Neutrinos decouple at $T \sim$ MeV.

After this $T_\nu = \frac{T_F \lambda_F}{\lambda}$ T_F: Freezeout temp.

λ_F: λ at freezeout.

e^+, e^-, γ remain in thermal equilibrium.

T: temperature of this system.

How does T evolve?

$s\lambda^3 = \text{constant} \Rightarrow s_F \lambda_F^3 = s\lambda^3$

E.M. interaction: $\frac{1}{m_W^2} \to \frac{1}{E^2} \sim \frac{1}{T^2}$

Apply this for $T << .5\,\text{MeV} \sim m_e \Rightarrow e^+e^-$ non-relativistic

$g_{eff}^{(1)} T_F^3 \lambda_F^3 = g_{eff}^{(2)} T^3 \lambda^3 = 2$ $g_{eff}^{(1)} = 2 + \frac{7}{8}(2+2) = \frac{11}{2}$

$T_F \lambda_F = T\lambda \left(\frac{4}{11}\right)^{1/3}$

$T_\nu = T\left(\frac{4}{11}\right)^{1/3}$

today $T_\nu = 2.73 \times \left(\frac{4}{11}\right)^{1/3}$

For massless neutrinos we have black body spectrum at temperature T_ν.

For massive neutrinos the distribution is more complicated but follows what we derived earlier.

The range $\cdot 1\,\text{MeV} \lesssim T \lesssim 1\,\text{MeV}$ is important for another reason.

We have excess baryons $\to$ in thermal equilibrium with the bath at temperature T.

Below T_F, $\not\exists$ neutrino freezeout, π is the bath that the baryons feel.

At $T \gtrsim \text{MeV}$, $(n_B \lambda^3)$ is constant but $n \leftrightarrow p$ transition is possible via weak interaction.

$n \to p + e^- + \bar{\nu}_e$, $n + \nu_e \rightleftarrows p + e^-, \cdots$

Nuclear binding energy ~ Mev/nucleon

$T <$ Mev, p, n will bind into Nuclei

p, n

→ By studying this process carefully we'll be able to predict the relative abundance of light elements (d, He^3, He^4, Li^3, ...)

Heavy elements are formed in stars.

Broad overview of nucleosynthesis

① Above $T > \text{MeV}$, when $n + \cdots \leftrightarrow p + \cdots$ is in equilibrium, we expect

$$\frac{n_n}{n_p} \sim e^{-(m_n - m_p)/T} \qquad m_n - m_p \sim \text{MeV}.$$

For $T \gg \text{MeV}$, $n_n \sim n_p$

For $T < \text{MeV}$, $\frac{n_n}{n_p}$ starts reducing.

At the same time, weak interaction falls out of equilibrium.

The reason that this number is not frozen is because n is not stable.

$$n \to p + e^- + \bar{\nu}_e$$

Half life of a free neutron ~ 13 minutes
~ 800 secs.

We need to compare this with the time scale that is involved as T varies between MeV and .1 MeV.

Rough estimate: $\left(\frac{\dot{\lambda}}{\lambda}\right)^2 = \frac{8\pi G}{3}\rho \simeq \frac{8\pi G}{3} \times T^4 \times \text{const.}$

$\lambda \sim \frac{1}{T}$ $\quad \left(\frac{\dot{T}}{T}\right)^2 \sim G T^4 \quad (\; G = 1$

$\sim T^4$

$\dot{T} = T^3 \qquad \frac{1}{T^2} \equiv t \qquad T \simeq t^{-1/2}$

$T = 1\,\text{MeV} = \frac{1\,\text{MeV}}{10^{19}\,\text{GeV}} \sim 10^{-22}$ in natural units.

$t \sim 10^{-44}\, \underbrace{t_{pl}}_{5\times 10^{44}\,\text{sec.}} \sim 1$ sec. $\qquad t \sim 100$ sec for $T \sim 1$ MeV

n ~~half~~ life

② Formation of bound state.

The beginning of the process requires some non-equilibrium process.

But once the formation starts, almost all n's get bound inside He^4 (very few remain in other light nuclei like D, He^3, ...) $\Rightarrow$ $\dfrac{n_n}{n_p} = \dfrac{2 n_{He^4} + \cdots}{2 n_{He^4} + n_H + \cdots}$

$$\Rightarrow \frac{n_{He^4}}{n_H} = \frac{n_n}{2(n_p - n_n)}$$

$\Rightarrow$ If we know n_n/n_p at the time of bound state formation then we know n_{He^4}/n_H.

Details:

Step I. We need to find time dependence of T and T_ν

T → Temperature of the bath

T_ν → determines the number of ambient $\nu, \bar{\nu}$ → affects $n \leftrightarrow p$ transition.

Steps for determining T, T_ν.

① Use $S\lambda^3 =$ const. to find $T \leftrightarrow \lambda$ reln.

② $T_\nu = T_F \lambda_F / \lambda$ to find $T_\nu \leftrightarrow \lambda$ reln.

③ $$\left(\frac{\dot{\lambda}}{\lambda}\right)^2 = \frac{8\pi G}{3}\left(\rho_{e^+e^-\gamma} + \rho_\nu\right)$$

$\rho_{e^+e^-\gamma} \to T$, $\rho_\nu \to T_\nu$

$\Rightarrow \lambda$ as fn. of t.

①, ② $\Rightarrow T, T_\nu$ as fn. of t

$$\text{(1)}\quad s_{e^+e^-\gamma} = \frac{2}{3} a_B T^3 \times \underset{\gamma}{2} + \frac{p_{e^+e^-} + \rho_{e^+e^-}}{T}$$

$$= \frac{4}{3} a_B T^3 + \frac{1}{2\pi^2} * (\overset{e^-}{2} + \overset{e^+}{2}) \int_0^\infty k^2\, dk \left[\ln\left(1 + e^{-\sqrt{k^2+m^2}/T}\right) + \frac{1}{T}\sqrt{k^2+m^2}\, \frac{e^{-\sqrt{k^2+m^2}/T}}{1 - e^{-\sqrt{k^2+m^2}/T}} \right]$$

$$=$$