Suppose that some particle freezes out at temp. T_F $(\lambda = \lambda_F)$.

interaction rate < expansion rate

$\downarrow$

At $T = T_F$, we have thermal distribution.

$$n(k)\,dk = \frac{g}{2\pi^2} k^2 dk \frac{e^{-\sqrt{k^2+m^2}/T_F}}{1 - e^{-\sqrt{k^2+m^2}/T_F}}, \quad e(k) = \sqrt{k^2+m^2}$$

Q. How does the spectrum look at later time?

We saw that a particle of energy $\sqrt{k^2+m^2}$ at $\lambda = \lambda_F$ has energy $\tilde{e} = \sqrt{\tilde{k}^2+m^2}$ where

$$\tilde{k} = k\lambda_F/\lambda.$$

At λ, the number of particles in the range $(\tilde{k}, \tilde{k}+d\tilde{k})$ is

$$\tilde{n}(\tilde{k})\, d\tilde{k} = \left(\frac{\lambda_F}{\lambda}\right)^3 n(k)\, dk$$

$$= \left(\frac{\lambda_F}{\lambda}\right)^3 k^2\, dk \; \frac{e^{-\sqrt{k^2+m^2}/T_F}}{1-e^{-\sqrt{k^2+m^2}/T_F}}, \qquad \tilde{k} = k\,\frac{\lambda_F}{\lambda}$$

$$= \tilde{k}^2\, d\tilde{k} \; \frac{e^{-\sqrt{\tilde{k}^2\lambda^2/\lambda_F^2+m^2}/T_F}}{1-e^{-\sqrt{\tilde{k}^2\lambda^2/\lambda_F^2+m^2}/T_F}} = \tilde{k}^2\, d\tilde{k} \; \frac{e^{-\sqrt{\tilde{k}^2+\tilde{m}^2}/\tilde{T}}}{1-e^{-\sqrt{\tilde{k}^2+\tilde{m}^2}/\tilde{T}}}$$

$$\tilde{T} = \frac{T_F\,\lambda_F}{\lambda}, \quad \tilde{m} = \frac{m\,\lambda_F}{\lambda}$$

$$\tilde{n}(\tilde{k})\, d\tilde{k} = \tilde{k}^2\, d\tilde{k} \quad \frac{e^{-\sqrt{\tilde{k}^2+\tilde{m}^2}/\tilde{T}}}{1-e^{-\sqrt{\tilde{k}^2+\tilde{m}^2}/\tilde{T}}}$$

$$\tilde{k} = \frac{k\lambda_F}{\lambda}, \quad \tilde{T} = \frac{T\lambda_F}{\lambda}, \quad \tilde{m} = \frac{m\lambda_F}{\lambda}, \quad e = \sqrt{\tilde{k}^2+\tilde{m}^2}$$

$$\rho = \int_0^\infty \tilde{n}(\tilde{k})\, d\tilde{k}\, \sqrt{\tilde{k}^2+\tilde{m}^2} \Rightarrow$$ the result as a function of λ which can be substituted into the Friedmann eqn.:

$$\tilde{n}(\tilde{k})\, d\tilde{k} = \tilde{k}^2\, d\tilde{k} \;\frac{e^{-\sqrt{\tilde{k}^2+\tilde{m}^2}/\tilde{T}}}{1 - e^{-\sqrt{\tilde{k}^2+\tilde{m}^2}/\tilde{T}}}$$

$$\tilde{k} = \frac{k\lambda_F}{\lambda}, \quad \tilde{T} = \frac{T_F\lambda_F}{\lambda}, \quad \tilde{m} = \frac{m\lambda_F}{\lambda}, \quad e = \sqrt{\tilde{k}^2+\tilde{m}^2}$$

Simple limits: (i) $\tilde{T} \gg m > \tilde{m}$

Dominant contribution comes from $\tilde{k} \sim \tilde{T} \gg m$

$\Rightarrow$ We can set $m=0$, $\tilde{m}=0$ in various eqs.

$\rightarrow$ thermal distr. of massless particles.

Extreme case: CMB

(2) Other limit: $\tilde{T} << m$.

$e^{-\sqrt{\tilde{k}^2+\tilde{m}^2}/\tilde{T}}$ For $k >> \tilde{T}$, this factor is suppressed.

$k \lesssim \tilde{T}$. $\tilde{k} << m$

$e = \sqrt{\tilde{k}^2+m^2} \simeq m$

In this case the particles become non-relativistic. $p=0$ matter.

$\rho = m\, n_F \lambda_F^3/\lambda^3$

Note: If dark matter consists of massive elementary particles, it had decoupled some time in the past.

$$\rho_{DM}\Big|_{today} = m n_F \lambda_F^3 / \underbrace{\lambda^3}_{=1 \text{ today}} = m n_F \lambda_F^3$$

Suppose a stable massive particle decouples at $T \gtrsim m \rightarrow$ mass of the particle.

$n_F \sim n_r\Big|_{T_F}$ r After this # density of and the decoupled particle falls off as $1/\lambda$

For $T > m$, the energy densities in the decoupled particles and γ are also of the same order (both $\sim \frac{1}{\lambda^4}$)

when $T < m$, energy density in $\gamma \sim \frac{1}{\lambda^4}$

but energy density in the decoupled particle $\sim \frac{1}{\lambda^3}$ $\Rightarrow$ start to dominate over energy density of radiation

If this happens at all, the universe must have ~~already~~ entered matter dominated phase.

Dark matter particles would have to be produced when they are non-relativistic so that the number density at the time of production is exponentially suppressed.

What we described is thermal freezeout.

But we could have chemical freezeout without thermal freezeout.

→ A system can remain in thermal equilibrium but fall out of chemical equilibrium

Rate of some processes that violate some 'conservation laws' become smaller than the expansion rate.

$\Rightarrow$ We get new conservation laws.

Effectively in the action we drop certain terms that become unimportant.

$\sim$ New action has additional conservation laws.

$\Rightarrow$ Rewrite the eqs. again.

The earlier description will provide the initial condition for the new $\tilde{n}_{(a)}$'s.

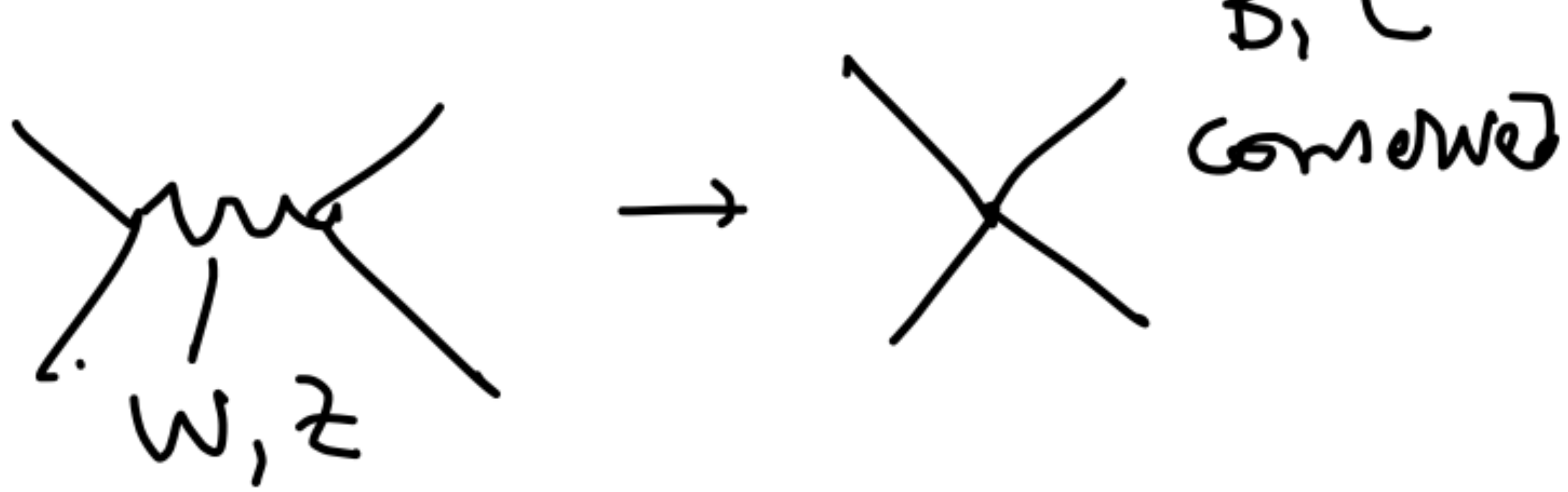
Example: Baryon number is anomalous in the standard model. (also lepton no.)

$B-L$ is conserved.

At $T \gtrsim 100\,\text{GeV}$, $B-L$ is conserved but not B, L

For $T << 100\,\text{GeV}$, B, L become approximate conserved quantities.

SM $\to$ Fermi theory



How to calculate the freeze out temp.

Consider a particle moving with speed v & suppose that n is the density of particle with which it can scatter & exchange energy.

σ: scattering <u>cross section</u>.

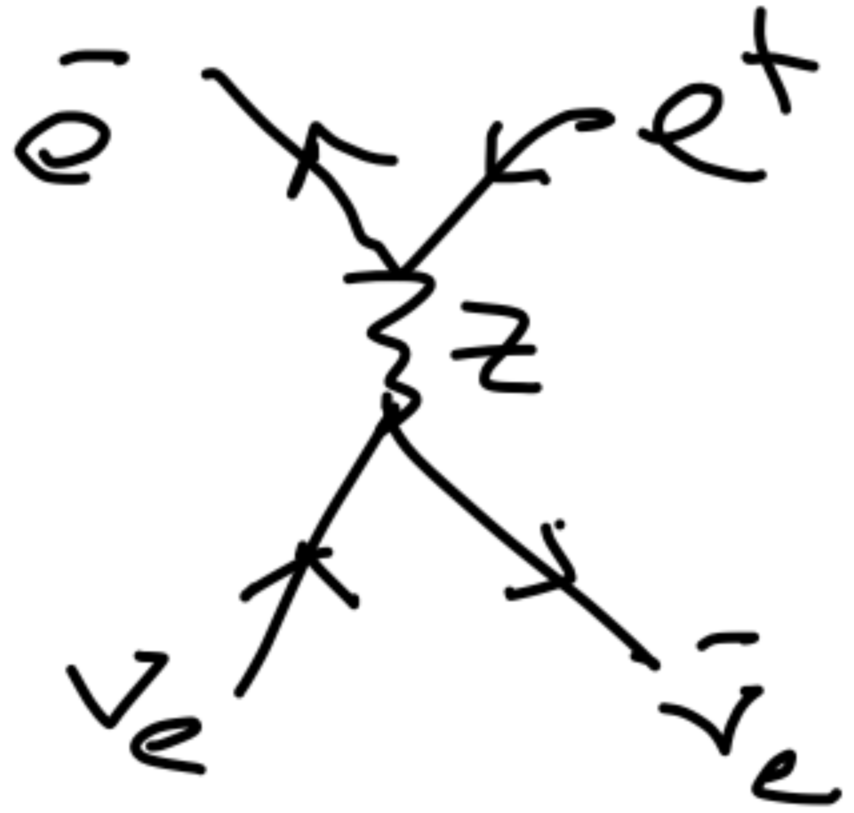
How many scattering will the particle have in time δt?

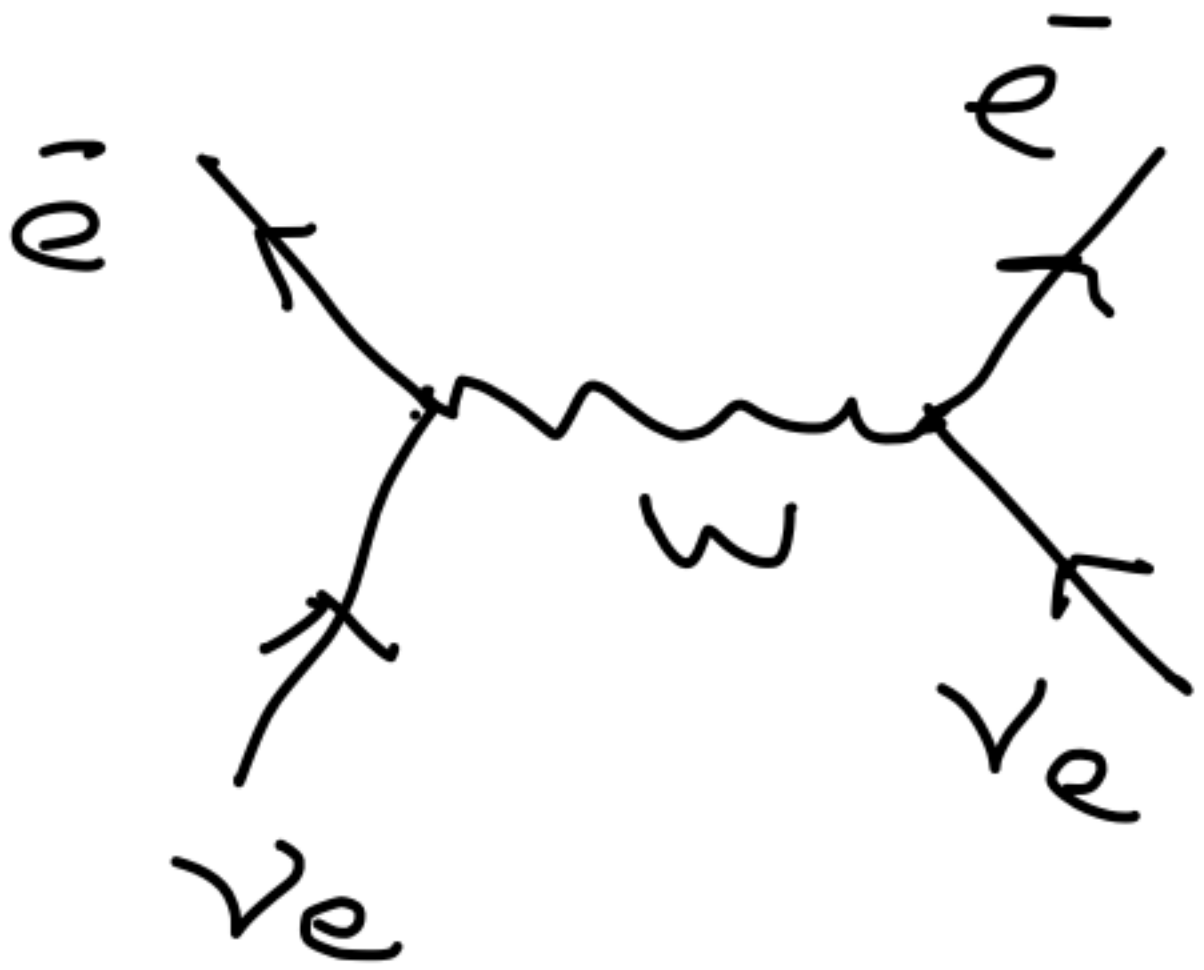
$\sigma v \delta t\, n$ $\Rightarrow$ For equilibrium need $\boxed{\sigma v n > \frac{\dot{\lambda}}{\lambda}}$.

Neutrino decoupling

weak interaction $\quad \mathrm{GeV} \gg T \gtrsim \mathrm{MeV}$

$$\nu_e \bar{\nu}_e \rightleftarrows e^+ e^- , \quad \nu_e e^+ \leftrightarrows \nu_e e^+ , \quad \nu_e e^- \rightleftarrows \nu_e e^-$$





etc.

$$\sigma \propto \left(\frac{g_W^2}{m_W^2, m_Z^2} \right)^2$$

$$\sim \left(\frac{1}{250\,\mathrm{GeV}} \right)^4$$

$$\sigma \propto \frac{1}{(250\,\text{GeV})^4} E^2 \rightarrow \text{energy of } \nu_e, e, \text{etc.}$$

$$\sim T$$

$$[L^2] \sim [M^{-2}]$$

$$\sigma \sim \frac{T^2}{(250\,\text{GeV})^4}$$

$$\sigma n v \sim \frac{T^5}{(250\,\text{GeV})^4}$$

$$n \sim T^3 \qquad v \sim 1$$

$$\frac{\dot{\lambda}}{\lambda} = \left(\frac{8\pi G}{3}\rho\right)^{1/2}$$ by Friedmann eq.

(ignored $\frac{k}{a_0^2 \lambda^2}$ term)

$$\sim \left(\frac{T^4}{m_{pl}^2}\right)^{2} \sim \frac{T^2}{m_{pl}}$$

$\hbar = c = 1$

$G = \frac{1}{m_{pl}^2}$

$\downarrow$

dimension M^{-2}

For equilibrium

$$\frac{T^5}{(250\,\mathrm{Gev})^4} \gtrsim \frac{T^2}{m_{pl}}$$

$$T^3 \gtrsim \frac{(250\,\mathrm{Gev})^4}{10^{19}\,\mathrm{Gev}} = \frac{10^{10}(\mathrm{Gev})^4}{10^{19}\,\mathrm{Gev}} \sim 10^{-9}(\mathrm{Gev})^3$$

$T_F \sim 10^{-3}\,\mathrm{Gev} \sim \mathrm{MeV}.$

After the freeze out neutrinos follow black body spectrum with

$$\boxed{T_\nu = \frac{\lambda_F T_F}{\lambda}} \rightarrow \text{assuming massless neutrinos.}$$

At $\lambda = \lambda_F$, $T_\gamma = T_\nu$

For $\lambda > \lambda_F$, $T_\nu = \dfrac{\lambda_F T_F}{\lambda}$, $\boxed{T_\gamma = \dfrac{\lambda_F T_F}{\lambda}}$

as long as all particles in equilibrium are relativistic. As T falls below $m_e \sim .5$ MeV.