For i-th species

$$\left.\begin{matrix} n_i \\ \bar{n}_i \end{matrix}\right\} = \frac{g_i}{2\pi} \int_0^\infty dk\, k^2 \, \frac{e^{-\frac{1}{T}\sqrt{k^2+m_i^2} \pm \frac{\mu_i}{T}}}{1+e^{-\frac{1}{T}\sqrt{k^2+m_i^2} \pm \frac{\mu_i}{T}}}$$

Suppose: $\frac{n_i - \bar{n}_i}{n_r} = \alpha << 1 \quad (\alpha \sim 10^{-9})$

Relativistic $\quad T \gtrsim m_i$

$$\left.\begin{matrix} n_i - \bar{n}_i \propto \frac{\mu_i}{T} \\ n_i, \bar{n}_i \sim n_r \end{matrix}\right\} \Rightarrow \frac{n_i - \bar{n}_i}{n_r} \sim \frac{\mu_i}{T} \sim \alpha$$

Small $\alpha \Rightarrow$ Small $\frac{\mu_i}{T}$

For the non-relativistic case $T \ll m_i$

$$n_i \simeq g_i \, e^{-(m_i - \mu_i)/T} \left(\frac{m_i T}{2\pi}\right)^{3/2}$$

$$\bar{n}_i \simeq g_i \, e^{-(m_i + \mu_i)/T} \left(\frac{m_i T}{2\pi}\right)^{3/2}$$

$$n_r \sim T^3$$

$$\frac{n_i - \bar{n}_i}{n_r} \sim \left(e^{\frac{\mu_i}{T}} - e^{-\frac{\mu_i}{T}}\right) e^{-m_i/T} \left(\frac{m_i}{2\pi T}\right)^{3/2} \sim \alpha$$

$$e^{\frac{\mu_i}{T}} - e^{-\frac{\mu_i}{T}} \sim e^{\frac{m_i}{T}} \left(\frac{2\pi T}{m_i}\right)^{3/2} \alpha \sim 1 \text{ for}$$

For $T < \frac{m_i}{20}$, $\frac{\mu_i}{T}$ is not small.

For $\alpha \sim 10^{-9}$ $\frac{m_i}{T} \sim 20$

For $T \lesssim \frac{m_i}{20}$ | $\frac{\mu_i}{T}$ is not small.

For n. $T \sim 50\,\mathrm{MeV}$

e: $T \sim \frac{.5}{20}\,\mathrm{MeV} \sim .025\,\mathrm{MeV}$.

For $T \sim \mathrm{MeV}$, $\frac{\mu_n}{T}$ not small.

$$e^{\frac{\mu_i}{T}} - e^{-\frac{\mu_i}{T}} \sim e^{m_i/T} \times \left(\frac{m_i}{2\pi T}\right)^{3/2} \times \alpha$$

$T \sim \mathrm{MeV}$ $\quad e^{1000} \Rightarrow \frac{\mu_i}{T} \sim 1000$

n_i

Goal: Study a system of $p, n, e^{\pm}, \nu_e, \bar{\nu}_e, \gamma$

At $T \gtrsim$ MeV the system is in equilibrium.

3 conserved charges:

1. Electric charge: $n_p - n_{\bar{p}} - n_{e^-} + n_{e^+}$
2. Baryon number: $n_p - n_{\bar{p}} + n_n - n_{\bar{n}}$
3. Lepton number: $n_{e^-} - n_{e^+} + n_{\nu_e} - n_{\bar{\nu}_e}$

In grand canonical ensemble, introduce in the exponent:

$$\tilde{\mu}_1(n_p - n_{\bar{p}} - n_{e^-} + n_{e^+}) + \tilde{\mu}_2(n_p - n_{\bar{p}} + n_n - n_{\bar{n}})$$

$$+\ \tilde{\mu}_3(n_{e^-} - n_{e^+} + n_{\nu_e} - n_{\bar{\nu}_e})$$

$\Rightarrow$

$$\mu_p = -\mu_{\bar{p}} = \tilde{\mu}_1 + \tilde{\mu}_2, \quad \mu_n = -\mu_{\bar{n}} = \tilde{\mu}_2$$

$$\mu_{e^-} = -\mu_{e^+} = -\tilde{\mu}_1 + \tilde{\mu}_3, \quad \mu_{\nu_e} = -\mu_{\bar{\nu}_e} = \tilde{\mu}_3$$

$$\bar{\nu}_\mu + p \rightleftharpoons \mu^+ + n$$

100 MeV.

We'll take μ_n, μ_e, μ_ν to be independent.

Ex. $\boxed{\mu_p = \mu_n - \mu_e + \mu_\nu}$

At $T \gtrsim$ MeV.

$$n_n = \frac{1}{2\pi^2} \times 2 \times \int_0^\infty dk\, k^2 \frac{e^{-\frac{\sqrt{k^2+m^2}-\mu_n}{T}}}{1+e^{-\frac{\sqrt{k^2+m^2}-\mu_n}{T}}}$$

$$\simeq 2\, e^{-\frac{m_n-\mu_n}{T}} \left(\frac{m_n T}{2\pi}\right)^{2/3}$$

$$n_{\bar{n}} = 2\, e^{-\frac{m_n+\mu_n}{T}} \left(\frac{m_n T}{2\pi}\right)^{2/3} \rightarrow \sim e^{-2000}$$

$$n_p = 2\, e^{-\frac{m_p-\mu_p}{T}} \left(\frac{m_p T}{2\pi}\right)^{2/3}, \quad n_{\bar{p}} \sim e^{-2000}$$

$$= n_n\, e^{-\frac{m_p-m_n}{T} + \frac{\mu_p-\mu_n}{T}} = n_n\, e^{\frac{Q}{T} - \frac{\mu_e}{T} + \frac{\mu_\nu}{T}}$$

$$Q = m_n - m_p$$

$$\frac{n_n}{n_p} = e^{-\frac{Q}{T} + \frac{\mu_e}{T} - \frac{\mu_\nu}{T}} \qquad Q = m_n - m_p \simeq 1.293\,\text{MeV}$$

For $T <$ MeV, $\frac{n_n}{n_p}$ begins to drop.

However at this stage ν's fall out of equilibrium. $T_\nu \neq T$.

$\Rightarrow$ The formula is not valid.

Ex. After decoupling, if $\mu_\nu \neq 0$ at T_F then the number distribution of ν's remain thermal with $\hat{T}_\nu = \frac{T_F \lambda_F}{\lambda}$ and $\mu_\nu = \frac{\mu_F \lambda_F}{\lambda}$.

For $T < T_F$, we have a neutrino system with (T_ν, μ_ν) and $e_+ e_- \gamma$ system with (T, μ_e)

p, n are in contact with this system.

Goal: Find how $\frac{n_n}{n_p}$ evolves with time.

Step 1. Find the rate of the process $n \to p$. $R(n \to p)$

$n + e^+ \to p + \bar{\nu}_e$, $n + \nu_e \to p + e^-$, $n \to p + e^- + \bar{\nu}_e$

Step 2 Find the rate of the process

$p \to n$ $R(p \to n)$

$p + \bar{\nu}_e \to n + e^+$, $p + e^- \to n + \nu_e$, $p + e^- + \bar{\nu}_e \to n$.

Step 3

$$\frac{d}{dt}(n_n \lambda^3) = -R(n \to p)\, n_n \lambda^3 + R(p \to n)\, n_p \lambda^3$$

$$\frac{d}{dt}(n_p \lambda^3) = -\frac{d}{dt}(n_n \lambda^3)$$

Example: Take the process $n + e^+ \to p + \bar{\nu}_e$

Rate of this per neutron (taken to be at rest)

$$\equiv \sigma \, v_{e^+} \, n_{e^+}$$

$$\Downarrow$$

$$2\pi^2 A E_\nu^2 / v_{e^+}$$

→ includes sum over both spins of e^+

$$\downarrow$$

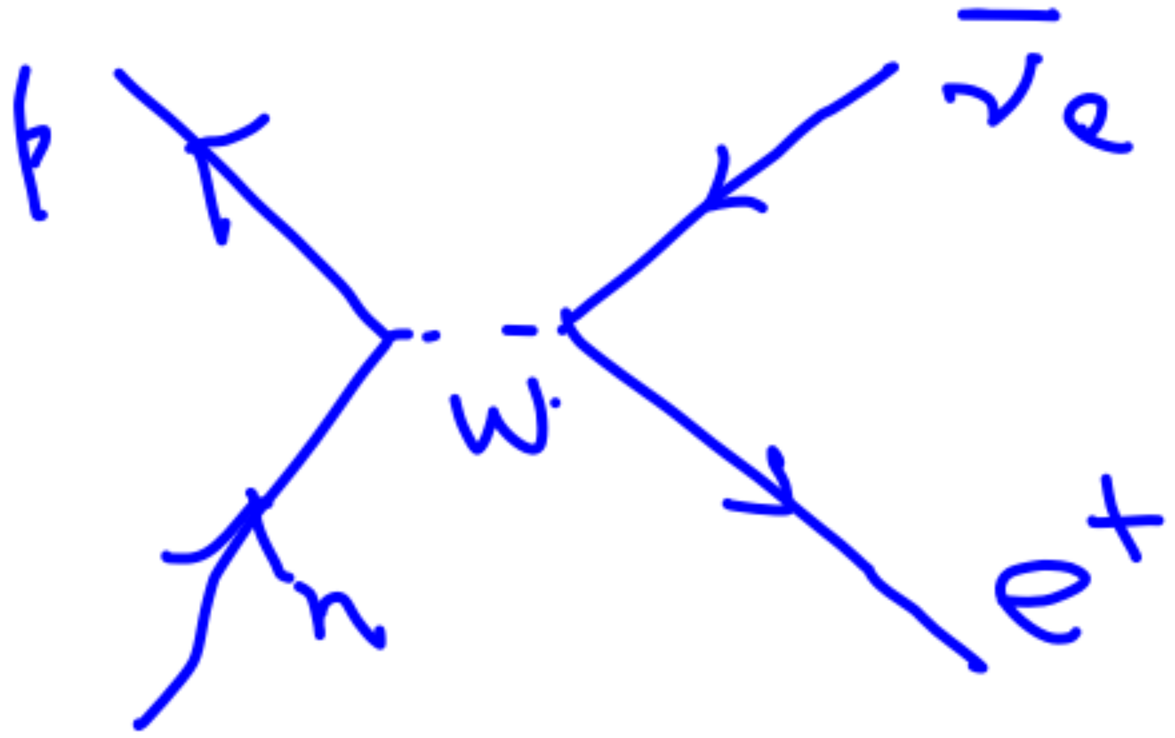
$$\frac{G_{weak}^2 (1+3g_A^2) \cos^2\theta_c}{2\pi^3}$$

E_ν: energy of $\bar{\nu}_e$ produced in the collision

$$G_{weak} \sim \frac{g_W^2}{m_W^2}$$

$g_A = 1.257$, $G_{weak} = 1.166 \times 10^{-5} (\text{GeV})^{-2}$ $\cos\theta_c = .975$

$$\bar{\psi}_p \gamma^\mu (1+g_A\gamma^5)\psi_n \quad \bar{\psi}_e \gamma_\mu (1+\gamma_5)\psi_{\nu_e} \times G_{weak} \times \cos\theta_c$$



up to an overall factor.

Contribution to $n \to p$ transition rate per neutron from $n + e^+ \to p + \bar{\nu}_e$:

$$\frac{1}{2\pi^2}\int p_e^2\, dp_e \, \frac{e^{-\frac{E_{e^+}+\mu_e}{T}}}{1+e^{-\frac{E_{e^+}+\mu_e}{T}}} \times 2\pi^2 A E_\nu^2 \left(1 - \frac{e^{-(E_\nu+\mu_\nu)/T_\nu}}{1+e^{-(E_\nu+\mu_\nu)/T_\nu}}\right)$$

probability that the positron state is actually occupied.

probability that the final state is unoccupied.

$$p_e = \sqrt{E_e^2 - m_e^2} = \sqrt{(q+Q)^2 - m_e^2}$$

$$q = -E_\nu$$

$$E_{e^+} = m_p + E_\nu - m_n = -Q - q \geq m_e$$

$$q < -Q - m_e$$

Ex. Check that the rate of $n \to p$ via

$n + e^+ \to p + \bar{\nu}_e$ is

$$A \int_{-\infty}^{-Q-m_e} dq \left(1 - \frac{m_e^2}{(q+Q)^2}\right)^{1/2} \frac{q^2 (q+Q)^2}{\left(1 + e^{-\frac{q+Q}{T} + \frac{\mu_e}{T}}\right)\left(1 + e^{\frac{q-\mu_\nu}{T_\nu}}\right)}$$

$n + \nu_e \to p + e^-$, $n \to p + \bar{\nu}_e + e^-$

Result: Same integral over different range of q.

$q \geq 0$

$-Q + m_e \leq q \leq 0$

$0 \geq -q \geq Q - m_e$

Total rates:

$$R(n \to p) = \int_{\substack{-\infty \\ |q+Q| \geq m_e}}^{\infty} dq \left(1 - \frac{m_e^2}{(q+Q)^2}\right)^{1/2} \frac{q^2 (q+Q)^2}{\left(1+e^{-\frac{q+Q-\mu_e}{T}}\right)\left(1+e^{\frac{q-\mu_\nu}{T_\nu}}\right)}$$

$$R(p \to n) = \int_{\substack{-\infty \\ |q+Q| \geq m_e}}^{\infty} dq \left(1 - \frac{m_e^2}{(q+Q)^2}\right)^{1/2} \frac{q^2 (q+Q)^2}{\left(1+e^{-\frac{q-\mu_\nu}{T_\nu}}\right)\left(1+e^{\frac{q+Q-\mu_e}{T}}\right)}$$

Ex: If $T_\nu = T$ $\quad R(p \to n) = e^{-\frac{Q}{T} + \frac{\mu_e - \mu_\nu}{T}} R(n \to p)$

$\Rightarrow$ In equilibrium: $\quad n_n R(n \to p) = n_p R(p \to n)$

$$\Rightarrow \boxed{n_n = n_p \, e^{-\frac{Q}{T} + \frac{\mu_e - \mu_\nu}{T}}}$$

derived earlier from equilibrium stat mech.