Comparison between 10-d and 4-d viewpoint

~~Only~~ Assume that ρ is the only Kahler modulus ($h_{1,1}=1$)

Results in 4-d:

$$V = e^{+K}\left(G^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2\right)$$

$$D_i W = \partial_i W + \partial_i K\, W.$$

$$K = -3\ln(-i(\rho-\bar{\rho})) - \ln(-i(\tau-\bar{\tau}))$$

$$- \ln\left(-\frac{i}{2}\int \Omega\wedge\bar{\Omega}\right)$$

$$W = \int G^{(3)}\wedge\Omega, \qquad G^{(3)} = F^{(3)} - \tau H^{(3)}.$$

$i: \rho, \alpha$

$\to$ complex structure, τ.

$$D_\rho W = \partial_\rho W - \frac{3}{\rho-\bar{\rho}} W = -\frac{3}{\rho-\bar{\rho}} W$$

Substitute into V:

$$V = e^{+K} G^{\alpha\bar{\beta}} D_\alpha W \overline{D_\alpha W}$$

$\to$ independent of ρ.

Minimization: $D_\alpha W = 0$.

Unbroken SUSY:

$D_i W = 0 \quad \forall i \rightarrow$ general result from Sugra.

$D_\rho W = 0 \Rightarrow W = 0.$

$\Rightarrow$ SUSY condition: $W=0,\ D_\alpha W = 0.$

Now compare with results from 10-d:

① We have already assumed that we have solved for background $\tau_{10\text{-}d}$ and metric on CY_3. ($F^{(3)}, H^{(3)}$ do not enter explicitly)

Source of trace reverse Einstein eq. involves:

$$\tilde{T}^{loc}_{mn} - \frac{1}{8}\tilde{g}_{mn}(\tilde{T}^{loc})^m{}_m$$

Vanish for 3-brane (only $\tilde{T}_{\mu\nu} \neq 0$).

② In this background switch on $F^{(3)}, H^{(3)}$.

eqs. of motion $\Rightarrow *_6 G^{(3)} = i G^{(3)}$

$\Downarrow$

$G^{(3)}$ is imaginary anti-self-dual.

Vanishing of total D3-brane charge

$$\int d^6y \sqrt{\det \tilde{g}} \left[\frac{i}{12\tau_2} \tilde{g}^{mm'} \tilde{g}^{nn'} \tilde{g}^{pp'} G_{mnp} \, {}_6{*}\bar{G}_{m'n'p'} + 2\kappa_{10}^2 e^{-6A} T_3 \rho_3^{loc} \right] = 0$$

$\Downarrow$

satisfy by place localized D3-branes (O3 planes).

$\Rightarrow$ will not affect the metric $\tilde{g}_{mn}$.

Finally solve eq. for $\alpha = e^{4A}$

α from $F^{(5)}$; e^{4A}: Warp factor of the metric

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A} \tilde{g}_{mn}(y) dy^m dy^n$$

/ physical.

Some mathematical results on CY_3:

① ${}_6{*}G = iG$

$\Rightarrow G$ is a linear combination of $(0,3)$ and $(2,1)$ forms.

② If Ω is a $(3,0)$ form then $\partial_A \Omega$ is an l.c. of $(3,0)$ and $(2,1)$ forms.

A: complex structure

③ Unbroken SUSY

$G^{(3)}$ is a $(2,1)$ form.

(no $(0,3)$ component).

~~Examine~~

$$W = \int G^{(3)} \wedge \Omega$$

$$\partial_A W = \int G^{(3)} \wedge \partial_A \Omega$$

$$K = -\ln\left(-\frac{i}{2}\int \Omega \wedge \bar{\Omega}\right)$$

$$\partial_A K = -\frac{\int \partial_A \Omega \wedge \bar{\Omega}}{\int \Omega \wedge \bar{\Omega}}$$

$$\Rightarrow D_A W = \partial_A W + \partial_A K \, W$$

$$= \int G^{(3)} \wedge \partial_A \Omega - \frac{\int \partial_A \Omega \wedge \bar{\Omega} \int G^{(3)} \wedge \Omega}{\int \Omega \wedge \bar{\Omega}}$$

$G^{(3)}$ has $(0,3)$ & $(2,1)$ component.

$\partial_A \Omega$ has $(3,0)$ and $(2,1)$ compnet.

$\Rightarrow$ Only the $(0,3)$ component of $G^{(3)}$ contributes

$$c \underset{\bar{\Omega}}{\Downarrow}$$

c — some constant.

$$D_A W = c \int \bar{\Omega} \wedge \partial_A \Omega - \frac{\int \partial_A \Omega \wedge \bar{\Omega} \; c \int \bar{\Omega} \wedge \Omega}{\int \Omega \wedge \bar{\Omega}}$$

$$= c \int \bar{\Omega} \wedge \partial_A \Omega - c \int \bar{\Omega} \wedge \partial_A \Omega = 0.$$

$$G^{(3)} = F^{(3)} - \tau \mathcal{H}^{(3)}$$

$$\partial_\tau W = \int \partial_\tau G^{(3)} \wedge \Omega$$

$$= - \int \mathcal{H}^{(3)} \wedge \Omega$$

$$D_\tau W = \partial_\tau W + \partial_\tau k \, W$$

$$= - \int \mathcal{H}^{(3)} \wedge \Omega - \frac{1}{\tau - \bar{\tau}} \int G^{(3)} \wedge \Omega$$

$$= - \int \frac{(\bar{G}^{(3)} - G^{(3)})}{\tau - \bar{\tau}} \wedge \Omega - \frac{1}{\tau - \bar{\tau}} \int G^{(3)} \wedge \Omega$$

$$= - \frac{1}{\tau - \bar{\tau}} \int \bar{G}^{(3)} \wedge \Omega \quad = 0$$

$(3,0)$

$(3,0) + (1,2)$

Thus eq. of motion from 10-d perspective $\Rightarrow$ 4-d eq. of motion.

<u>SUSY</u>

$G^{(3)}$ has only $(2,1)$ component.

$$\Rightarrow W = \int G^{(3)} \wedge \Omega = 0$$

$\rightarrow$ agrees with 4-d result.

So far we have fixed complex structure moduli & τ.

~~What is the order of the masses?~~

$\cancel{x = e^{-K} \times e \ \text{independent term}}$

$\cancel{e^{3(\alpha-\bar{\alpha})} \ln(\alpha-\bar{\alpha})^{3}}$

$\cancel{R \gg}$

For having control we need τ_2 large.

— Not true in general.

But for appropriate choice of fluxes τ_2 may be moderately large.

Invoke this mechanism.

e_0 is still not determined.

Not possible using perturbative means.

KKLT: Use non-perturbative effects.

LVS: Use α' corrections.

KKLT

Non-perturbatively generated superpotential

e.g. Euclidean D3 on 4-cycle of CY_3.

Area $\propto R^4 \propto \mathrm{Im}\,\rho$

$$\Delta W \propto e^{\overset{\text{constant}}{i c \rho}}$$

Look for susy solns. to $D_\alpha W = 0$, $D_\rho W = 0$

$\Downarrow$

Zeroeth order soln. exists.

ΔW corrects it.

Solve & 'integrate out' the τ and complex structure moduli.

$$\Rightarrow \quad W_{eff} = W_0 + A e^{i c \rho}$$

Constants.

$$K = -3 \ln(-i(\rho - \bar{\rho}))$$

$D_\rho W_{eff} = 0$

$\Rightarrow \partial_\rho W_{eff} + \partial_\rho K \, W_{eff} = 0.$

$$icA e^{ic\rho} - \frac{3}{\rho - \bar{\rho}} (W_0 + A e^{ic\rho}) = 0.$$

Take c, A, W_0 real.

Look for soln. $\rho = i\sigma$ (σ real.)

$$\Rightarrow \quad cA e^{-c\sigma} + \frac{3}{2\sigma} (W_0 + A e^{-c\sigma}) = 0$$

$$\Rightarrow W_0 = -\frac{2\sigma}{3} cA e^{-c\sigma} - A e^{-c\sigma}$$

$$= -A e^{-c\sigma} \left(1 + \frac{2c\sigma}{3}\right)$$

$$V = e^K (|D_\rho W|^2 - 3|W|^2)$$

$$= -\frac{c^2 A^2 e^{-2c\sigma}}{6} \Big|_{\sigma = \sigma_{cr}}$$

(σ_{cr} soln.)

$\rightarrow$ AdS vacuum.

Now we ~~...~~ have fixed all moduli.

$\rightarrow$ need to lift it to dS.

Strategy: Adjust fluxes so that we have large warping.

~~...~~ Placing D3 - brane at places of small $g_{\mu\nu}(y) \Rightarrow$ small tension.

$\sqrt{dit}g$ small.

Place $\overline{D3}$ -brane there to cancel the negative V & produce small +ve V.

(Adjust fluxes to satisfy D3-brane charge $= 0$ condition)