Deforming away from constant T configuration:
For comparison with orientifold we need to keep the coupling small everywhere (almost) on the base.

\[ j = \frac{(24)^3 \cdot f^3}{4f^3 + 27g^2} \]

Need \( \Delta \) Small.

\[ 4f^3 + 27g^2 \leq 0. \]

\[ 4f^3 + 27g^2 = 0. \]

\[ f = -3h^2 \quad g = -2h^3 \]

Parametrize \( f \) and \( g \) by

\[ f = -3h^2 + c\eta \]

\[ g = -2h^3 + 2c\eta + c^2x \]

\( c \): Small number.

\( \eta \): Polynomial of degree 8

\( x \): Polynomial of degree 12.

\( h \): Polynomial of degree 4.
\[ \Delta = 4f^3 + 27g^2 \]

\[ = c^2 \left( \eta^2 (4c\eta - 9h^2) + 54h(c\eta - 2h^2)x \right) \]

\[ \times \frac{1}{3x} + 27c^2x^2 + 3 \]

\[ \lim_{c \to 0} \Delta = c^2 (-9h^2) (\eta^2 + 12hx) \]

Zeros at \( h = 0 \): \( \implies 4 \) double zeros

\[ \eta^2 + 2hx = 0 \]: \( \implies 16 \) single zeros.

\[ \deg 4 \quad \deg 12 \]

\[ \delta(x) = \frac{4(2h^3)}{c^2(-9h^2)(\eta^2 + 12hx)} \]

As long as \( |h| < \left( \frac{c}{2} \right)^{1/2} \),

\[ \frac{\text{num.}}{\text{den.}} = \frac{N}{D} = c^2 \]

\[ \implies \delta(x) \text{ large} \implies 0 \text{ in } t \text{ large.} \]

As long as we keep away from \( h = 0 \), we can use an appropriate duality frame in which \( \delta(x) \) is large.
Monodromy around zeros of $f - \Delta$:

Single zero at $z = z_0$ (1x),

\[ j(z(t+1)) \sim \frac{A}{z-z_0}. \]

\[ \text{Logarithm:} \]

\[ \ln j(z(t+1)) \sim \ln |z-z_0| + \ln A \]

as $z$ goes around $z_0$.

$\ln j(z(t+1))$ increases by $2\pi i$.

At the core: $j(z) \to 0$. $z \to \infty$

$\sim j \sim e^{2\pi i z}$

$\ln j$ increases by $2\pi i$.

$z \to z$ increases by $1$.

Monodromy: \[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\] (or its S^1 action).

Away from zeros of $h$, $j(z(t))$ is large. $z$ can take I'm & large.
Monodromy around all the zeroes of $\Delta$ from $\eta^2 + 12h x = 0$. De

\[
\begin{pmatrix}
\eta & 1 \\
0 & 1
\end{pmatrix}
\]

a D7-brane at each zero of $\eta^2 + 12h x = 0$.

Next we consider the monodromy around the zeroes of $h$.

Double zero.

\[
\tilde{c} = \left( \frac{\eta - 3h^2}{c_1 - 3h^2} \right)^3 \frac{c_1}{c_1 - 12h x}
\]

Calculating monodromy around $h = 0$ directly is difficult since the numerator is small and can also have zeroes, need $h > 0$.

So use indirect method.
Take a circle of radius $r \approx 1$ ending a zero of $h$ but no other zeroes of $\Delta$.

This circle lies in a region where $\text{Im} \frac{1}{z}$ is large.

3. Monodromy $= \pm T^h$

\[
T^h = \begin{pmatrix} e^h & \frac{1}{e^h - 1} \\ 0 & 1 \end{pmatrix}
\]

On the other hand, we known that the zeroes at $h=0$ are actually two single zeroes of $\Delta h(\pm 1)$ which are close for $\epsilon \to 0$.

Monodromy around each:

\[
MTM^{-1} \quad NTN^{-1}
\]

\[
\Delta h \approx 2.2 / \text{matrix}
\]

\[
MTM^{-1} \quad NTN^{-1} = \pm T^h
\]
In the region $1/h > \leq 1/c \sqrt{h}$

\[ i(x) \sim \frac{h^4}{e^{2(\eta^2 + 2\eta x)}} \]

As we go along the curve:

\[ i(x+\epsilon) \sim (2-\eta_x)^4 \]

\[ i \frac{\eta_x}{2} \]

\[ -2\eta_x \approx 4 \ln (2-\eta_x) \]

\[ \omega = \frac{\eta_x}{2} \ln (2-\eta_x) + \cdots \]

As $\eta$ goes around $\eta_0$:

\[ \omega \approx \omega + \frac{\eta_x}{2\pi} \times 2\pi + 4 \]

Monodromy $\approx \pm \left( \begin{matrix} 1 & -1 \\ 0 & 1 \end{matrix} \right)$

$+ \omega - \rho$

$MTM^{-1} NTN^{-1} = \pm \left( \begin{matrix} 1 & -\eta \\ 0 & 1 \end{matrix} \right)$
$M, N \in SL(2, \mathbb{R})$ matrices.

$$T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$  

**General solution:**

$$MTM^{-1} = \begin{pmatrix} 1-b & b^2 \\ -1 & 1+b \end{pmatrix}$$ for $b \in \mathbb{R}$.

$$NTN^{-1} = \begin{pmatrix} 1-b & \frac{12}{2} \\ -1 & 3+b \end{pmatrix}.$$

$$MTM^{-1} NTN^{-1} = -T^{-4}$$

$\Rightarrow$ **Monotony**

$\Rightarrow$ $h = 0$ $\Rightarrow$ solution is orientifold plane.

$$T = \frac{2}{\sqrt{5}} \ln \left( 2-2e^{i\pi} \right) + \text{const}.$$  

As $T \to \infty$, $T \to \frac{4\pi}{\sqrt{-2}} \times (-\infty) \to -\infty$.

Solution breaks form.
F-theory provides a resolution.

Correct soln.

\[5(\tau+1) = \frac{4(2x)^3}{(c^2 - 3h^2)^3} \]

\[\Delta = c^2 \varepsilon^2 \eta^2 (4c^2 - 3h^2) + 54h(c^2 - 2h^2)x^2 + 27c^2 \varepsilon^2 \eta^2 \]

\[\Delta = c^2(-c^2)(\eta^2 + 12h^2)\]  

Double zero at \( x = 0 \) is an artifact of \( C \to 0 \) limit.

For small but finite \( C \), the double zero is split into a pair of single zeroes.

Pair of 9-branes

\( \mathcal{SL}(2,2) \) conjugate to \( D7 \) by matrices \( M \) and \( N \).