$$\tilde{n}_{(\ell)} = \sum_\alpha C_\alpha^{(\ell)} (n_\kappa - \bar{n}_\kappa), \quad n_B = \sum_\alpha b_\alpha (n_\alpha - \bar{n}_\alpha)$$

$$M_{k\ell} = \sum_\alpha C_\alpha^{(k)} C_\alpha^{(\ell)} \bar{g}_\alpha, \quad \vartheta_k = \sum_\alpha b_\alpha C_\alpha^{(k)} \bar{g}_\alpha.$$

g_α for F, $2g_\alpha$ for B.

Relevant conserved charges are $B-L$, Y.

$\tilde{n}_Y = 0$ since Y is gauge charge.

Particle	B	L	$Y = I_3 - Q$	$\bar{g}$
u_L	$1/3$	0	$-1/6$	3
d_L	$1/3$	0	$-1/6$	3
u_R	$1/3$	0	$-2/3$	3
d_R	$1/3$	0	$1/3$	3
ν_L	0	1	$1/2$	1
e_L	0	1	$1/2$	1
e_R	0	1	1	1
ϕ^+	0	0	$-1/2$	2
ϕ^0	0	0	$-1/2$	2

Colour

$$M_{kl} = \sum_\alpha C_\alpha^{(k)} C_\alpha^{(l)} \bar{g}_\alpha$$

$$M_{B-L,B-L} = \Big(\tfrac{1}{3} \times \tfrac{1}{3} \times 3 + \tfrac{1}{3} \times \tfrac{1}{3} \times 3 + \tfrac{1}{3} \times \tfrac{1}{3} \times 3 + \tfrac{1}{3} \times \tfrac{1}{3} \times 3 + (-1)(-1)\,1 + (-1)(-1)\,1 + (-1)(-1)\,1 \Big) \times N_g$$

$\times N_g$, $N_g = 3$

$N_d = \frac{13}{3} N_g$, $N_d = 1$

Ex.

$$M = \begin{array}{cc} & \begin{array}{cc} B-L & \quad\quad Y \end{array} \\ \begin{array}{c} B-L \\ \\ Y \end{array} & \begin{pmatrix} \frac{13}{3} N_g & -\frac{8}{3} N_g \\ & \\ -\frac{8}{3} N_g & \frac{10}{3} N_g + N_d \end{pmatrix} \end{array} \underset{N_d = 1}{\overset{N_g = 3}{=}} \begin{pmatrix} 13 & -8 \\ -8 & 11 \end{pmatrix}$$

$$\vartheta = \left(\frac{4}{3} N_g , \ -\frac{2}{3} N_g \right) = (4, -2).$$

$$M^{-1} = \frac{1}{79} \begin{pmatrix} 11 & 8 \\ 8 & 13 \end{pmatrix}$$

$$n_B = \vartheta M^{-1} \begin{pmatrix} \tilde{n}_{B-L} \\ 0 \end{pmatrix}$$

$$= (4 \ \ -2) \ \frac{1}{79} \tilde{n}_{B-L} \begin{pmatrix} 11 \\ 8 \end{pmatrix}$$

$$= \frac{1}{79} \tilde{n}_{B-L} \times 28$$

$$n_B = \frac{28}{79} \tilde{n}_{B-L}.$$

Possible origin of B-L violation at high energy:

SU(5) GUT does not violate B-L:

e.g. $p \to e^+ + \pi^0$

π^0: $B=0,\ L=0$

e^+: $B=0,\ L=-1$

p: $B=1,\ L=0$

B-L is conserved.

There is a hint of B-L violation from low energy physics itself.

We know that neutrinos have mass.

$\rightarrow$ SM cannot be correct. $\bar{\psi} = \psi^\dagger \gamma^0$

Simplest possibility: Introduce right handed ν's, ν_{Ri} $(i = 1, 2, 3)$ that are neutral under $SU(3) \times SU(2) \times U(1)$.

Can introduce coupling of the form

$$\sum_{i,j=1}^{3} \underbrace{\lambda_{ij}}_{\text{constants}} \left(\bar{\nu}_{Li} \;\; \bar{e}_{Li}\right) \begin{pmatrix} \phi^{0*} \\ \phi^{+*} \end{pmatrix} \nu_{Rj} + h.c. = \sum_{i,j} \lambda_{ij} \left(\phi^{0*} \bar{\nu}_{Li} \nu_{Rj} + \phi^{+*} \bar{e}_{Li} \nu_{Rj} \right)$$

$\langle \phi^0 \rangle = v$

$\Rightarrow$ generates a mass term

$$v \sum_{i,j} \lambda_{ij} \bar{\nu}_{Li} \nu_{Rj} + \text{hermitian conjugate}$$

$\rightarrow$ Dirac mass.

The new term does not violate B-L symmetry since we can assign $L=1$ to ν_{Ri}.

What other terms can we add?

Majorana mass - maintain gauge invariance

$$\sum_{i,j} M_{ij} \overline{\nu^c_{Ri}} \nu_{Rj}$$ → Lorentz invariant

→ $L=2$ - breaks $(B-L)$ symmetry

ν^c_R : charge conjugate of ν_R

↳ left handed fermion.

→ Gauge invariant term, violates B-L.

ν_{Ri} carries $L=1$, ν^c_{Ri} has $L=-1$, $\overline{\nu^c_{Ri}}$ has $L=1$

In the presence of this term, the neutrino mass matrix has the structure:

$$\begin{array}{c|cc} & \nu_L^c & \nu_R \\ \hline \nu_L & 0 & \lambda v \\ \nu_R^c & \lambda v & M \end{array} \qquad \begin{pmatrix} 0 & \lambda v \\ \lambda v & M \end{pmatrix} \xrightarrow[\text{for large } M]{} \text{ ev's are } M, \; -\frac{\lambda^2 v^2}{M}.$$

If $M \gg v$, then one ev. is large. the other ev. is $\ll v$. $\boxed{\text{Seesaw mechanism}}$

$\to$ natural mechanism for getting small ν mass.

Effectively, after the Higgs has got vev, this gives a mass term of the form:

$$\gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\sum_{i,j} m_{ij} \bar{\tilde{\nu}}^c_{Li} \tilde{\nu}_{Lj} \qquad m \sim \frac{\lambda^2 v^2}{M}$$

Effectively the light neutrinos acquire Majorana mass.

→ Violates Lepton no. even at low energy (not useful for B production)

At high energy out of equilibrium decay of the heavy neutrinos could produce B-L violation.

→ need appropriate amount of C and CP violation.

- Possible mechanism for baryogenesis.

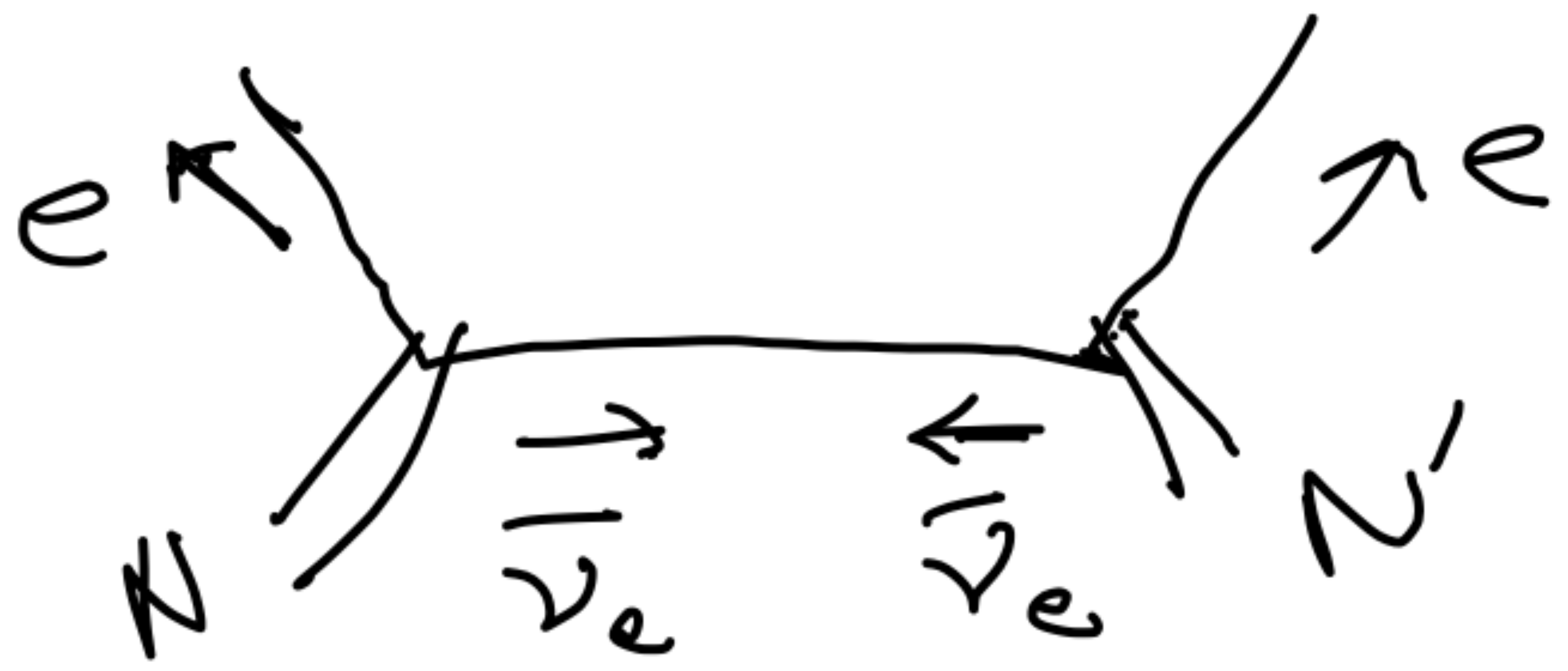
- Baryogenesis via leptogenesis.

Can it be tested at low energy?

One looks for neutrinoless double β decay.

$$N \to N' + e^- + \bar{\nu}_e \,, \quad N \to N' + e^+ + \nu_e$$

$$N \to N' + \underbrace{e^- + e^-}_{L=2} \quad \text{or} \quad N \to N' + e^+ + e^+$$



$$\overline{\nu^c_e}\, \nu^c_e + h.c.$$

→ If seen these will signal lepton no. violation.

$$(\bar{\nu}^c_L \; \bar{e}^c_L) \begin{pmatrix} \phi^* \\ \phi^* \end{pmatrix} (\phi \quad \phi) \begin{pmatrix} \nu_{Lc} \\ e^c_L \end{pmatrix} \times \frac{\lambda^2}{M} \quad c$$

→ effective gauge invariant term in the Lagrangian.

$$\langle \phi^0 \rangle = v$$

$$\longrightarrow \frac{\lambda^2 v^2}{M} \bar{\nu}^c_L \nu_L$$

Need the extra particle of mass M whose decay produces B-L at high energy.