Example

F-theory on K3

We had taken

$$y^2 = x^3 + f(z)\,x + g(z)$$

$f(z)$: Polynomial of degree 8

$g(z)$: Polynomial of degree 12.

• Parametrization:

$$f(z) = -3h^2 + \overbrace{c}^{\text{constant}}\,\underbrace{\eta}_{\text{degree } 8}$$

$$g(z) = -2\underbrace{h^3}_{\text{degree } 4} + c\,h\,\eta + c^2\underbrace{\chi}_{\text{degree } 12.}$$

In $c \to 0$ limit we have the base

$S^2 \to$ Tetrahedron.

Following the general logic we would say that this is an orientifold of

$$\xi^2 = h(z)$$

by $(-1)^{F_L} \cdot \Omega \cdot \mathcal{I}_2 \to \xi \to -\xi$.

What is the space $\xi^2 = h(z)$?

$$h(z) = \prod_{i=1}^{4} (z - z_i).$$

$$\xi^2 = \prod_{i=1}^{4} (z - z_i).$$

$\Rightarrow$ Double cover of S^2.

branch points at z_1, z_2, z_3, z_4.

x x

(Note now we are talking about the base & not the fibers).

This is a torus.

$\Rightarrow$ Orientifold of IIB on $T^2/(-1)^{F_L}\cdot\Omega\cdot\mathcal{I}_2$

$\mathcal{I}_2$: $\xi \to -\xi$ corresponds to $\omega \to -\omega$

if we describe the torus by

$$\omega = \omega + 1 = \omega + \tau.$$

Since every F-theory compactification on CY_{n+1} has a region in its moduli space where it can be described as IIB on $CY_n / (-1)^{F_L} \cdot \Omega \cdot \mathbb{Z}_2$, we shall from now on consider only orientifolds.

Unbroken SUSY: $\mathcal{N}=1$ in $D=4$.

IIB on $CY_3 \Rightarrow \mathcal{N}=2$

orientifold $\Rightarrow \mathcal{N}=1$

Equivalently F-theory on $CY_4 \Rightarrow \frac{32}{8} = 4$ SUSY

$\Downarrow$

$\mathcal{N}=1$ in $D=4$.

Low energy effective field theory of the moduli fields must be described by general $\mathcal{N}=1$ sugra.

→ controlled by Kahler potential and superpotential

(Also $f_{ab}(\Phi)$ and the killing vector $K_i^a(\Phi)$ generating gauge symmetry if gauge fields are present).

What are the massless fields?

① Closed string fields are subset of fields on IIB on CY_n which are invariant under $(-1)^{F_L} \cdot \Omega \cdot \sigma_2$.

② Open string fields living on D7 branes (& D3 branes if D3 and O3 are present).

Focus on closed string fields.

→ moduli fields ⇒ no superpotential.

(This will change in the presence of fluxes!)

① Complex structure moduli:

→ Part of vector multiplet in IIB on CY_n.

Arise from metric degrees of freedom.

Only a subset invariant under $\mathcal{I}_2$.

$(-1)^{F_L} \cdot \Omega$ leaves it invariant.

$\Rightarrow$ This subset survives in the orientifold.

$$K_{com.} = -\ln\left(\frac{-i}{2}\int \Omega\wedge\bar{\Omega}\right)$$

fn. of complex structure moduli invariant under $\mathcal{I}_2$.

$\to$ same as in heterotic.

Hyper multiplets in IIB on CY_3:

$C^{(0)}, e^{-2\Phi}, B^{(2)}_{\mu\nu}, C^{(2)}_{\mu\nu}$

dualized to scalars.

Projected out by $(-1)^{F_L} \cdot \Omega \cdot \mathcal{I}_2$

trivial

The correct complex scalar that forms part of chiral multiplet is

$$\tau = C^{(0)} + i e^{-\Phi}$$

Comparing kinetic term for τ with the general form of action:

$$-\frac{\partial_\mu \tau \, \partial^\mu \bar{\tau}}{(\tau-\bar{\tau})^2} \quad \text{vs.} \quad -\partial_i \partial_{\bar{j}} K \, \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^j$$

$$\Rightarrow \quad K_\tau = -\ln[-i(\tau-\bar{\tau})]$$

IIB on CY_3 also has other hypermultiplets associated with (1,1) forms $\omega^\alpha_{m\bar{n}}$.

divide into

$\omega^{(+)a}_{m\bar{n}}$, $\omega^{(-)s}_{m\bar{n}}$

even under $\mathcal{I}_2$ odd under $\mathcal{I}_2$

In IIB on K3 hypermultiplet fields are:

$$g_{m\bar{n}} = \sum_\alpha \phi_\alpha \, \omega^\alpha_{m\bar{n}}$$

$$B^{(2)}_{m\bar{n}} = \sum_\alpha \xi_\alpha \, \omega^\alpha_{m\bar{n}}$$

$$C^{(2)}_{m\bar{n}} = \sum_\alpha \eta_\alpha \, \omega^\alpha_{m\bar{n}}$$

$$C^{(4)}_{m\bar{n}\mu\nu} = \sum_\alpha (\chi_\alpha)_{\mu\nu} \, \omega^\alpha_{m\bar{n}}$$ dualize to scalar.

For heterotic on CY_3, ϕ_α and $\xi_{\bar{\alpha}\alpha}$

survived to give complex scalars.

Now $\underbrace{\phi_a, \chi_a}$, $\underbrace{\xi_s, \eta_s}$ survive.

compact moduli.

ϕ_a describes non-compact piece.

(has periodic identification)

Open string fields from D-branes.

→ compact moduli

Scalars describing position of ~~D7~~ D3 in

CY_3 or Wilson line on D7.

Once we ~~introduce fluxes~~ break

SUSY all the moduli will

acquire potential.

Danger: Non-compact moduli may

run off to ∞. ⇒ out of control.

→ We need to stabilize them before breaking SUSY.

For ~~non~~ compact moduli this problem does not exist.

For this reason we shall only consider the compact moduli (ϕ_a, χ_a).

combine to complex scalar.

For simplicity consider the case where there is only one ϕ_a.

→ The size modulus.

$$g_{m\bar{n}} = e^{2u} \tilde{g}_{m\bar{n}}$$

The Kahler modulus.

fixed metric on CY_3.

Its SUSY partner.

$$F_{m\bar{n}\mu\nu} = \chi_{\mu\nu} J_{m\bar{n}}$$

dualize to a scalar b.

$\rho = b + i e^{4u}$ is the chiral scalar.

$K_\rho = -3 \ln[-i(\rho - \bar{\rho})]$

Note: $\text{Im}\,\rho \propto \mathcal{V}^{2/3}$

$\mathcal{V} \propto e^{6u} \to$ Volume of CY_3 in Canonical metric.

$K_\rho = -2 \ln \mathcal{V}$

Compare that with heterotic on CY_3:

$K = -\ln \mathcal{V} \to$ volume in string metric

Similarly:

$K_\tau = -\ln(-i(\tau - \bar{\tau})) = -\ln(e^{-\Phi}) +$ constant

Φ: 10-d dilaton.

In heterotic:

$K_S = -\ln(-i(S - \bar{S})) = -\ln(e^{-2\Phi^{(4)}})$

$\Phi^{(4)}$: dilaton in 4-d