Can a decoupled massless particle have $T > T_{CMB}$.

— not possible if the decoupled particle was in thermal equilibrium with ordinary matter just before decoupling.

However in high energy physics we can have more complicated scenarios.

- There could be a "hidden sector" that decouples from the ordinary sector early in the history of the universe.

As time evolves some particles in the hidden sector may become non-relativistic raising the temp. of the lighter hidden sector particles.

Same may happen in the ordinary sector.

If there are large no. of particles in the hidden sector which become non-relativistic before the decoupling of the particle of interest, then its temp. may be raised so high that $T > T_{CMB}$.

Dark matter → many types of candidates.
Cosmology tells us that they are non-relativistic at the time of structure formation.
→ allows for large range of masses.
Most popular version – "weakly interacting" massive particles (WIMPs)
→ interact with ordinary matter with weak interaction strength, mass $\gtrsim 100$ GeV.

Must be stable or have life-time $\gtrsim$ age of the universe.

They decouple at some point

When they decouple they must be non-relativistic.

Suppose they were relativistic.

$n_D \sim n_r$ at the time of decoupling.

$\swarrow$ Dark matter

$n_D \propto \frac{1}{\lambda^3}$, $n_r \propto \frac{1}{\lambda^3}$ except that n_r jumps when some particle goes non-relativistic.

once these become non-relativistic,

$\rho_D \propto \frac{1}{\lambda^3}$ $\rho_r \propto \frac{1}{\lambda^4}$

$\Rightarrow$ ρ_D dominates over ρ_r..

$\rightarrow$ inconsistent with what we know.

n_D must be $\ll n_\gamma$.

$\rightarrow$ possible if D was non-relativistic at the time of decoupling.

$n_D \propto e^{-m_D/T} \rightarrow$ small for $T \ll m_D$.

As T goes below m_D, n_D begins to fall due to exponential suppression.

$n_D = \bar{n}_D$ unless somehow $n_D - \bar{n}_D$ was created.

We'll assume that D is its own anti-particle so that $n_{\bar{D}} = n_D = n$.

As T falls, n gets depleted by $D + D \to$ light particles.

thermalize.

Annihilation rate per particle

$$n \sigma v \to \text{constant as } v \to 0.$$

$$\frac{d}{dt}(n\lambda^3) = -(n\lambda^3)\, n\sigma v + \text{rate of production from thermal bath.}$$

$$\frac{d}{dt}(n\lambda^3) = -n\lambda^3 (n\sigma v)$$

$$\lambda = c\, t^{1/2}$$

$$\times \frac{1}{(n\lambda^3)^2}$$

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{1}{n\lambda^3}\right) = \frac{1}{\lambda^3}\sigma v$$

$$\simeq c^{-3}\sigma v\, t^{-3/2}$$

$$\frac{1}{n\lambda^3} = K - 2C^{-3}\sigma v\, t^{-1/2}$$

$\longrightarrow$ constant as $t \to \infty$

$n\lambda^3$ reaches a constant value as $t \to \infty$.

In contrast, if n followed thermal equilibrium formula, then

$$n = \text{const.} \times T^{3/2} \times e^{-m_D/T}$$

$$\lambda \propto T^{-1}$$

$$n\lambda^3 = \text{const.} \times T^{-3/2} \times e^{-m_D/T}$$

$$T \sim \lambda^{-1} \sim t^{-1/2}$$

$$1/T \sim t^{1/2}$$

$\Rightarrow$ falls of exponentially.

Full analysis:

$$\frac{d}{dt}(n\lambda^3) = -n\lambda^3 n\sigma v + \text{rate of production}$$

In equilibrium, $\frac{d}{dt}(n_{eq}\lambda^3) = 0$

equilibrium value.

rate of production from bath

$$= n_{eq}\lambda^2 n_{eq}\sigma v$$

$$\frac{d}{dt}(n\lambda^3) = -(n^2 - n_{eq}^2)\lambda^3\sigma v$$

$$n_{eq} = (2s_D+1)\,\frac{1}{2\pi^2}\int_0^\infty k^2\,dk\,\left(e^{\sqrt{k^2+m_D^2}/T}\mp 1\right)^{-1}$$

$k/T = y$

$T/m_D = x$

$$= (2s_D+1)\,\frac{1}{2\pi^2}\,T^3\int_0^\infty y^2\,dy\,\left(e^{\sqrt{y^2+x^{-2}}}\mp 1\right)^{-1}$$

$$\frac{d}{dt}(n\lambda^3) = -(n^2-n_{eq}^2)\,\lambda^3\,\sigma v \qquad \lambda = \frac{c}{T}$$

$$\frac{d}{dt}\Big(\underbrace{\frac{n}{T^3}}_{=u}\Big) = -(n^2-n_{eq}^2)\,\frac{1}{T^3}\,\sigma v$$

$$\boxed{\frac{du}{dt} = -(u^2-u_{eq}^2)\,T^3\,\sigma v} \qquad u_{eq}^{(x)} = \frac{n_{eq}}{T^3}$$

$$\frac{du}{dt} = -\left(u^2 - u_{eq}^2\right) T^3 \sigma v$$

$$\frac{\dot{T}}{T} = -\frac{\dot{\lambda}}{\lambda} = -\sqrt{\frac{8\pi G \rho}{3}}$$

$\rho \rightarrow \frac{1}{2} a_B T^4 \mathcal{N}$

$a_B \rightarrow \pi^2/15$

$\mathcal{N} \rightarrow$ effective no. of dof

1 for B, $\frac{7}{8}$ for F.

$$\frac{\dot{T}}{T^3} = -\sqrt{\frac{4\pi^3 G \mathcal{N}}{45}}$$

$x = \frac{T}{m_D}$

$\times \frac{1}{\dot{T}}$

$$\Rightarrow \frac{du}{dT} = +\left(u^2 - u_{eq}^2\right) \sqrt{\frac{45}{4\pi^3 G \mathcal{N}}} \, \sigma v$$

$$\boxed{\frac{du}{dx} = \left(u^2 - u_{eq}(x)^2\right) m_D \sqrt{\frac{45}{4\pi^3 G \mathcal{N}}}}$$

Boundary condition : $u = u_{eq}(x)$ for large x.

$u_D = u_{eq}(0)$

$\Downarrow$

$\left(\frac{n_D}{T_D^3}\right) \to$ relic density of dark matter.

n_0 : observed dark matter density ~~today~~,

$n_0 \lambda_0^3 = n_D \lambda_D^3 \quad \Big| \quad K \lambda_D T_D = \lambda_0 T_0$

$\lambda_0^3 = 1$.

$n_0 = n_D \lambda_D^3 = n_D \left(\frac{T_0}{K T_D}\right)^3$

$= u_D \left(\frac{T_0}{K}\right)^3$

$\hookrightarrow$ The factor by which T_r is enhanced after D. - decoupling.

Generalization of this procedure can be used to calculate other relic densities like (B-L).

As T falls below $M_{\text{Heavy}\,\nu}$'s the densities of heavy ν begins to fall.

Same kind of diff. eq. allows us to calculate their densities.

Once this process is out of equilibrium, then the decay of the heavy ν to lighter particles will produce B-L.

prefers one sign over the other.

(need C and CP violation)

$$m_\nu \sim \frac{\lambda^2 v^2}{M} \sim \lambda^2 \frac{(250\,\text{GeV})^2}{M}$$

$v = 250$ GeV.

$M_W, M_Z \sim g v$

$m_\nu = .1$ eV.

$$M = \lambda^2 \frac{(250\,\text{GeV})^2}{.1\,\text{eV}} = \lambda^2 (250)^2 \times 10^{10}\ \text{GeV}$$