$$\frac{dx}{dt} = -\alpha x^2 + \beta \frac{1}{f(T)} (1-x-y)$$

$n_0 = .73\, n_B$

$x = n_e/n_0$, $y = \frac{n_{1s}}{n_0}$

$$\frac{dy}{dt} = (\Gamma_{2s} + 3\Gamma_{2p} P) \frac{1}{f(T)} (1-x-\cancel{y}) - \varepsilon y$$

We have described how to calculate the coefficients.

$\alpha, \Gamma_{2s}, \Gamma_{2p}$ need to be calculated in the presence of the thermal bath.

Recall that for processes with ν final state we had to multiply by $\frac{1}{1+e^{-E_\nu/T}}$ $\Rightarrow$ Prob. that the final neutrino state is unoccupied

For bosons we have a similar factor

$$\frac{1}{1-e^{-E_\gamma/T}} \Rightarrow \text{enhances the process.}$$

$\Rightarrow$ induced process.

Not very important for Γ_{2s}, Γ_{2p}

$$E_\gamma = (B_1 - B_2) \qquad \exp\left(-\frac{B_1-B_2}{T}\right) \ll 1$$

Not for $e^- p \to H + \gamma \Rightarrow$ For this we need to account for the factor.

α: Involves transition to any (n, l) state with $n \geq 2$

See Weinberg's book for more detailed discussion.

$$\frac{dx}{dt} = \cdots, \quad \frac{dy}{dt} = \cdots, \quad \frac{d}{dt}(1-x-y) = \cdots$$

Compare this with what we would get by assuming thermal equilibrium.

$\Downarrow$

Sets all the RHS to 0.

$X(T)$, $Y(T)$. T is time dependent.

t dependence of T is much slower than the reaction rates.

$$\frac{dx}{dt} = \underset{\Lambda_1}{\uparrow}(\cdots), \quad \frac{dy}{dt} = \underset{\Lambda_2}{\uparrow}(\cdots), \quad \frac{d}{dt}(1-x-y) = \underset{\Lambda_3}{\uparrow}(\cdots)$$

In the limit $\Lambda_1, \Lambda_2, \Lambda_3 \to \infty$, the soln. will be given by that obtained assuming thermal equilibrium.

<u>HW I</u>: Solve the coupled eqs. for $\frac{dx}{dt}, \frac{dy}{dt}$.

$$1 \longrightarrow 2 \longrightarrow 3$$

$e^- p$ n, l, $n \geq 2$ $1s$

$$\frac{1}{f(T)}(1-x-y)\left(\beta + \Gamma_{2s} + 3P\,\Gamma_{2p} + \varepsilon f(T)\right) = \alpha x^2 n_0 + \varepsilon(1-x).$$

$$\frac{d}{dt}(1-x-y) \simeq 0$$

R.H.S. of $\frac{dx}{dt} + \frac{dy}{dt}$ can be set to zero.

$$-\alpha x^2 n_0 + \frac{\beta}{f(T)}(1-x-y) = -\left(\Gamma_{2s} + 3P\,\Gamma_{2p}\right)\frac{1}{f(T)}(1-x-y)$$

$$+\, \varepsilon y$$

$$-\,\varepsilon(1-x-y) + \varepsilon(1-x)$$

$$\frac{dx}{dt} = -\alpha x^2 n_0 + \frac{\beta}{\xi(T)}(1-x-y)$$

$$= -\alpha x^2 n_0 + \beta \frac{\alpha x^2 n_0 + \varepsilon(1-x)}{\Gamma_{2s}+3P\Gamma_{2p}+\beta+\cancel{\varepsilon\,\xi}(T)}$$

$$\varepsilon = \left(\Gamma_{2s}+3P\Gamma_{2p}\right) e^{-(B_1-B_2)/T} \ll \Gamma_{2s}+3P\Gamma_{2p}$$

$$\frac{dx}{dt} = -\alpha x^2 n_0 \frac{\Gamma_{2s}+3P\Gamma_{2p}}{\Gamma_{2s}+3P\Gamma_{2p}+\beta} + \frac{\Gamma_{2s}+3P\Gamma_{2p}}{\Gamma_{2s}+3P\Gamma_{2p}+\beta}(1-x)$$

$$\times \beta \times e^{-(B_1-B_2)/T}$$

$$\beta = \alpha \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-B_2/T}$$

$$\frac{dX}{dt} = \frac{\Gamma_{2s} + 3P\,\Gamma_{2p}}{\Gamma_{2s} + 3P\,\Gamma_{2p} + \beta}\,\alpha\left[-X^2 n_0 + \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-B_1/T}(1-X)\right]$$

$$= \frac{\Gamma_{2s} + 3P\,\Gamma_{2p}}{\Gamma_{2s} + 3P\,\Gamma_{2p} + \beta}\, n_0\left[-X^2 + S^{-1}(1-X)\right]$$

This eq. is driving X ~~to~~ towards equilibrium value.

$$S = n_0\left(\frac{m_e T}{2\pi}\right)^{-3/2} e^{B_1/T}$$

$S X^2 = (1-X) \rightarrow$ condition derived assuming thermal equilibrium.

Overall one finds that rate of decrease of X is slower than the one we get assuming thermal equilibrium.

For $\Omega_B h^2 = .02$, thermal equilibrium

$\Rightarrow X = .00493$ at $3000K$.

Diff. eq. $X = .154$ at $3000K$.

Given $X(t)$, how do we decide which surface we should call the last scattering surface?

Define Opacity $G(T)$: The fraction of CMB photons at time $t(T)$ that undergoes at least one scattering before it reaches us.

Higher opacity $\Rightarrow$ less visibility.

Consider a CMB photon at time t.

Define $q(t',t)$ as the probability that it does not scatter till time t'.

$$\frac{dq(t',t)}{dt'} = -\left(\sigma_T\, n_e(t') \cancel{+\sigma_c\, n_e(t')}\right) q(t',t)$$

elastic scattering.

$$q(t_0,t) = \exp\left(-\int_t^{t_0} \left(\sigma_T\, n_e(t') \cancel{+\sigma_c\, n_e(t')}\right) dt'\right)$$

today

$$O(T) = 1 - q(t_0,t)$$

$1 - G(T) = q(t_0, t)$: Fraction of the CMB photons at t that reaches us with no scattering in between.

$\rightarrow$ can also be interpreted as the fraction of CMB photons ~~today~~ that had ~~their~~ last scattering before t. (at temp $> T$)

Fraction of CMB photons that had their last scattering in the temp range $(T, T+dT)$. $\qquad T(t), \; t(T)$

$$= G'(T)\, dT.$$

$$G(T) = 1 - \exp\left(-\int_t^{t_0} \sigma_T\, n_{\cancel{fs}\,e}(t')\, dt'\right).$$

$$G'(T)\, dT = -\exp\left(-\int_t^{t_0} \sigma_T\, n_{\cancel{fs}\,e}(t')\, dt'\right)$$

$$\sigma_T\, n_{\cancel{fs}\,e}(t)\, dt \rightarrow \left(\frac{dT}{dt}\right)^{-1} dT$$

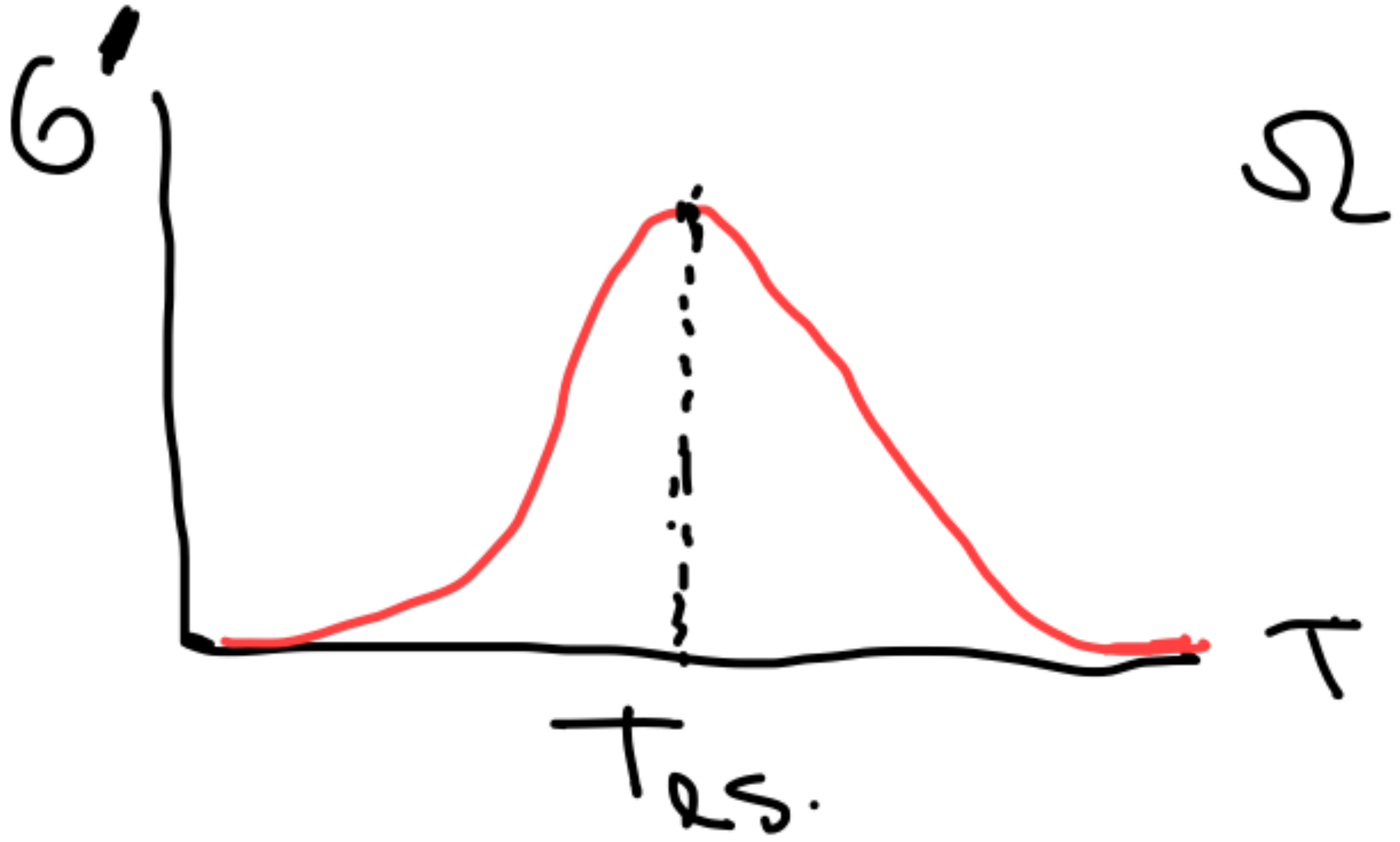
$$T = \frac{T_0}{\lambda}$$

$$\dot{T} = -\frac{T_0}{\lambda^2}\dot{\lambda} = -T H(t)$$

$$= \frac{(1-G(T))\, \sigma_T\, n_{e\,\cancel{fs}}(t)}{dT/(T H(t))}$$

$$G'(T) = (1 - G(T))\, \sigma_T\, n_{\cancel{\text{is}}\,e}(t)\, \frac{1}{H(t)\, T}$$

Recall that $G'(T)\, dT$ is the fraction of CMB photons whose last scattering happened in temp. range $(T, T+dT)$.



$\Omega_B h^2 = .02$ the peak is at $2954\,K \pm 253\,K$.

This part of the study of cosmology is based on SM physics.

$1 \div 10\,\mathrm{MeV}$

Thermal equilibrium erases memory of the past (mostly).

Some data need to be provided as initial condition:

① Values of conserved charges

baryon no.
lepton no.

② Relic density of particles that have fallen out of equilibrium.

dark matter

Do we have non-zero lepton no?

→ We don't know.

We do not know if there is a difference between $\nu, \bar{\nu}$ numbers.

If we want to explain the origin of dark matter or non-zero baryon no. we have go beyond SM.

— Incorporate in the QFT new fields with new interactions. ⇒ Will be studied next.

The other thing we cannot study within standard cosmology is the origin of fluctuations.

→ Requires going beyond SM (inflation).