Yesterday we analyzed $g_{\mu\nu}$ eq.

Today $\to$ other field eqs.

$$\tilde{F}^{(5)} = dC^{(4)} - \frac{1}{2} C^{(2)} \wedge H + \frac{1}{2} B \wedge F^{(3)}.$$

Bianchi identity $\equiv$ eq. of motion

$$d\tilde{F}^{(5)} = -F^{(3)} \wedge H^{(3)} + \text{localized sources.}$$

$\downarrow$ 3-branes.

Ansatz: $\tilde{F}^{(5)} = (1+*)\, d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$

$$\Rightarrow d\tilde{F}^{(5)} = e^{-8A}[\tilde{\nabla}^2\alpha - 8\tilde{g}^{mn}\partial_m A\, \partial_n \alpha]$$
$$dy^4 \wedge \cdots \wedge dy^9.$$

$$-F^{(3)} \wedge H = -\frac{1}{2i\tau_2} G \wedge \bar{G} \quad \Big| \quad G \equiv G^{(3)} = F^{(3)} - \tau H^{(3)}$$

$$\Rightarrow \tilde{\nabla}^2\alpha = i e^{2A} \frac{1}{12\tau_2} G_{mnp} \underbrace{*_6 \bar{G}^{mnp}}_{\text{6-d Hodge dual.}}$$
$$+ 2e^{-6A}\partial_m\alpha\, \partial^m(e^{4A})$$
$$+ 2\kappa_{10}^2 e^{2A} T_3 \rho_3^{loc.}$$

ρ_3^{loc} $\downarrow$ 3-brane charge.

T_3: D3-brane tension

$$\left(\sqrt{-\det g^{(6)}_{(int)}}\right)^{-1} \delta^{(6)}(y - y_i)$$

$y_i \to$ 3-brane location.

Recall:

$$\tilde{\nabla}^2 e^{4A} = e^{2A} \frac{1}{12\tau_2} G_{mnp} \overline{G}^{mnp} + e^{-6A} \partial_m \alpha \partial^m \alpha$$
$$+ e^{-6A} \partial_m (e^{4A}) \partial^m (e^{4A})$$
$$+ \frac{1}{2} \kappa_{10}^2 e^{2A} (T_m{}^m - T_\mu{}^\mu)^{loc.}$$

$$\tilde{\nabla}^2 (e^{4A} - \alpha) = \frac{e^{2A}}{24\tau_2} |iG - *_6 G|^2$$
$$+ e^{-6A} \partial_m (e^{4A} - \alpha) \partial^m (e^{4A} - \alpha)$$
$$+ 2\kappa_{10}^2 e^{2A} \left[\frac{1}{4} (T^m{}_m - T^\mu{}_\mu)^{loc} - T_3 \rho_3^{loc} \right]$$

l.h.s. integrates to 0.

First two terms are +ve definite.

We restrict to sources for which

$$\frac{1}{4} (T^m{}_m - T^\mu{}_\mu)^{loc} - T_3 \rho_3^{loc} \geq 0.$$

(True for D3, $\overline{D3}$, O3, D7, $\overline{D7}$, O7 etc)

$$\Rightarrow *_6 G = iG, \quad \alpha = e^{4A}, \quad \frac{1}{4} (T^m{}_m - T^\mu{}_\mu)^{loc} - T_3 \rho_3^{loc} = 0.$$

Using these conditions:

$$\tilde{\nabla}^2 \alpha = 2\alpha^2 \frac{i}{12\tau_2} \tilde{g}^{mm'} \tilde{g}^{nn'} \tilde{g}^{pp'} G_{mnp} *_6 \bar{G}_{m'n'p'}$$
$$+ 2\alpha^{-1} \tilde{g}^{mn} \partial_m \alpha \partial_n \alpha + 2 K_{10}^2 e^{2A} T_3 \rho_3^{loc}$$

$$\Rightarrow -\tilde{\nabla}^2(\alpha^{-1}) = \frac{i}{12\tau_2} \tilde{g}^{mm'} \tilde{g}^{nn'} \tilde{g}^{pp'} G_{mnp} *_6 \bar{G}_{m'n'p'}$$
$$+ 2 K_{10}^2 e^{-6A} T_3 \rho_3^{loc}$$

$$\Rightarrow \int d^6y \sqrt{\det \tilde{g}} \Big[\frac{i}{12\tau_2} \tilde{g}^{mm'} \tilde{g}^{nn'} \tilde{g}^{pp'}$$
$$G_{mnp} *_6 \bar{G}_{m'n'p'} + 2K_{10}^2 e^{-6A} T_3 \rho_3^{loc} \Big] = 0$$

$\Downarrow$

Condition for vanishing of total 3-brane charge.

Eqs. for 3-form field strengths:

$$\underbrace{dF^{(3)} = dH = 0}_{\text{Bianchi}} \Rightarrow dG^{(3)} = -d\tau \wedge H^{(3)}$$

eq.

E.O.M. $d\left(\frac{e^{4A}}{\tau_2} *_6 (|\tau|^2 H - \tau_1 F^{(3)})\right) - F^{(3)} \wedge d\alpha = 0$

$d\left(\frac{e^{4A}}{\tau_2} *_6 (-\tau_1 H + F^{(3)}\right) + H \wedge d\alpha = 0.$

→ Equivalent to real and imaginary parts of

$$d\Lambda + \frac{i}{\tau_2} d\tau \wedge \mathrm{Re}(\Lambda) = 0$$

$$\Lambda = e^{4A} *_6 G - i\alpha G.$$

<u>Recall</u> $e^{4A} = \alpha$ $\qquad *_6 G = iG.$

$\Rightarrow \quad \Lambda = 0.$

$\Rightarrow \quad C^{(2)}_{mn}, \quad B^{(2)}_{mn}$ eq. automatically satisfied.

Eq. of motion for τ and g_{mn} remain:

$$\tilde{\nabla}^2 \tau = \frac{1}{i\tau_2} \tilde{g}^{mn} \partial_m \tau \partial_n \tau - \frac{4\kappa_{10}^2}{\sqrt{-\det g}} \tau_2^2 \frac{\delta \tilde{S}^{loc}}{\delta \bar{\tau}}$$

$\tilde{S}^{(loc)}$: action for localized sources in the background metric $\tilde{g}_{mn}$.

$$\tilde{R}_{mn} = \kappa_{10}^2 \Big[\frac{1}{2\tau_2^2} \partial_{\{m} \tau \partial_{n\}} \bar{\tau} + \tilde{T}^{loc}_{mn} - \frac{1}{8} \tilde{g}_{mn} (\tilde{T}^{loc})^m{}_m \Big]$$

Note: $\tilde{g}_{mn}$ and $\tilde{\tau}$ eq. does not depend explicitly on the background fluxes.

1. Solve the combined eqs. involving $\tilde{g}_{mn}$, τ eq. in presence of localized sources.

2. Choose $F^{(3)}$, $H^{(3)}$ in $H^3(M)$ s.t. $G = F^{(3)} - \tau H^{(3)}$ is imaginary self-dual:

$$*_6 G = i G$$

3. Ensure that

$$\int \left(H^{(3)} \wedge F^{(3)} + 2K_{10}^2 T_3 \rho_3^{loc} \right) = 0.$$

4. Find $\alpha = e^{4A}$ by solving the appropriate Laplace eq. with sources.