S.300

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Inclusion of gravity

Euclidean action:

$$S_E = \int d^4x \sqrt{\det g}\left(\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + V(\phi)\right)$$

$$+ S_{EH}$$

$x^0 = \tau$ → Euclidean time, x^i: $i = 1, 2, 3$ → space.

Look for $O(4)$ invariant solution:

$$\xi = \sqrt{\tau^2 + \vec{x}^2}$$

$$\phi = \phi(\xi), \quad ds^2 = d\xi^2 + \rho(\xi)^2 d\Omega_3^2$$

$\rho(\xi) = \xi$ → Flat space $\mathbb{R}^4$

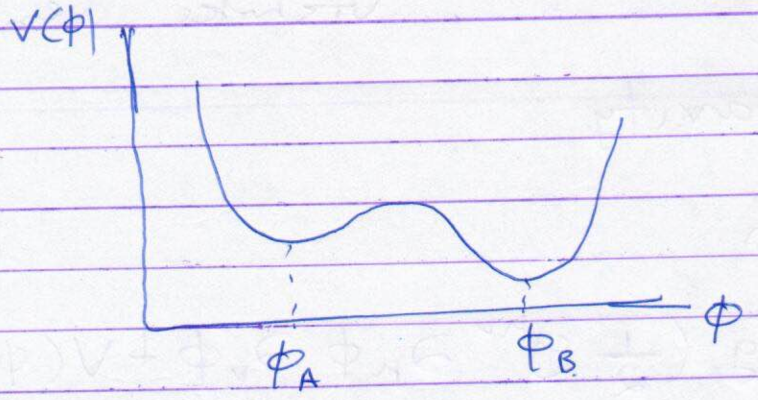
$$S_E = 2\pi^2 \int d\xi \left\{ \rho^3 \left(\frac{1}{2}\phi'^2 + V(\phi)\right) - \frac{3}{\kappa}\rho(\rho'^2 + 1) \right\} \qquad ' \equiv \partial_\xi$$

Eqs. of motion:

$$\phi'' + \frac{3\rho'}{\rho}\phi' = \frac{dV}{d\phi}$$

$$\rho'^2 = 1 + \frac{1}{3}\kappa\rho^2\left(\frac{1}{2}\phi'^2 - V\right), \quad \kappa = 8\pi G$$

↓ Analog of Freedman eqn.



False vacuum.

Thin wall approximation: $\phi_A - \phi_B = \epsilon$ small.

ϕ motion: Particle in potential $-V(\phi)$ with friction $\frac{3\rho'}{\rho}$.

Boundary conditions:

$\phi \to \phi_A$ as $\rho \to \infty$

$\to \phi_B$ as $\rho \to 0$.

We need to delay rolling from $\rho = 0$ by large amount so that the friction term can be neglected.

Define $U(\phi)$ s.t. $V(\phi) = U(\phi) + \epsilon W(\phi)$

$U(\phi_A) = U(\phi_B)$

Solve $\phi''(\xi) = \dfrac{\partial V(\phi)}{\partial \phi}$

$\rightarrow$ Motion in potential $-V(\phi)$.

Starts at ϕ_B as $\xi \to -\infty$ and reaches ϕ_A as $\xi \to \infty$.

Translation invariance in ξ.

$\Rightarrow$ general soln.

$\phi = F(\xi - \bar{\xi})$

Choose $\bar{\xi}$ such that $\phi = \frac{\phi_+ + \phi_-}{2}$ at $\xi = \bar{\xi}$.

Note: Here translation invariance in ξ is a symmetry of the full problem.

$\rightarrow$ part of general coordinate invariance.

$\Rightarrow$ Even in the full problem ξ can be changed at will.

$\bar{\rho} \equiv \rho(\bar{\xi}) \rightarrow$ Physical

$\rightarrow$ Value of ρ when $\phi = \frac{\phi_+ + \phi_-}{2}$.

Goal : Determine $\bar{\rho}$. by extremizing

$$S_E(\phi) - S_E(\phi = \phi_A).$$

$$S_E(\phi) = 2\pi^2 \int_0^\infty \left\{ \rho^3 \left(\tfrac{1}{2}\phi'^2 + V\right) - \frac{3}{\kappa}\rho(\rho'^2+1) \right\} d\xi.$$

$$\underset{\text{using } \rho \text{ eq.}}{=} 2\pi^2 \int_0^\infty \left[\rho^3\left(\tfrac{1}{2}\phi'^2 + V\right) - \frac{3}{\kappa}\rho\left\{2 + \tfrac{1}{3}\kappa\rho^2\left(\tfrac{1}{2}\phi'^2 - V\right)\right\}\right]$$

$$= 2\pi^2 \int_0^\infty d\xi \left(2\rho^3 V - \frac{6}{\kappa}\rho\right)$$

$$= 4\pi^2 \int_0^\infty d\xi\, \rho\left(\rho^2 V - \frac{3}{\kappa}\right)$$

$$B = S_E(\phi) - S_E(\phi_A)$$

$$= 4\pi^2 \int_0^\infty d\xi \left[\rho\left(\rho^2 V - \frac{6}{\kappa}\right) - \rho_0\left(\rho_0^2 V(\phi_A) - \frac{3}{\kappa}\right)\right]$$

Again divide the integration region into 3-parts.

Outside: $\rho \gg \bar{\rho}$

$\phi \to \phi_A \Rightarrow \rho(\xi) \to \rho_0(\xi)$

$\Rightarrow$ Integrand $\to 0$

Inside: $\rho \ll \bar{\rho}$, $\rho_0 \ll \bar{\rho}$

$\phi =$ constant (ϕ_B or ϕ_A)

$\underbrace{\rho'^2 = (1 - \frac{1}{3} \kappa \rho^2 V)}_{\text{constant.}}$

$$d\xi = \left(1 - \frac{1}{3}\kappa\rho^2 V\right)^{-1/2} d\rho$$

$$\Rightarrow B_{\text{inside}} = 4\pi^2 \int d\rho \left(1 - \frac{1}{3}\kappa\rho^2 V_B\right)^{-1/2} \left(\rho^3 V - \frac{3}{\kappa}\rho\right)$$

$$- (B \to A)$$

$$= \left(-\frac{3}{\kappa}\right) 4\pi^2 \int_0^{\bar{\rho}} d\rho\, \rho \left(1 - \frac{1}{3}\kappa\rho^2 V_B\right)^{1/2}$$

$$- (B \to A)$$

$$= -\frac{12\pi^2}{K} \cdot \frac{1}{2} \left(-\frac{3}{K V_B}\right) \cdot \frac{2}{3} \left(1 - \frac{1}{3} K \rho^2 V_B\right)^{3/2} \Big|_0^{\bar\rho}$$

$$- (B \to A)$$

$$= \frac{12\pi^2}{K^2} \left[\frac{1}{V_B} \left\{ \left(1 - \frac{1}{3} K \bar\rho^2 V_B\right)^{3/2} - 1 \right\} \right.$$

$$\left. - \frac{1}{V_A} \left\{ 1 - \frac{1}{3} K \bar\rho^2 V_B\right)^{3/2} - 1 \right\} \Big]$$

Note: $\frac{d}{d\bar\rho} B_{inside} = -\frac{12\pi^2 \bar\rho}{K}$

$$\times \left[\left(1 - \frac{1}{3} K \bar\rho^2 V_B\right)^{1/2} - \left(1 - \frac{1}{3} K \bar\rho^2 V_A\right)^{1/2} \right]$$

B_{wall} : Contribution from $\rho \sim \bar\rho$ region.

Set smoothly varying functions like ρ to be $\bar\rho$.

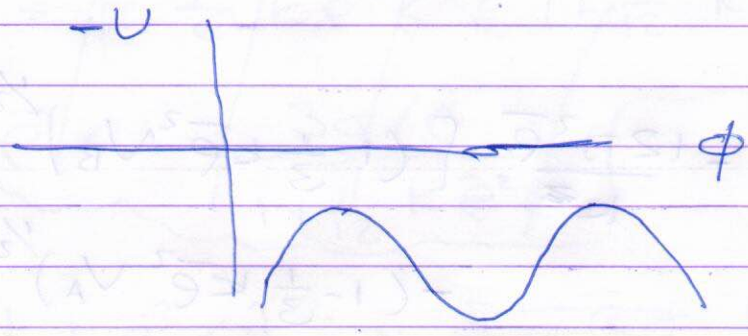
$$B_{inside} = -4\pi^2 \bar\rho^3 \int d\xi \, (V(\phi) - V_A)$$

$$= 2\pi^2 \bar\rho^3 \sigma$$

$$\sigma = \cancel{2\pi^2\rho^3} \overset{-2}{} \int d\xi \left(V(\phi) - V_A \right)$$

$$= \cancel{2\pi^2\rho^3}\, -2 \int d\xi \left(U(\phi) - U_A \right).$$

Recall particle motion in $-U(\phi)$.



$$\phi'' - U'(\phi) = 0.$$

$$\frac{d}{d\xi}\left(\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 - U(\phi) \right) = 0.$$

$$\Rightarrow \frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 - \left(U(\phi) - U(\phi_A) \right) = \text{Constant} = 0.$$

$$\sigma = \int d\xi \left(\underbrace{\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2}_{\text{KE}} - \underbrace{\left(U(\phi) - U(\phi_A) \right)}_{\text{PE}} \right)$$

→ Total energy of the kink

(ignore gravity here).

$B = 2\pi^2 \bar{e}^3 \sigma$

$+ \frac{12\pi^2}{K^2} \left[\frac{1}{V_B} \left\{ \left(1 - \frac{1}{3} K \bar{e}^2 V_B\right)^{3/2} - 1 \right\} \right.$

$\left. - \frac{1}{V_A} \left\{ \left(1 - \frac{1}{3} K \bar{e}^2 V_A\right)^{3/2} - 1 \right\} \right]$

Determine $\bar{e}$ using

$\frac{dB}{d\bar{e}} = 0.$

$6\pi^2 \bar{e}^2 \sigma - \frac{12\pi^2 \bar{e}}{K} \left[\left(1 - \frac{1}{3} K \bar{e}^2 V_B\right)^{1/2} - \left(1 - \frac{1}{3} K \bar{e}^2 V_A\right)^{1/2} \right] = 0$

$\bar{e}\sigma = \frac{2}{K} \left[\left(1 - \frac{1}{3} K \bar{e}^2 V_B\right)^{1/2} - \left(1 - \frac{1}{3} K \bar{e}^2 V_A\right)^{1/2} \right]$

Note: $K \to 0$ limit:

$\bar{e}\sigma = \frac{2}{K} \frac{1}{2} \cdot \frac{1}{3} K \bar{e}^2 (V_A - V_B)$
$\sim \epsilon$

$= \frac{1}{3} \bar{e}^2 \epsilon$

$\Rightarrow \bar{e} = 3\sigma/\epsilon \rightarrow$ old result in absence of gravity.

$V_B = V_A - \epsilon$

$\bar{e}\sigma = \frac{2}{K} \left(1 - \frac{1}{3} K \bar{e}^2 V_A\right)^{1/2} \left[\left(1 + \frac{\frac{1}{3} K \bar{e}^2 \epsilon}{1 - \frac{1}{3} K \bar{e}^2 V_A}\right)^{1/2} - 1 \right]$

Different cases:

de-Sitter → Minkowski:

KKLT → ($\bar{\rho} \to \infty$) vacuum.

$$V_A = \epsilon, \quad V_B = 0.$$

$$\bar{\rho}\,\sigma = \frac{2}{\kappa}\left[1 - \left(1 - \frac{1}{3}\kappa\bar{\rho}^2\epsilon\right)^{1/2}\right]$$

$$\cancel{= \frac{2}{\kappa}\frac{1}{6}\kappa\bar{\rho}^2\epsilon\left(1+\frac{1}{12}\kappa\bar{\rho}^2\epsilon\right)}$$

$$\cancel{\bar{\rho} = \frac{3\sigma/\epsilon}{1+\frac{1}{12}\kappa\bar{\rho}^2\epsilon}}$$

$$\left(1 - \frac{1}{3}\kappa\bar{\rho}^2\epsilon\right)^{1/2} = \cancel{\frac{2}{\kappa}\bar{\rho}\sigma}\; 1 - \frac{\kappa}{2}\bar{\rho}\sigma$$

$$1 - \frac{1}{3}\kappa\bar{\rho}^2\epsilon = 1 - \kappa\bar{\rho}\sigma + \frac{\kappa^2}{4}\bar{\rho}^2\sigma^2$$

$$\kappa\bar{\rho}\sigma = \bar{\rho}^2\left(\frac{\kappa^2}{4}\sigma^2 + \frac{1}{3}\kappa\epsilon\right)$$

$$\bar{\rho} = \frac{\cancel{\kappa}\,\sigma}{\frac{\kappa^{\cancel{2}}}{4}\sigma^2 + \frac{1}{3}\epsilon} = \frac{3\sigma/\epsilon}{1 + \frac{3}{4}\frac{\kappa\sigma^2}{\epsilon}}$$

Substituting in the expression for B we can compute B.

B → very large (Due to the small ϵ).

$$B = \frac{27\pi^2\sigma^4}{2\epsilon^3}\,\frac{1}{\left(1+\frac{3}{4}\frac{\kappa\sigma^2}{\epsilon}\right)^2}$$

Half life $\sim 1/B$ is large.

Minkowski $\rightarrow$ AdS. $V_B = -\epsilon$, $V_A = 0$

approximately $\overline{D3}$ gone.

our vacuum

$$\bar{e}\sigma = \frac{2}{k}\left(1-\frac{1}{3}k\bar{e}^2 V_A\right)^{1/2}\left[\left(1+\frac{\frac{1}{3}k\bar{e}^2\epsilon}{1-\frac{1}{3}k\bar{e}^2 V_A}\right)^{1/2}-1\right]$$

$$= \frac{2}{k}\left[\left(1+\frac{1}{3}k\bar{e}^2\epsilon\right)^{1/2}-1\right]$$

$$\Rightarrow \quad 1+\frac{1}{3}k\bar{e}^2\epsilon = \left(1+\frac{k}{2}\bar{e}\sigma\right)^2$$

$$= 1+k\bar{e}\sigma+\frac{1}{4}k^2\bar{e}^2\sigma^2$$

$$\Rightarrow \quad \cancel{k}\cancel{\bar{e}}\sigma = \frac{1}{3}\cancel{k}\bar{e}^2\epsilon\left(1-\frac{3}{4}\frac{k\sigma^2}{\epsilon}\right)$$

$$\bar{e} = \frac{3\sigma}{\epsilon}\,\frac{1}{1-\frac{3}{4}\frac{k\sigma^2}{\epsilon}}$$

Note: No soln. if

$$\epsilon < \frac{3}{4}k\sigma^2$$

$\Rightarrow$ The zero energy bubble cannot form

$\Rightarrow$ Decay cannot take place.

$$B = \frac{27\pi^2\sigma^4}{2\epsilon^3} \frac{1}{\left(1 - \frac{3}{4}\frac{\kappa\sigma^2}{\epsilon}\right)^2}$$

$\to \infty$ as $\epsilon \to \frac{3}{4}\kappa\sigma^2$

$\Rightarrow$ Decay probability $\sim e^{-B} \to 0$

AdS $\to$ AdS.

$$U_A = -\Lambda \qquad U_B = -\Lambda - \epsilon$$

$$\bar{\rho}\,\sigma = \frac{2}{\kappa}\left(1 + \frac{1}{3}\kappa\bar{\rho}^2\Lambda\right)^{1/2}$$

$$\left[\left\{1 + \underbrace{\frac{\frac{1}{3}\kappa\bar{\rho}^2\epsilon}{1 + \frac{1}{3}\kappa\bar{\rho}^2\Lambda}}_{\text{small}}\right\}^{1/2} - 1\right]$$

$$= \frac{2}{\kappa}\left(1 + \frac{1}{3}\kappa\bar{\rho}^2\Lambda\right)^{1/2} \frac{1}{2} \frac{\frac{1}{3}\kappa\bar{\rho}^2\epsilon}{\left(1 + \frac{1}{3}\kappa\bar{\rho}^2\Lambda\right)}$$

$$\Rightarrow \left(1 + \frac{1}{3}\kappa\bar{\rho}^2\epsilon\right)^{1/2}$$

$$= \frac{\cancel{2}}{\cancel{\kappa}}\frac{1}{\cancel{2}}\frac{1}{\bar{\rho}\sigma}\frac{1}{3}\kappa\bar{\rho}^2\epsilon = \frac{\epsilon\bar{\rho}}{3\sigma}$$

$$1+\frac{1}{3}\kappa\bar{\rho}^2\epsilon = \frac{\epsilon^2\bar{\rho}^2}{9\sigma^2}$$

$$1 = \frac{1}{3}\kappa\bar{\rho}^2\epsilon\left(-1+\frac{\epsilon^2}{9\sigma^2}\cdot\frac{3}{\kappa\epsilon}\right)$$

$$= \frac{1}{3}\kappa\bar{\rho}^2\epsilon\left\{\frac{\cancel{3}\epsilon}{3\cancel{3}\kappa\sigma^2}-1\right)$$

$$\bar{\rho}^2 = \frac{3}{\kappa\epsilon}\,\frac{1}{\frac{\cancel{3}\epsilon}{3\kappa\sigma^2}-1}$$

$$= \frac{3}{\cancel{\kappa}\epsilon}\,\frac{3\cancel{\kappa}\sigma^2}{\epsilon}\left(1-\frac{3\kappa\sigma^2}{\epsilon}\right)$$

$$= \frac{9\sigma^2}{\epsilon^2}\,\frac{1}{1-\frac{3\kappa\sigma^2}{\epsilon}}$$

Need $\epsilon > 3\kappa\sigma^2$

We shall use this to show that SUSY vacua cannot decay.