

Photon freezeout at 10^5 K.

Matter-Radiation equality:

$$\frac{1}{2} a_B T^4 \left(2 + 6 \times \frac{7}{8} \times \left(\frac{4}{11} \right)^{4/3} \right) = \Omega_m \frac{3 H_0^2}{8 \pi G} \left(\frac{T}{T_0} \right)^3$$

$\parallel$
 $\pi^2/15$

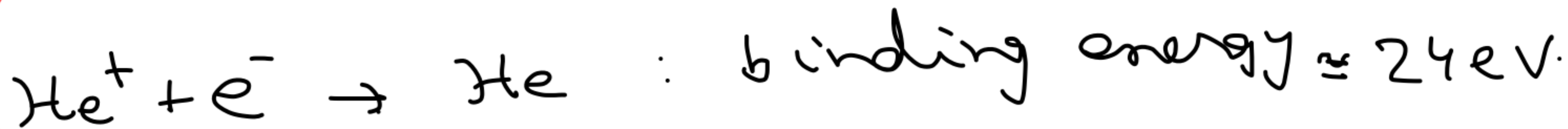
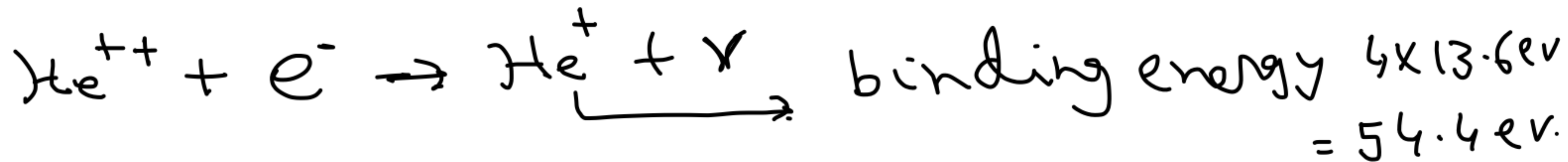
$$\Rightarrow T = 6.6 \times 10^4 \text{ K} \quad \Omega_m h^2$$

$\approx 10^4 \text{ K} \quad \cdot 3 \quad \searrow$

$$H_0 = 100 h \text{ km/s/Mpc.}$$

$$\Downarrow$$
$$10^4 \times 8.6 \times 10^{-5} \text{ eV} \approx 0.86 \text{ eV.}$$

Recombination: ions $\rightarrow$ atoms.



$\rightarrow$ remove certain fraction of free electrons.

$$X_n = .13, \quad X_p = 1 - X_n \approx .87$$

.13 : fraction of protons in He.
.74 : fraction of protons that are free.

$\left. \begin{array}{l} .13 \\ .74 \end{array} \right\} \text{fraction of electrons get bound}$

Consider a temperature T when the He atoms have already formed.

Important particles: p, e^-, γ , various states of H-atom (n, l) states.

n_e : no. density of free electrons

n_p : no density of free protons.

$n_{n,l}$: no density of H atoms in n, l state

$n_e = n_p$ due to charge neutrality.

Conserved charges

$$\text{Total \# electrons} = (n_e + \sum_{n,l} n_{n,l}) \lambda^3$$

$$\text{Total \# protons} = (n_p + \sum_{n,l} n_{n,l}) \lambda^3$$

Assume thermal equilibrium.
In grand canonical ensemble:

$$\tilde{\mu}_1 (n_e + \sum_{n,l} n_{n,l}) + \tilde{\mu}_2 (n_p + \sum_{n,l} n_{n,l})$$

$$= \mu_e n_e + \mu_p n_p + \sum_{n,l} \mu_{n,l} n_{n,l}$$

$$\Rightarrow \mu_e = \tilde{\mu}_1, \quad \mu_p = \tilde{\mu}_2, \quad \mu_{n,l} = \tilde{\mu}_1 + \tilde{\mu}_2 = \mu_e + \mu_p$$

$$n_\alpha = g_\alpha e^{-(m_\alpha - \mu_\alpha)/T} \left(\frac{m_\alpha T}{2\pi} \right)^{3/2} \quad \alpha = e^-, p$$

↓
degeneracy.

(n, l)

$$g_e = 2, \quad g_p = 2, \quad g_{n,l} = 2 \times 2 \times (2l+1).$$

$$n_e = 2 e^{-(m_e - \mu_e)/T} \left(\frac{m_e T}{2\pi} \right)^{3/2}$$

$$n_p = 2 e^{-(m_p - \mu_p)/T} \left(\frac{m_p T}{2\pi} \right)^{3/2}$$

$$n_{n,l} = 2 \times 2 \times (2l+1) \times e^{-(m_e + m_p - B_{n,l} - \mu_e - \mu_p)/T} \times \left(\frac{m_p T}{2\pi} \right)^{3/2}$$

$$= (2l+1) n_e n_p \left(\frac{m_e T}{2\pi} \right)^{-3/2} e^{B_{n,l}/T}$$

binding energy
↑

Use $n_e = n_p$

$$n_{n,l} = (2l+1) n_p^2 e^{B_{n,l}/T} \left(\frac{m_e T}{2\pi} \right)^{-3/2}$$

$$\frac{n_{n,l}}{n_{1s}} = (2l+1) e^{(B_{n,l} - B_{1s})/T} \approx (2l+1) e^{-\frac{13.6 \text{ eV}}{T} \times \left(1 - \frac{1}{n^2}\right)}$$

$\downarrow$
 $\frac{3}{4}$ for $n=2$

$$B_{1s} = 13.6 \text{ eV.}$$

$$B_{n,l} = \frac{13.6}{n^2}$$

$$T \sim 3000 \text{ K} \sim 3 \times 10^3 \times 8.65 \times 10^{-5}$$

$$= 2.4 \times 10^{-1} \text{ K}$$

$$\therefore e^{-\frac{13.6 \text{ eV}}{T}} \sim e^{-13.6 \times \frac{10}{2.4}} \sim e^{-60} \ll 1.$$

Define: $X = \frac{n_p}{n_p + n_{1,s}}$

$$1 - X = \frac{n_{1,s}}{n_p + n_{1,s}}$$

$$n_{n,l} = (2l+1) e^{B_{n,l}/T} \left(\frac{m_e T}{2\pi} \right)^{-3/2} \times \underbrace{n_e n_p}_{n_p^2}$$

$$1 - X = e^{B_{1,s}/T} \left(\frac{m_e T}{2\pi} \right)^{-3/2} \frac{n_p^2}{n_p + n_{1,s}}$$

$$= e^{B_{1,s}/T} \left(\frac{m_e T}{2\pi} \right)^{-3/2} X^2 \underbrace{(n_p + n_{1,s})}_{\cdot 74 \times \frac{\Omega_B}{m_p} \frac{3 H_0^2}{8\pi G} \left(\frac{T}{T_0} \right)^3}$$

$$1 - X = X^2 S$$

13.6 eV

157894 K/T

$$S = 1.75 \times 10^{-22} \left(\frac{T}{1 \text{ K}} \right)^{3/2} \Omega_B h^2 e$$

analog of ϵ in nucleosynthesis.

For $\Omega_m h^2 = 0.2$

($\Omega_m = .3$, $h = .9$)
 $h^2 = .81$

T (in K)	X
4500	.998
4000	.950
3800	.634
3600	.290
3400	.094
3200	.024

The actual process forms out of thermal equilibrium & hence is slower.

Thermal equilibrium $\Rightarrow$

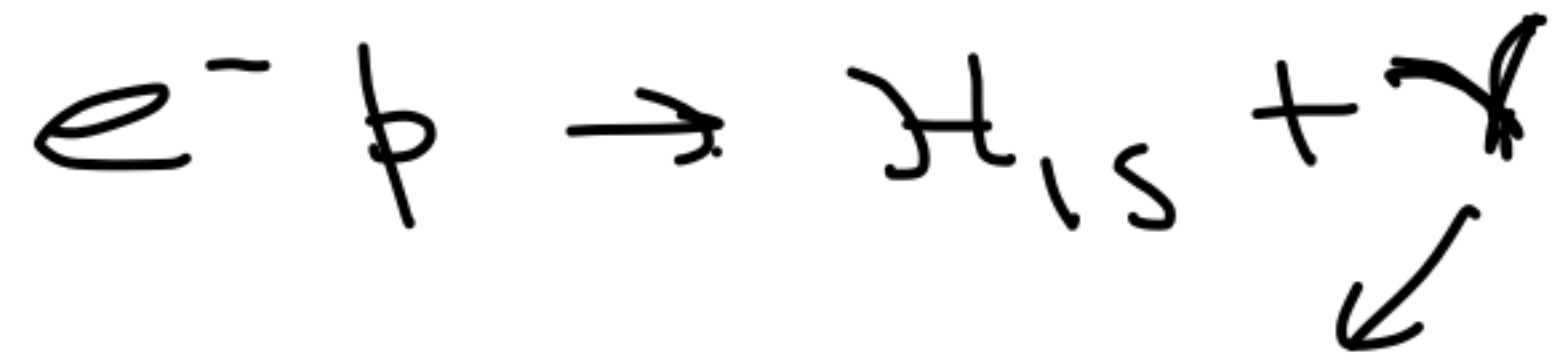
$e^- + p \rightarrow H + \gamma$ with certain frequency.

$\gamma + H \rightarrow e^- + p$ | why is density of H
so low at $T \sim 13.6 \text{ eV}$.

As T falls: For $\gamma + H \rightarrow e^- + p$, the γ needs

to have energy $\sim B_{\text{H}}$.
If $T \ll B_{\text{H}}$ then the fraction of γ with this energy $\propto e^{-\frac{B_{\text{H}}}{T}}$

Consider the process



has energy $\sim 13.6 \text{ eV}$.

How many such photons are present in the bath? $T \sim 3000 \text{ K}$

$$\exp(-13.6/T) \sim e^{-60} \sim 10^{-20}$$

No. of such photons in the bath $\sim n_r \times 10^{-20}$
of such photons produced in $e^- p$ binding $\sim 10^{-10} n_r$

The 13.6 eV photon cannot lose energy very fast.

→ cannot lose energy efficiently by scattering with electrons.

→ can lose energy due to redshift. but that also slow.

This photon will eventually hit an H which has formed in IS state and either kick the electron out or excite it to some (n, l) state.

Conclusion: we cannot populate $1s$ state by direct electron capture.

→ Mostly populated by $2p \rightarrow 1s + \gamma$

Another process: $2s \rightarrow 1s + \gamma + \gamma$
(slow)

Full treatment of e^-p recombination is done numerically.

$\frac{d}{dt}(n_{n,l} \lambda^3) = \text{rate of production} - \text{rate of annihilation}$

$\frac{d}{dt}(n_e \lambda^3), \quad \frac{d}{dt}(n_p \lambda^3) \rightarrow \text{not independent eqs.}$

arxiv: astro-ph/9909275

$\rightarrow$ uses 300 energy levels.

We'll use a simpler version

Regard this as a combination of 3 systems:

- ① Free e^- , p pair.
- ② All states in level (n, l) for $n \geq 2$
→ assumed to be in thermal equilibrium
- ③ H-atom in the 1s state.

Justification for treating (n, l) states with $n \geq 2$ in thermal equilibrium:

Energy difference $|E_{n, l} - E_{n', l'}| < \frac{13.6 \text{ eV}}{4}$
 γ that is produced during such transitions has $E < \frac{13.6 \text{ eV}}{4}$.

Fraction of γ in the bath with such E :
 $\exp\left(-\frac{13.6 \text{ eV}}{4T}\right) \quad T \sim 3000 \text{ K} \quad \left| \quad e^{-13.6 \text{ eV}/(4T)} \gg 10^{-10} \right|$