Deforming away from constant τ configuration:

For comparision with orientifold

we need to ~~g~~ keep the coupling small

everywhere (almost) on the base.

$$j = \frac{(24)^3 \, 4f^3}{\underbrace{4f^3+27g^2}_{=\Delta}}$$

Need Δ small.

$$4f^3+27g^2 \simeq 0.$$

$$4f^3+27g^2=0.$$

$$f=-3h^2 \qquad g=-2h^3$$

Parametrize f and g by

$$f=-3h^2+c\eta$$

$$g=-2h^3+ch\eta+c^2\chi$$

c: small number.

η: Polynomial of degree 8

χ: Polynomial of degree 12.

h: Polynomial of degree 4.

$\Delta = 4f^3 + 27g^2$

$$\underset{\mathcal{E}x}{=} c^2\{\eta^2(4c\eta - 9h^2) + 54h(c\eta - 2h^2)x + 27c^2x^2\}$$

$c \to 0$ $\quad \Delta = c^2(-9h^2)(\eta^2 + 12hx)$

Zeroes at $h = 0. \Rightarrow$ 4 double Zeroes

$\eta^2 + 12hx = 0. \Rightarrow$ 16 single Zeroes.

deg 4 deg. 12

$$j(\tau) = 4\cdot(24)^3 \frac{(c\eta - 3h^2)^3}{c^2(-9h^2)(\eta^2 + 12hx)}$$

As long as $|h| >> (c\eta)^{1/2}$,

Num. ~ 1 Den $\sim c^2$

$\Rightarrow j(\tau)$ large $\Rightarrow$ $\operatorname{Im}\tau$ large.

As long as we keep away from $h = 0$, we can use an appropriate duality frame in which $\operatorname{Im}\tau$ is large.

Monodromy around zeroes of Δ:

Single zero, at $z = z_0$ (1 7)

$$j(\tau(z)) \sim \frac{A}{z - z_0}$$

~~$\oint dz\, j(\tau(z)) =$~~

$$\ln j(\tau(z)) = \cancel{A}\, \ln(z - z_0) + \ln A$$

as z goes around z_0,

$\ln j(\tau(z))$ increases by $2\pi i$

~~Furthermore if $\tau(z)$~~

At the core: $j \to \infty$. $\tau \to i\infty$ (or its image).

$$\Rightarrow \quad j \simeq e^{2\pi i \tau}$$

$\ln j$ increase by $2\pi i$

$\Rightarrow$ τ increases by 1.

Monodromy: $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (or its $SL(2,\mathbb{Z})$ conjugation).

Away from zeroes of h, $j(\tau)$ is large.

$\Rightarrow$ <u>Can take</u> $\mathrm{Im}\,\tau$ large.

$\Rightarrow$ Monodromies around all the zeroes of Δ from $\eta^2 + 12hX = 0$ are

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

$\Rightarrow$ a D7-brane at each zero of $\eta^2 + 12hX = 0$.

Next we consider the monodromy around the zeroes of h.

Double zero.

$$j = \frac{(c\eta - 3h^2)^3}{c^2(-9h^2)(h^2 + 12hX)}$$

Calculating monodromy around $h=0$ directly is difficult since the numerator is small and can also have zeroes near $h=0$.

$\rightarrow$ Use indirect method.

Take a circle of radius ~ 1 enclosing a zero of h but no other zeroes of Δ.

This circle lie in a region where $\operatorname{Im}\tau_2$ is large.

$\Rightarrow$ Monodromy $= \pm T^n$

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

On the other hand we know that the zeroes at $h=0$ are actually two single zeroes of $j(\tau(t))$ which are close for $c=0$.

Monodromy around each:

$$\underbrace{MTM^{-1} \qquad NTN^{-1}}_{SL(2,\mathbb{Z}) \text{ matrices.}}$$

$$MTM^{-1}\, NTN^{-1} = \pm T^n$$

In the regime $|h| >> |c|^{1/2}$

$$j(\lambda) \sim \frac{h^4}{c^2(\eta^2 + 2hx)}$$

As we go around the curve:

$$j(\lambda(z)) \sim (z - z_0)^4$$

$$\parallel$$

$$e^{-2\pi i \tau}$$

$$-2\pi i \tau \simeq 4 \ln(z - z_0)$$

$$\tau = +\frac{4i}{2\pi} \ln(z - z_0) + \cdots$$

As z goes around z_0.

$$\tau \to \tau + \frac{4i}{2\pi} \times 2\pi i = \tau - 4$$

$$\text{Monodromy} = \pm \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}$$

$+$ or $-$?

$$MTM^{-1} NTN^{-1} = \pm \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}$$

M, N : $SL(2,\mathbb{Z})$ matrices.

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

General solution:

$$MTM^{-1} = -\begin{pmatrix} 1-p & p^2 \\ -1 & 1+p \end{pmatrix} \quad p \in \mathbb{Z}.$$

$$NTN^{-1} = -\begin{pmatrix} 1-p & (p+2)^2 \\ -1 & 3+p \end{pmatrix}$$

$$MTM^{-1}\, NTN^{-1} = -T^{-4}$$

$\Rightarrow$ Monodromy $-T^{-4}$

$\Rightarrow$ $h=0$ ~~plane~~ subspace is orientifold plane.

$$\tau \simeq \frac{4i}{2\pi} \ln(z - z_0) + \text{const.}$$

As $z \to z_0$, $\tau \to \frac{4i}{2\pi} \times (-i\infty)$

$\to -i\infty.$

Soln. breaks down.

F-theory provides a resolution.

Correct soln.

$$j(\tau(t)) = \frac{4(24)^3 (c\eta - 3h^2)^3}{\Delta}$$

$$\Delta = c^2 \{ \eta^2 (4c\eta - 9h^2) + 54h(c\eta - 2h^2)x + 27c^2x^2 \}$$

$$\simeq c^2(-9h^2)(\eta^2 + 12hx).$$

Double zero at $h=0$ is an artifact of $c \to 0$ limit.

For small but finite c, the double zero is split into a pair of single zeroes.

$\Downarrow$

pair of 7-branes

$\Downarrow$

$SL(2,\mathbb{Z})$ conjugates of D7

by matrices M and N.