Nucleosynthesis: Numerical analysis

$\lambda(t)$, $T(t)$, $T_\nu(t)$ are determined from:

① Energy conservation (entropy conservation)

② Friedmann eq.

③ $T_\nu \propto \frac{1}{\lambda}$ after decoupling.

$\rightarrow$ not affected by nucleosynthesis.

$$n_B(t) = \frac{\Omega_B}{m_p} \frac{3H_0^2}{8\pi G} \left(\frac{T(t)}{T_0}\right)^3 \rightarrow \text{determines } n_B(t).$$

n_i: number density of ith nucleus, includes p, n

$$\sum n_i A_i = n_B$$

$\hookrightarrow$ No. of protons + neutrons in the nucleus.

$$\frac{d}{dt}(n_i \lambda^3) = [\text{rate of production} - \text{rate of destruction}] \text{ of } i\text{th type nucleus.}$$

Define $Y_i = n_i / n_B$

Since $n_B \lambda^3 = \text{constant}$, we can divide both sides by $n_B \lambda^3$ and take $\frac{1}{n_B \lambda^3}$ inside the derivative.

$$\Rightarrow \quad \frac{dY_i}{dt} = f_i(\{Y_k\}, t) \rightarrow \text{known.}$$

Initial conditions

$T > \text{MeV}$, $Y_i \simeq 0$ for $i = d, H^3, He^3, He^4, \ldots$

Y_n, Y_p given by the thermal equilibrium values.

General rule: Y_i's will try to approach the thermal equilibrium values.

If the reactions rates are slow to establish thermal equilibrium then $Y_i(t)$'s do not quite have their equilibrium values.

Between 1 MeV and 10^9 K (.086 MeV), p, n, d, γ system is in thermal equilibrium but the heavier nuclei are not.

We expect that the densities of p, n, d are given by their equilibrium values for given $T(t)$, and $X_n(t)$.
$\hookrightarrow$ calculated earlier.

At $T = 10^9$ K, n_d becomes large enough to bring the heavier nuclei in equilibrium.

At $T \sim 10^9$, n_d is large enough for full equilibrium <u>assuming</u> that p, n, d, γ system is in equilibrium but others are not.

$n_n \sim 10^{-48}$. $n_d = n_n n_p (\cdots)$

n_d will remain more or less constant at this critical value till all the free n's are exhausted and effectively gone into He^4!

After this, n_d will fall a bit but eventually n_d/λ^3 gets frozen to this critical value.

At this stage $H^3 + d \rightarrow He^4 + n$

$He^3 + d \rightarrow He^4 + p$

also fall out of equilibrium &

$n_{H^3}\lambda^3$, $n_{He^3}\lambda^3$, $n_{He^4}\lambda^3$ gets frozen.

Results:

$$\frac{n_d}{n_H} = (2.57 \pm .03) \times 10^{-5},$$

$$\frac{n_{H^3} + n_{He^3}}{n_H} \simeq (1.1 \pm 0.2) \times 10^{-5}$$

Some inferences from nucleosynthesis!

(1) $\frac{\rho_{He}}{\rho_H}$ determined by $X_n(t)$ ↳ time of nucleosynthesis.

determined by requiring

$T(t) \sim 10^9 K.$

Suppose the universe had extra light decoupled particles at the time of nucleosynthesis.

→ contribute to ρ → increase $\frac{\dot{\lambda}}{\lambda}$

→ faster expansion → $10^9 K$ is reached at smaller t ⇒ larger X_n ⇒ larger $\frac{\rho_{He}}{\rho_H}$

② Lessons from $Y_d = \frac{n_d}{n_B}$

Recall: n_d is determined from:

$$\frac{n_d \sigma_d v}{\dot{\gamma}/\gamma} \sim 1$$

σ_d : x-section for

$d + d \to H^3 + p$

$He^3 + n$

$$n_d = \frac{1}{\langle \sigma_d v \rangle} \sqrt{\frac{8\pi G}{3} \frac{1}{2} a_B T^4 \left(2 + \frac{7}{8} \times 6 \times \left(\frac{T_\nu}{T}\right)^4\right)}$$

$$n_B = \frac{1}{m_p} \frac{3H_0^2}{8\pi G} \Omega_B \left(\frac{T}{T_0}\right)^3 \Rightarrow Y_d = \frac{n_d}{n_B} \propto \frac{1}{\Omega_B}$$

$\Omega_B = .04$ is compatible with observed Y_d.

Production of heavier nuclei:

$$He^3 + He^4 \to Be^7 + \gamma$$
$$He^3 + He^4 \to Li^7 + \gamma$$

Electromagnetic Δ hence slow.

Eventually they form atoms by capturing electrons.

Atomic Be^7 decays to Li^7 by electron capture. $Be^7 + e^- \to Li^7 + \nu_e$ | $Y_{Li} \sim 10^{-10}$

Since the final composition is mainly p and He^4, one could consider:

$p + He^4 \to$ some nucleus with $A=5$
$+ e^{\pm}, \gamma$.

$He^4 + He^4 \to$ some nucleus with $A=8$
$+ e^{\pm}, \gamma$.

There is no stable nucleus with $A = 5, 8$. | $He^4 + He^4 \to \left.\begin{matrix} Li + p \\ Be + n. \end{matrix}\right\}$ Not allowed due to energy balance condition.

We now move forward in time.

1. Photons go out of equilibrium.
2. Universe goes from radiation to matter domination.
3. e^- binds to $He^{++} \rightarrow He^+ \rightarrow He$.
4. e^- binds to $p \rightarrow$ The universe becomes transparent.

CMB ~~today~~ gives an image of the universe at this time.

When $T \ll .5$ MeV, most of the $e^+ e^-$ disappear, but the extra e^-'s remain.

$n_{e^-} \sim 10^{-9} n_\gamma \rightarrow$ small no.

Nevertheless since $e^- \gamma \rightarrow e^- \gamma$ is electromagnetic scattering, it has reasonably large x-section and the photons can be kept in thermal equilibrium by this scattering.

Q. When does this stop?

$$n_{e^-} = e^{-(m_e-\mu)/T} \times \cdots$$

$$n_{e^+} = e^{-(m_e+\mu)/T} \times \cdots$$

$\gamma e^- \to \gamma e^-$ scattering x-section

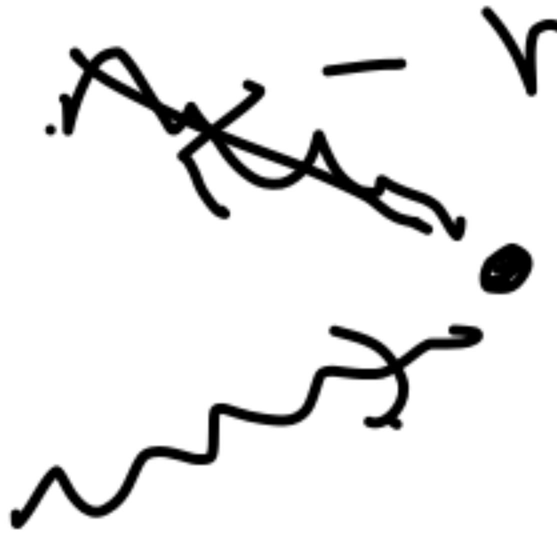
$$\sigma \simeq .67 \times 10^{-24}\ cm^2$$

Rate of scattering $= \sigma n_e c$

Naive guess: γ-freeze out happens when $\sigma n_e c = \frac{\dot{a}}{a} \Rightarrow$ Needs modification.

$\gamma e^- \to \gamma e^-$

γ has energy $\sim T$, momentum $\sim T$



e^- momentum $\sim T$

$m_e \gg T$

Momentum $\sim T$

e^- gets a momentum $\sim T$.
kinetic energy of order $\frac{T^2}{m_e}$ $\Rightarrow$ γ transfers an energy of order $\frac{T^2}{m_e}$ to e^-.

We need γ to transfer $\sim T$ energy / Hubble time.

$n + p \rightleftarrows d + \gamma$

Ex. n, p can remain in equilibrium even by elastic collision with γ around $T \sim \text{MeV}$.

Conclusion: We need $\frac{m_e}{T}$ collisions per Hubble time.

$$n_e \to .87\, n_B = .87 \times \frac{\Omega_B}{m_p} \frac{3H_0^2}{8\pi G} \times \left(\frac{T}{T_0}\right)^3$$

$$\Rightarrow \frac{\sigma n_e c}{\sqrt{\frac{8\pi G}{3} \frac{1}{2} a_B T^4 \left(2 + 6 \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3}\right)}} \sim \frac{m_e}{T}$$

Ex. Need $T > 1.5 \times 10^4 \, K \times (\underbrace{\Omega_b}_{.04} \underbrace{h^2}_{.7})^{-1/2}$

$\sim 10^5 \, K$

$H_0 = 100 \, h \, km/s/Mpc$

Below $10^5 K$, γ goes out of thermal equilibrium but it still has large no. of scatter / Hubble time.

Energy distribution continues to follow Planck distr. formula with $T \propto \frac{1}{\lambda}$.

e^- continues to be in thermal equilibrium with the photon bath since the # of collisions of e^-, p is 10^9 times that of γ.

$T \sim 10^5 K \sim 8.6 eV$.

$$\frac{m_e}{T} = \frac{.5 \times 10^6}{8.6} \sim 10^5$$