Issues: In SM baryon no. is anomalous.

→ not conserved at high temp. $T \gtrsim 100$ GeV.

Electroweak phase transition is 2nd order.

⇒ the universe does not go out of equilibrium.

→ no baryon no. generation.

— any baryon no. produced in the early universe will be wiped out.

Only way out: Some exactly conserved quantity in SM was produced in the early universe - necessary but not sufficient in general.

Conserved charges in SM:

(1) $B-L$ $\hookrightarrow$ lepton no.

$\Downarrow$

not anomalous

1 for $e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau$

-1 for $e^+, \bar{\nu}_e, \mu^+, \bar{\nu}_\mu, \tau^+, \bar{\nu}_\tau$

0 for everything else.

② $L_i - L_j$ $\quad i, j : e, \mu, \tau$

$L_e - L_\mu$: $\quad 1$ for $e^-, \nu_e, \mu^+, \bar{\nu}_\mu$

$\qquad\qquad -1$ for $e^+, \bar{\nu}_e, \mu^-, \nu_\mu$

Similarly $L_e - L_\tau$

$Y = I_3 - Q$

$u: \frac{1}{2} - \left(\frac{2}{3}\right) = -\frac{1}{6}$

$d = -\frac{1}{2} + \frac{1}{3} = -\frac{1}{6}$

③ Gauge charges: $SU(3) \times SU(2) \times U(1)$

$SU(3) \rightarrow$ ignore

$SU(2) \rightarrow I_3 = \begin{pmatrix} \frac{1}{2} & \\ & -\frac{1}{2} \end{pmatrix}$

$U(1) \rightarrow Y \rightarrow$ weak hypercharge

$\begin{pmatrix} u \\ d \end{pmatrix}$ $\quad u$ has $I_3 = \frac{1}{2}$

$\qquad\quad d$ has $I_3 = -\frac{1}{2}$

$Q = I_3 - Y$

Q. Given the conserved charge densities $\tilde{n}_{(l)}$, how do we calculate the baryon no. density n_B?

Assumption: $\tilde{n}_{(l)}/n_r$ is small.

If $\tilde{n}_{(l)} = 0$ for every l, then $\tilde{\mu}_{(l)} = 0$.

$\Rightarrow$ If $\tilde{n}_{(l)}$'s are small, then $\tilde{\mu}_{(l)}$ must be small.

$$\tilde{n}_{(\ell)} = \sum_\alpha c^{(\ell)}_\alpha (n_\alpha - \bar{n}_\alpha), \quad \mu_\alpha = \sum_\ell c^{(\ell)}_\alpha \tilde{\mu}_{(\ell)} = -\bar{\mu}_\alpha$$

$$n_\alpha = \frac{g_\alpha}{2\pi^2}\int_0^\infty k^2\, dk\, \frac{1}{e^{\frac{\sqrt{k^2+m_\alpha^2}-\mu_\alpha}{T}} \mp 1} \longrightarrow \int_0^\infty \frac{k^2\, dk}{e^{\frac{k-\mu_\alpha}{T}} \mp 1}$$

$$\bar{n}_\alpha = \frac{g_\alpha}{2\pi^2}\int_0^\infty k^2\, dk\, \frac{1}{e^{\frac{k+\mu_\alpha}{T}} \mp 1}$$

$$n_\alpha - \bar{n}_\alpha = \frac{g_\alpha}{2\pi^2}\int_0^\infty k^2\, dk\, \frac{e^{\frac{k}{T}} \cdot 2\sinh\frac{\mu_\alpha}{T}}{\left(e^{\frac{k+\mu_\alpha}{T}} \mp 1\right)\left(e^{\frac{k-\mu_\alpha}{T}} \mp 1\right)}$$

$$\simeq \frac{g_\alpha}{2\pi^2}\cdot\frac{2\mu_\alpha}{T}\int_0^\infty k^2\, dk\, \frac{e^{k/T}}{\left(e^{k/T} \mp 1\right)^2}$$

$$n_\alpha - \bar{n}_\alpha = \frac{g_\alpha}{2\pi^2} \frac{2\mu_\alpha}{T} \int_0^\infty k^2 dk \frac{e^{k/T}}{(e^{k/T} \mp 1)^2}$$

$$\underset{x = \frac{k}{T}}{=} \frac{g_\alpha}{\pi^2} \frac{\mu_\alpha}{T} \cdot T^3 \int_0^\infty x^2 dx \frac{e^x}{(e^x \mp 1)^2}$$

$$= \frac{\mu_\alpha}{6} \bar{g}_\alpha T^2$$

$\frac{\pi^2}{3}$ for B

$\frac{\pi^2}{6}$ for F.

$\bar{g}_\alpha = g_\alpha$ for F

$= 2 g_\alpha$ for B.

$$n_\alpha - \bar{n}_\alpha = \frac{1}{6} \mu_\alpha \bar{g}_\alpha T^2$$

$$\tilde{n}_{(\ell)} = \sum_\alpha C_\alpha^{(\ell)} (n_\alpha - \bar{n}_\alpha) = \sum_\alpha C_\alpha^{(\ell)} \frac{1}{6} \mu_\alpha \bar{g}_\alpha T^2$$

$$= \frac{1}{6} T^2 \sum_\alpha C_\alpha^{(\ell)} \bar{g}_\alpha \sum_k C_\alpha^{(k)} \tilde{\mu}_{(k)}$$

$$= \frac{1}{6} T^2 \sum_k M_{\ell k} \tilde{\mu}_{(k)}, \qquad M_{\ell k} \equiv \sum_\alpha C_\alpha^{(\ell)} C_\alpha^{(k)} \bar{g}_\alpha$$

$$\tilde{\mu}_{(k)} = \frac{6}{T^2} \sum_\ell (M^{-1})_{k\ell}\, \tilde{n}_{(\ell)}$$

$$\tilde{\mu}_{(k)} = \frac{6}{T^2} \sum_{l} (M^{-1})_{kl}\, \tilde{n}_{(l)}$$

$$\mu_\alpha = \sum_k C^{(k)}_\alpha\, \tilde{\mu}_{(k)} = \frac{6}{T^2} \sum_{k,l} C^{(k)}_\alpha (M^{-1})_{kl}\, \tilde{n}_{(l)}$$

$$n_\alpha - \bar{n}_\alpha = \frac{1}{6} T^2 \bar{g}_\alpha \mu_\alpha = \bar{g}_\alpha \sum_{k,l} C^{(k)}_\alpha (M^{-1})_{kl}\, \tilde{n}_{(l)}$$

$$n_B = \sum b_\alpha (n_\alpha - \bar{n}_\alpha) = \sum_\alpha \sum_{k,l} b_\alpha \bar{g}_\alpha C^{(k)}_\alpha (M^{-1})_{kl}\, \tilde{n}_{(l)}$$

baryon no. of α-th particle

$$\vartheta_k = \sum_\alpha b_\alpha \bar{g}_\alpha C^{(k)}_\alpha$$

ϑ_k

$$= \sum_{k,l} \vartheta_k (M^{-1})_{kl}\, \tilde{n}_{(l)}$$

$$n_B = \sum_{k,l} \vartheta_k (M^{-1})_{kl}\, \tilde{n}_{(l)}$$

$$\vartheta_k = \sum_\alpha b_\alpha C_\alpha^{(k)} \bar{g}_\alpha, \qquad M_{kl} = \sum_\alpha \bar{g}_\alpha C_\alpha^{(k)} C_\alpha^{(l)}$$

$$n_B = \sum_\alpha b_\alpha (n_\alpha - \bar{n}_\alpha), \qquad \tilde{n}_{(l)} = \sum_\alpha C_\alpha^{(l)} (n_\alpha - \bar{n}_\alpha)$$

In SM: $B-L$, Y, I_3, $L_e - L_\mu$, $L_e - L_\tau$.

$\Rightarrow$ M is a 5×5 matrix, ϑ: 5 dim vector.

We'll argue that M is block diagonal and $\vartheta_k \neq 0$ only for $k = B-L, Y$.

2×2 ( ) ( )

$$M_{kl} = \sum_\alpha \bar{g}_\alpha C_\alpha^{(k)} C_\alpha^{(l)} \quad \Big| \quad \tilde{n}_{(l)} = \sum C_\alpha^{(l)} (n_\alpha - \bar{n}_\alpha)$$

Suppose that $k = B-L, Y$, $\quad l = I_3$.

Claim: For every α, either $C_\alpha^{(l)} = 0$

or there is an α' such that

$$C_{\alpha'}^{(k)} = C_\alpha^{(k)}, \quad C_{\alpha'}^{(l)} = -C_\alpha^{(l)}.$$

$$\begin{pmatrix} u \\ d \end{pmatrix}.$$

$SU(3) \times SU(2) \times U(1)$

Example: If $\alpha = u$, $\alpha' = d$.

$\Rightarrow \quad M_{kl} = 0$

$$M_{k\ell} = \sum_\alpha \bar{g}_\alpha C^{(k)}_\alpha C^{(\ell)}_\alpha , \quad \tilde{n}_{(\ell)} = \sum_\alpha C^{(\ell)}_\alpha (n_\alpha - \bar{n}_\alpha)$$

Suppose that $k = B-L, Y$, $\ell = L_e - L_\mu$.

For every α, either $C^{(\ell)}_\alpha = 0$ or there is an α'' such that

$$C^{(k)}_{\alpha''} = C^{(k)}_\alpha , \quad C^{(\ell)}_{\alpha''} = - C^{(\ell)}_\alpha .$$

$L_e - L_\mu$

$$\left.\begin{array}{l} e^- \leftrightarrow \mu^- , \quad \nu_e \leftrightarrow \nu_\mu \\ e^+ \leftrightarrow \mu^+ , \quad \bar{\nu}_e \leftrightarrow \bar{\nu}_\mu \end{array}\right\} \Rightarrow M_{k\ell} = 0$$

Same argument holds for $\ell = L_e - L_\tau$

Conclusion: M is block diagonal

$$\begin{array}{cc} & \begin{array}{ccccc} B-L & Y & I_3 & L_e-L_\mu & L_e-L_\tau \end{array} \\ \begin{array}{c} B-L \\ Y \\ I_3 \\ L_e-L_\mu \\ L_e-L_\tau \end{array} & \begin{pmatrix} \times & \times & 0 & 0 & 0 \\ \times & \times & 0 & 0 & 0 \\ 0 & 0 & & & \\ 0 & 0 & & & \\ 0 & 0 & & & \end{pmatrix} \end{array}$$

$$\vartheta_k = \sum_\alpha \bar{g}_\alpha \, b_\alpha \, C^{(k)}_\alpha \qquad \tilde{n}_{(k)} = \sum_\alpha C^{(k)}_\alpha (n_\alpha - \bar{n}_\alpha)$$

if $k = I_3$, then for every α, either $C^{(k)}_\alpha = 0$, or there is α' such that

$$b_{\alpha'} = b_\alpha, \quad C^{(k)}_{\alpha'} = - C^{(k)}_\alpha \Rightarrow \vartheta_k = 0$$

Same reasoning holds for $k = L_e - L_\mu, L_e - L_\tau$

$$\vartheta = (X \; X \; 0 \; 0 \; 0)$$

$$n_B = \sum_{k,l} \vartheta_k \left(M^{-1}\right)_{kl} \tilde{n}_{(l)}$$

non-zero for $k = B-L, Y$

must be $B-L$ or Y

$\tilde{n}_Y = 0$

$$n_B = \sum_{k=B-L,Y} \vartheta_k \left(M^{-1}\right)_{k,B-L} \tilde{n}_{(B-L)}.$$

$\tilde{n}_{L_e-L_\mu}$, $\tilde{n}_{L_e-L_\tau}$ may be non-zero.

$\boxed{g\, H \bar{\Psi} \Psi}$

Result in SM.

(For 3 generations, 1 Higgs doublet)

$79-28$

$$n_B = \frac{28}{79} \tilde{n}_{B-L}$$

$$n_L = n_B - \tilde{n}_{B-L} = -\frac{51}{79} \tilde{n}_{B-L} = -\frac{51}{28} n_B.$$

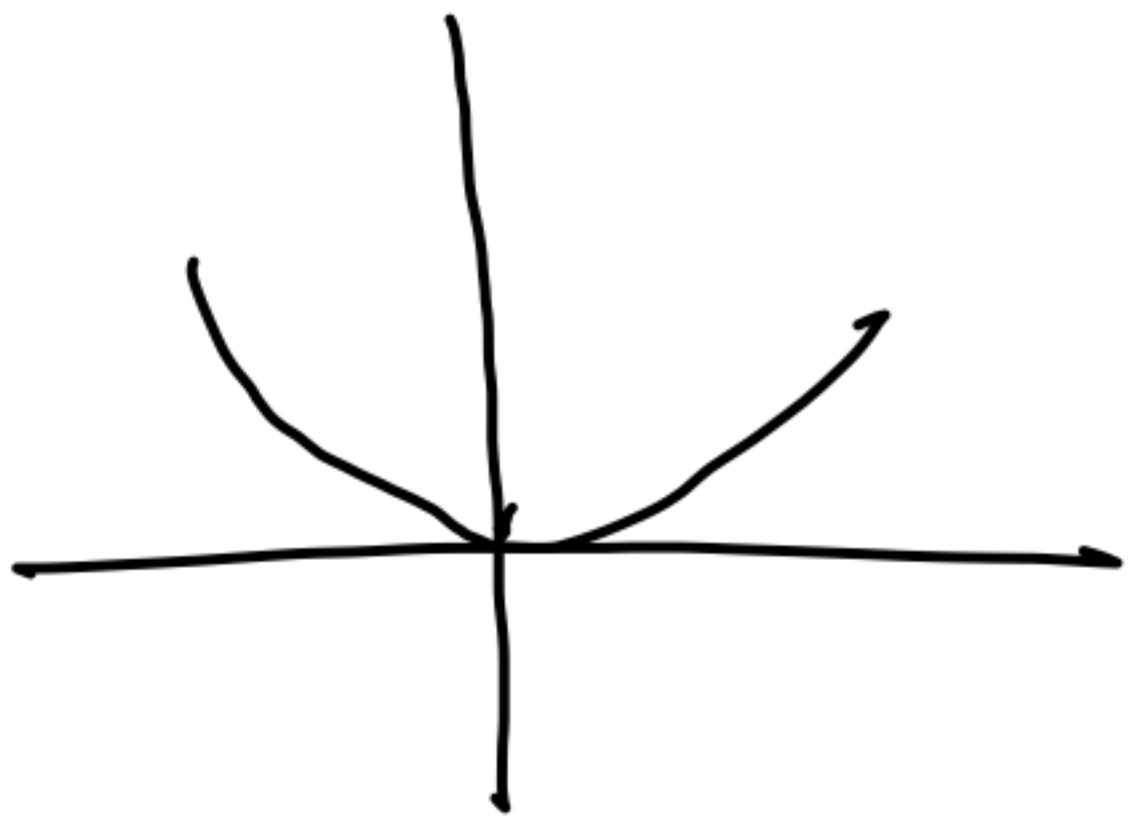
$$n_{e^-} - n_{e^+} = n_p = .87\, n_B.$$

$$n_\nu - \bar{n}_\nu = n_L - (n_{e^-} - n_{e^+}) = -\frac{51}{28} n_B - .87\, n_B$$

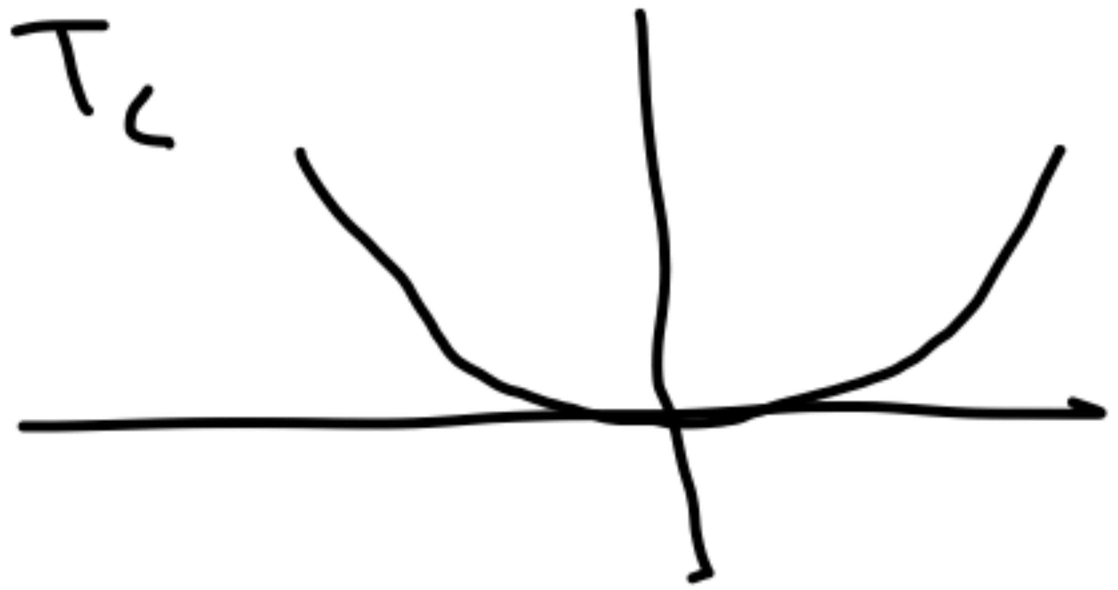
$\tilde{n}_{L_e - L_\mu}$, $\tilde{n}_{L_e - L_\tau}$

$d + u \to e^+ + \bar{u}$

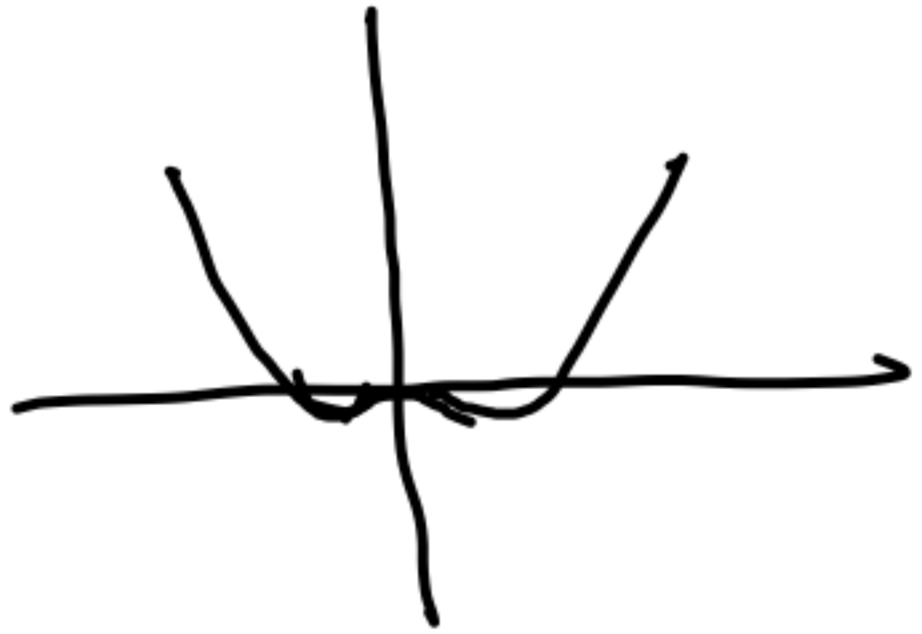
$B-L = \frac{2}{3} \qquad 1 - \frac{1}{3} = \frac{2}{3}$

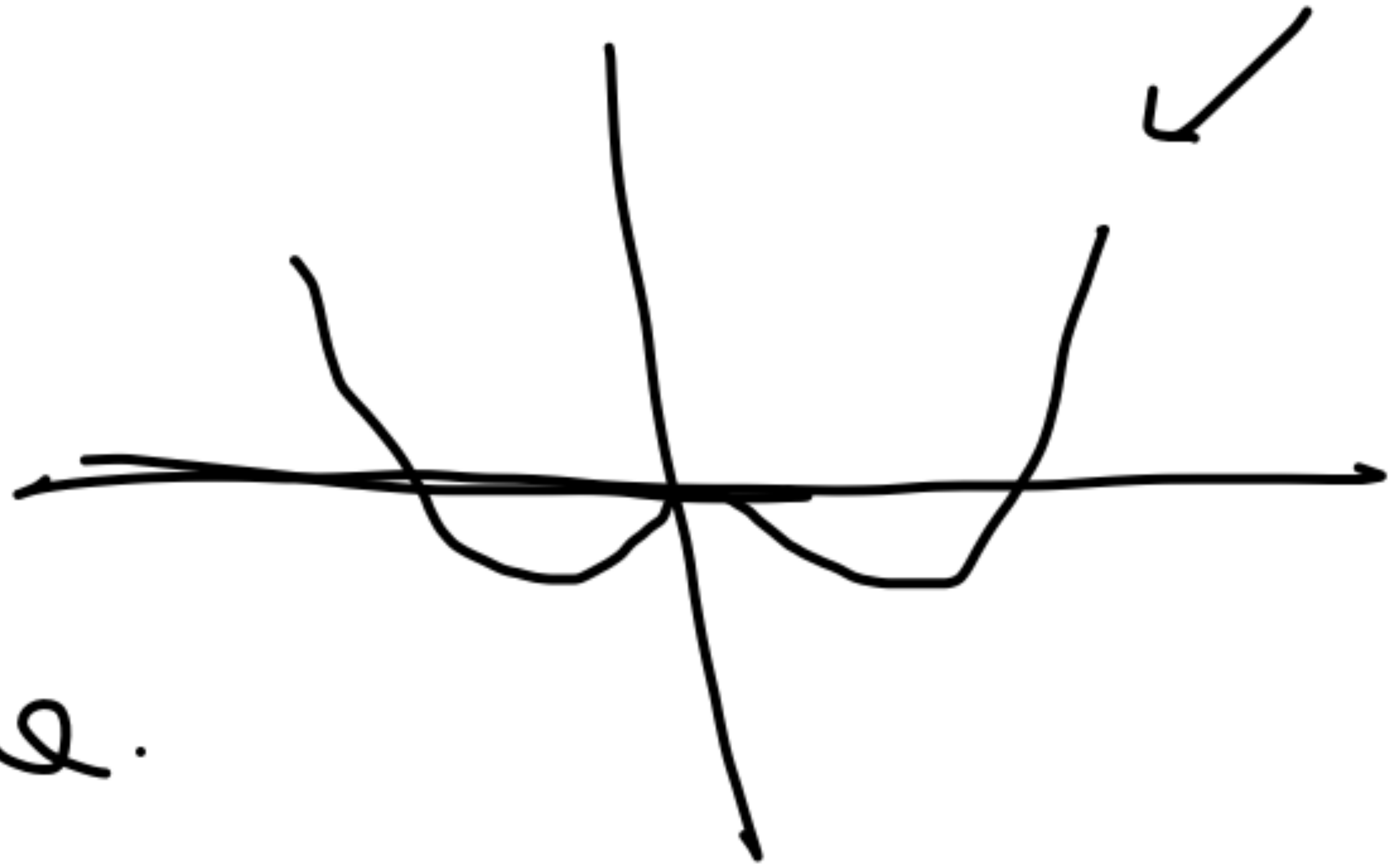


$T \to T_c$



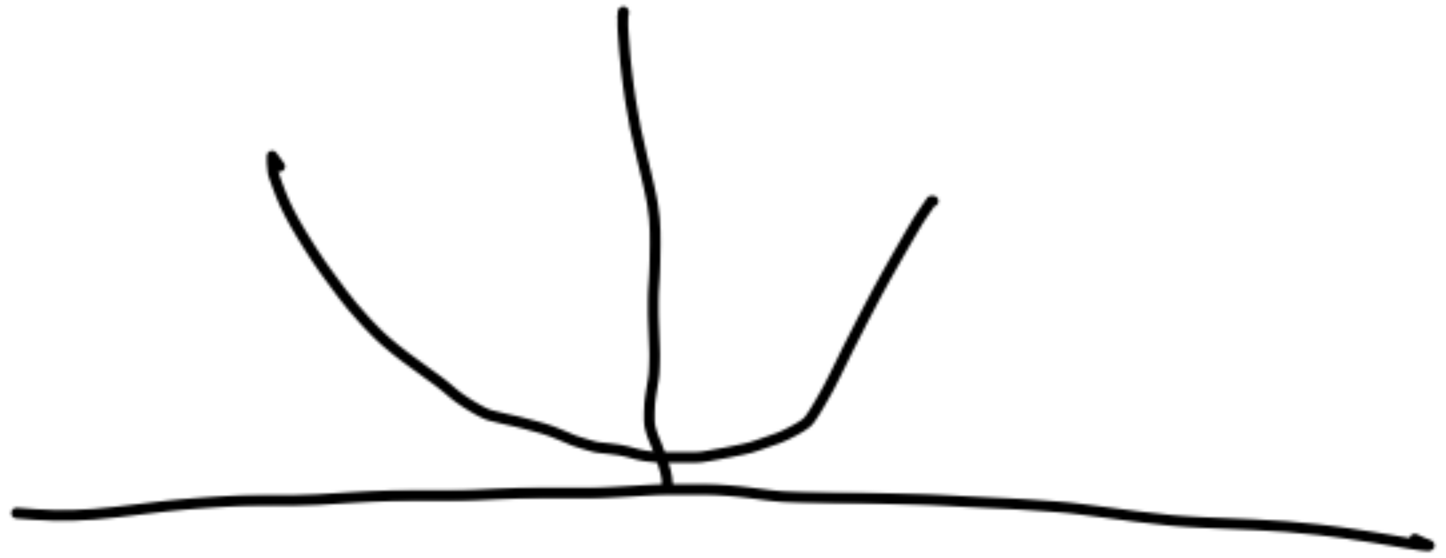
$T < T_c$

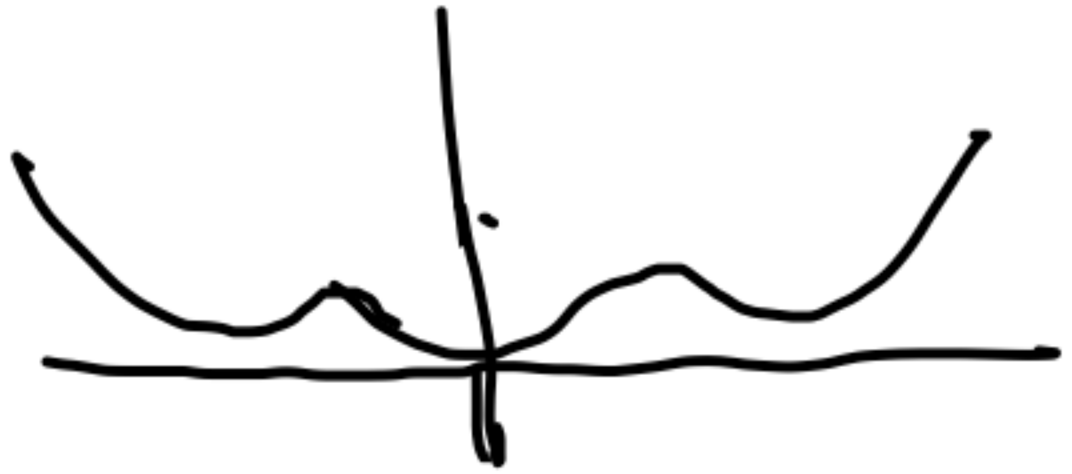




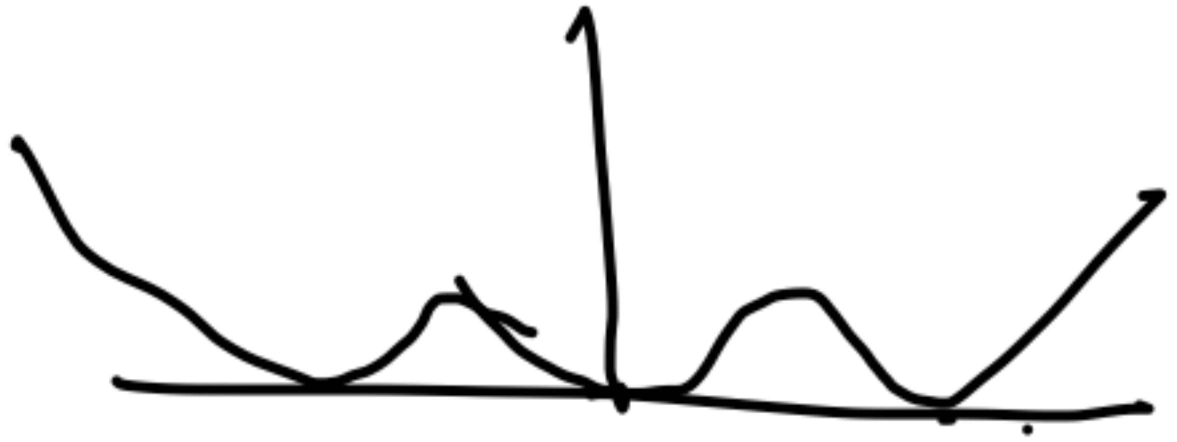
$\langle H \rangle$ changes continuously from 0 to non-zero value.

$\rightarrow$ 2nd order phase trs. $\Rightarrow$ the universe does not go out of equilibrium.





T_c



$T<T_c$

→ First order transition

Sudden jump drives it out of equilibrium.