In all 5 string theories and 11-d sugra the equations of motion for all background fields $= 0$ on $M \times \mathbb{R}^{3,1}$ is

$$R_{mn} - \frac{1}{2} R g_{mn} = 0 \qquad (\mathbb{R}^{4,1} \text{ for sugra})$$

$\rightarrow$ satisfied by a Calabi-Yau 3-fold.

If we have Calabi-Yau N-fold then $M \times \mathbb{R}^{9-2N,1}$ will be a solution.

However higher derivative corrections typically disturb this.

IIA, IIB: First corrections occur at order α'^3 (8-derivative terms in action)

$$R_{M_1N_1P_1Q_1} R_{M_2N_2P_2Q_2} R_{M_3N_3P_3Q_3} R_{M_4N_4P_4Q_4}$$

$\times$ tensor contraction.

$\Rightarrow$ affects e.o.m.

$$R_{mn} - \frac{1}{2} R g_{mn} = (\ldots)$$

Result: Given a Calabi-Yau manifold with given complex structure and Kahler class, we can always find a unique metric that satisfies the eq.

$$K = K_{\text{Ricci-flat}} + \underbrace{\Delta K}_{\text{a globally defined scalar function.}}$$

$\exists$ We still have soln. to e.o.m.

Note: Holonomy $\notin$ SU(3).

What about SUSY?

Result: Susy trs. laws are also modified by higher derivative corrections so that the new metric preserves SUSY.

– Can be understood by analyzing string world-sheet theory in this background

World-sheet fields: X^μ, Y^m → Scalars

$$\underbrace{\chi^\mu, \psi^m}_{\text{right-handed fermions}}, \quad \underbrace{\bar{\chi}^\mu, \bar{\psi}^m}_{\text{left handed fermions.}}$$

World sheet action (2,2) SUSY

$S[X^\mu, Y^m, \chi^\mu, \psi^m, \bar{\chi}^\mu, \bar{\psi}^m]$.

↓

is conformally invariant when e.o.m. are satisfied.

$$\underbrace{R_{mn} - \frac{1}{2} g_{mn} R}_{\text{one-loop } \beta\text{-fn.}} + \underbrace{\cdots}_{\text{Four-loop onwards.}} = 0.$$

Once we have world-sheet (2,2) SUSY + conformal invariance. we have superconformal invariance.

⇒ Existence of R-symmetry currents.

↓

can be used to construct space-time SUSY generators.

→ Thus it is linked to conformal invariance on world-sheet ⟷ e.o.m. in space-time

Heterotic string theories:

When all background field vanish then in Sugra a CY-manifold $\times \mathbb{R}^{3,1}$ is a solution.

What about higher derivative corrections?

→ begins at order α'.

① a term proportional to α' $R_{MNPQ} R^{MNPQ}$ in action.

② Defn. of H now is modified:

$$\tilde{H} = dB + K\,\Omega_3(A) - K\,\Omega_3(\omega)$$

from sugra — spin connection

$$K = \frac{\alpha'}{4}$$

→ ~~For~~ ~~Also~~ Pure CY_3 with Ricci flat metric stops solving eqs. of motion from order α' onwards.

A more serious problem

$$d\tilde{H} = -k\,\mathrm{Tr}\,(R \wedge R) + k\,\mathrm{Tr}\,(F \wedge F).$$

For $F = 0$.

$$\int_{\substack{4\text{-cycle} \\ \text{of } CY_3}} d\tilde{H} = -k \int \mathrm{Tr}\,(R \wedge R)$$

$\Downarrow$

Must vanish if $\tilde{H}$ is a globally defined 3-form

$\Downarrow$

not zero in general (topological, does not change under small deformation of the metric).

Thus we have an inconsistency that cannot be removed by small deformation of the metric.

Remedy: Switch on gauge fields A_μ^a such that

$$\int [\mathrm{Tr}\,(F \wedge F) - \mathrm{Tr}\,(R \wedge R)] = 0$$

over all 4-cycles of CY_3.

A simple solution:

Identify an $SU(3)$ subgroup of the gauge groups $E_8 \times E_8$ and $SO(32)$ and switch on A_μ^a inside that $SU(3)$ so that

$$A_\mu = \omega_\mu.$$

$SU(3)$ gauge field → spin connection associated with $SU(3)$ holonomy.

In that case:

$$\Omega_3(A) - \Omega_3(\omega) = 0.$$

$$\mathrm{Tr}(F \wedge F) - \mathrm{Tr}(R \wedge R) = 0.$$

$\Rightarrow$ $\tilde{H} = 0$ for $B = 0$.

But gauge fields now contribute to T_{mn}.

Exactly cancels the extra contribution from $\alpha R_{mnpq} R^{mnpq}$ term in the action.

e.g. $E_8 \supset SU(3) \times E_6$.

$\Rightarrow$ Embedding spin connection in $SU(3)$

we get surviving group $E_6 \times E_8$

grand unified gauge group — Hidden sector

can be further broken to

$SU(3) \times SU(2) \times U(1) \times (\cdots)$ by mechanism

to be described later.

$SO(32) \supset SO(6) \times SO(26)$

$\cup$

$SU(3) \times U(1)$

embedd in this.

Surviving group: $U(1) \times SO(26)$

broken at one loop by Fayet-Illiopoulos term.

World-sheet understanding:

X^μ, Y^m: scalars.

ψ^μ, ψ^m: right-moving fermions λ^a: $a = 1, \ldots 32$

left-moving fermions.

In $SO(32)$ λ^a transforms in the fundamental of $SO(32)$

In $E_8 \times E_8$, λ^a transform in $(16,1)+(1,16)$ of $SO(16)\times SO(16) \subset E_8 \times E_8$.

What is the effect of ~~...~~ switching on gauge connection in $SU(3)$?

$\lambda^1, \cdots \lambda^6$ feels it.

$y^m, \psi^m, \lambda^1, \cdots \lambda^6$ form an interacting ~~...~~ SUSY σ-model.

$\Rightarrow$ Identical to the interacting part of type II σ-model.

Rest is free.

$\Rightarrow$ automatically superconformal

Same arguments which imply space-time SUSY ~~&~~ stability under higher derivative corrections for type II hold here.