ex. For IIB supergravity:

\[ T_{\mu\nu} = \frac{1}{2\xi_{10}} \partial_{\mu} \partial_{\nu} \left[ -\frac{1}{2\xi_{10}} \partial_{\rho} \partial_{\sigma} \epsilon_{\rho\sigma\sigma} + \frac{1}{2} e^{-2A} \partial_{\rho} \partial_{\sigma} \epsilon_{\rho\sigma\sigma} - \frac{1}{12\xi_{2}} \partial_{\rho} \partial_{\sigma} \partial_{\tau} \partial_{\xi} \epsilon^{\rho\sigma\tau\xi} \right] \]

(indices raised and lowered by \( g_{\mu\nu}, g^{\mu\nu} \))

\[ T_{\mu\nu} = \frac{1}{2\xi_{10}} \partial_{\mu} \partial_{\nu} \left[ -\frac{1}{2\xi_{10}} \partial_{\rho} \partial_{\sigma} \epsilon_{\rho\sigma\sigma} + \frac{1}{2} e^{-2A} \partial_{\rho} \partial_{\sigma} \epsilon_{\rho\sigma\sigma} - \frac{1}{12\xi_{2}} \partial_{\rho} \partial_{\sigma} \partial_{\tau} \partial_{\xi} \epsilon^{\rho\sigma\tau\xi} \right] \]

Strategy: Calculate \( R_{\mu\nu} \) & \( R_{\mu\nu} \) from the metric ansatz & substitute into e.o.m.
\[ R_{uv} = k_{10}^2 \left( T_{uv} - \frac{1}{8} T^m_{\mu \nu} g_{\mu \nu} \right) \]

\[ - r_{uv} e^{4A} \]

\[ 2^2 A = e^{-2A} \]

\[ \frac{1}{12} \chi_{uv} \frac{G_{mn}}{G_{mn}} \]

\[ + \frac{1}{4} e^{-10A} 2^m \times \frac{2^m}{x} + \frac{1}{8} \]

\[ \frac{1}{16} e^{-2A} (T^m_{\mu} - T^m_{\nu}) \frac{G_{mn}}{G_{mn}} \]

\[ + \frac{1}{12} k_{10}^2 e^{2A} (T^m_{\mu} - T^m_{\nu}) \frac{G_{mn}}{G_{mn}} \]

**Note:** Indices are still raised and lowered by \( g_{\mu \nu}, g_{\mu \nu} \) except in \( \partial \).

\[ \int d^4 y \int \sqrt{-g} e^{2A} = 0 \]

\[ \phi = \int d^4 y \int \sqrt{-g} \left[ e^{2A} \frac{1}{12} \chi_{uv} \frac{G_{mn}}{G_{mn}} + e^{-6A} 2^m \times \chi^{2^m} + e^{-6A} 2^m \times \frac{G_{mn}}{G_{mn}} \right] \]

\[ + \frac{1}{12} k_{10}^2 e^{2A} (T^m_{\mu} - T^m_{\nu}) \frac{G_{mn}}{G_{mn}} \]

\[ \text{In absence of } (Tmn)_{\mu \nu \rho}, \ G_{mn} = 0 \text{, } A = \text{constant} \]