Conditions for successful inflation:

① $(V'(\phi))^2 << 24\pi G (V(\phi))^2$ } slow roll.

② $V''(\phi) << 8\pi G\, V(\phi)$

③ $-8\pi G \int_{\phi_i}^{\phi_f} \frac{V(\phi)}{V'(\phi)}\, d\phi > 60$

④ $V(\phi) << M_{pl}^4 \simeq G^{-2}$

Try $V(\phi) = g\phi^\alpha$

① $g^2 \alpha^2 \phi^{2\alpha-2} << 24\pi G\, g^2 \phi^{2\alpha}$

$\Rightarrow \phi^2 >> \alpha^2/(24\pi G)$

② $\phi^2 >> \alpha(\alpha-1)/8\pi G$

③ $+\frac{8\pi G}{\alpha} \frac{1}{2}(-\phi_f^2 + \phi_i^2) > 60$

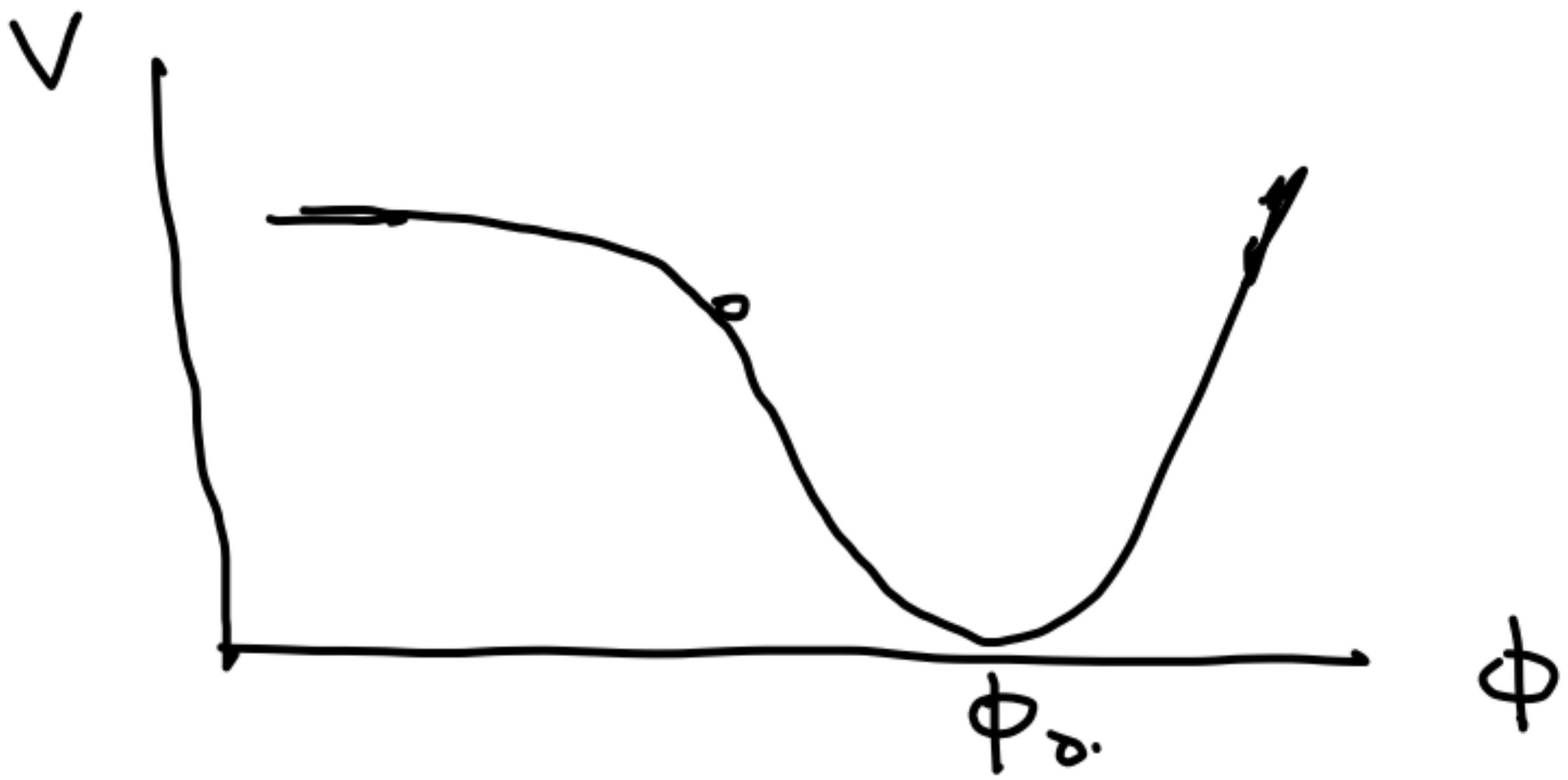
④ $g\,\phi^\alpha << G^{-2}$ → can be satisfied by taking g small.

Even $\frac{1}{2}m^2\phi^2$ can do this for small enough m.

One possible issue: Since we need to take ϕ to be large, if $V(\phi)$ contains other terms with larger powers of ϕ, they could violate these conditions.

$\rightarrow$ We need to make sure that this does not happen.

Reheating:



Technically we can defining the beginning of reheating to be the time when the slow roll conditions fail.

Approximate the potential as

$$V(\phi) = \frac{1}{2} M^2 (\phi - \phi_0)^2 \quad \text{during reheating}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} M^2 (\phi-\phi_0)^2, \quad p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2} M^2 (\phi - \phi_0)^2$$

$$\langle p \rangle \approx 0$$

Cosmological constant gets converted to non-relativistic matter.

Physical interpretation: Large collection of almost 0 momentum ϕ particle.

$\Rightarrow$ Not the beginning of the FRW universe.

— should be radiation dominated.

Need to transfer the energy density to radiation (includes standard model particles).

We assume that ϕ has coupling to standard model particles.

As a result ϕ is unstable and decay to light particles.

Γ : Total decay rate of ϕ.

During reheating $\rho_\phi(t) = \rho_f \left(\frac{a_f}{a}\right)^3 e^{-\Gamma(t-t_f)}$.

f : beginning of reheating (end of inflation).

$$\frac{d}{dt}(\rho_\phi \lambda^3) = -\Gamma(\rho_\phi \lambda^3)$$

$\rho_R(t)$: Energy density of radiation.

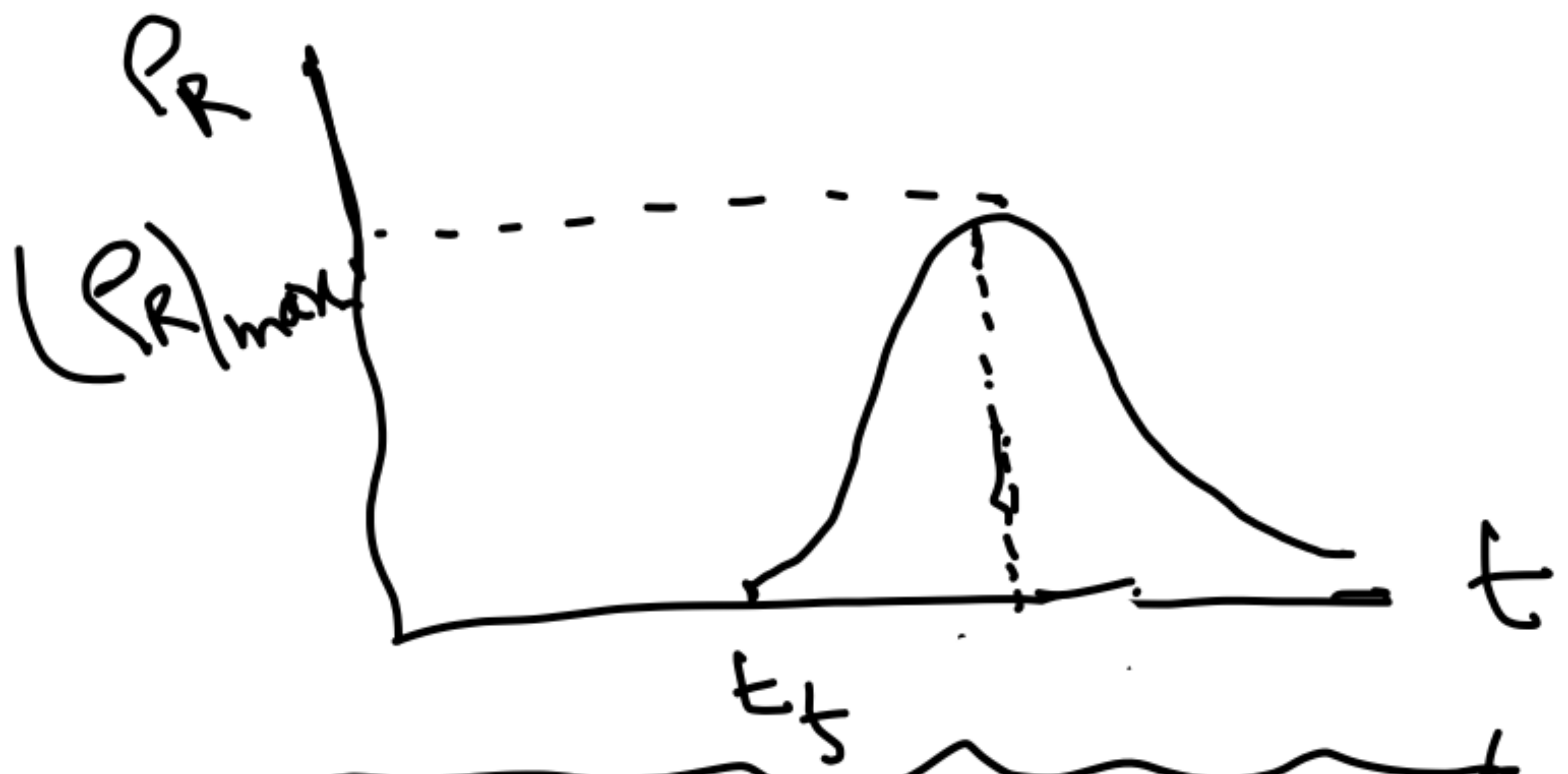
$$\frac{d}{dt}\left((\rho_R+\rho_\phi)\lambda^3\right) = -3\lambda^2\dot{\lambda}(\underbrace{p_\phi}_{=0} + \underbrace{p_R}_{=\frac{\rho_R}{3}})$$

$$= -\lambda^2\dot{\lambda}\rho_R$$

$$\frac{d}{dt}(\rho_R\lambda^3) + \lambda^2\dot{\lambda}\rho_R = -\frac{d}{dt}(\rho_\phi\lambda^3) = \Gamma\rho_\phi\lambda^3$$

$$\frac{1}{\lambda(t)}\frac{d}{dt}(\rho_R\lambda^4) = \Gamma\rho_f\lambda_f^3 e^{-\Gamma(t-t_f)} \to \rho_R\lambda^4 = \Gamma\rho_f\lambda_f^3\int_{t_f}^{t} e^{-\Gamma(t'-t_f)}\lambda(t')dt'$$

$$\rho_R(t) = \Gamma\rho_f\lambda_f^3\lambda(t)^{-4}\int_{t_f}^{t} e^{-\Gamma(t'-t_f)}\lambda(t')\,dt'$$



If we need some events after inflation e.g. B–L production, it has to happen at $T \lesssim T_R$

Define the reheating temperature T_R such that

$$\frac{1}{2} a_B T_R^4 \mathcal{N} = (\rho_R)_{max}$$

effective no. of dof

1 / boson

7/8 / fermion

Extreme cases: $\rho_R = \Gamma \rho_f \lambda_f^3 \lambda(t)^{-4} \int_{t_f}^{t} e^{-\Gamma(t'-t_f)} \lambda(t')\, dt'$

① $\Gamma \gg H(t_f)$

$$\rho_R = \Gamma \rho_f \lambda_f^3 \lambda(t)^{-4} \lambda_f \underbrace{\int_{t_f}^{t} e^{-\Gamma(t'-t_f)} dt'}_{1/\Gamma}$$

$$\simeq \rho_f \lambda_f^4 \lambda(t)^{-4}$$

systematic extension in $1/\Gamma$ is possible.

$\rightarrow \rho_R$ is maximum at $t \simeq t_f$. As if the whole energy density is converted to radiation instantaneously.

$$\rho_R = \Gamma \rho_f \lambda_f^3 \, \lambda(t)^{-4} \int_{t_f}^{t} e^{-\Gamma(t'-t_f)} \lambda(t') \, dt'$$

2 $\Gamma \ll H(t_f)$

$$\rho_R \simeq \Gamma \rho_f \lambda_f^3 \, \lambda(t)^{-4} \int_{t_f}^{t} \lambda(t') \, dt'.$$

In matter dominated universe:

$$\left(\frac{\dot{\lambda}}{\lambda}\right)^2 = \frac{8\pi G}{3} \rho_f \lambda_f^3 \, \lambda^{-3}$$

Integrate: $$\frac{2}{3}\left(\lambda^{3/2} - \lambda_f^{3/2}\right) = \left(\frac{8\pi G}{3} \rho_f \lambda_f^3\right)^{1/2} (t - t_f)$$

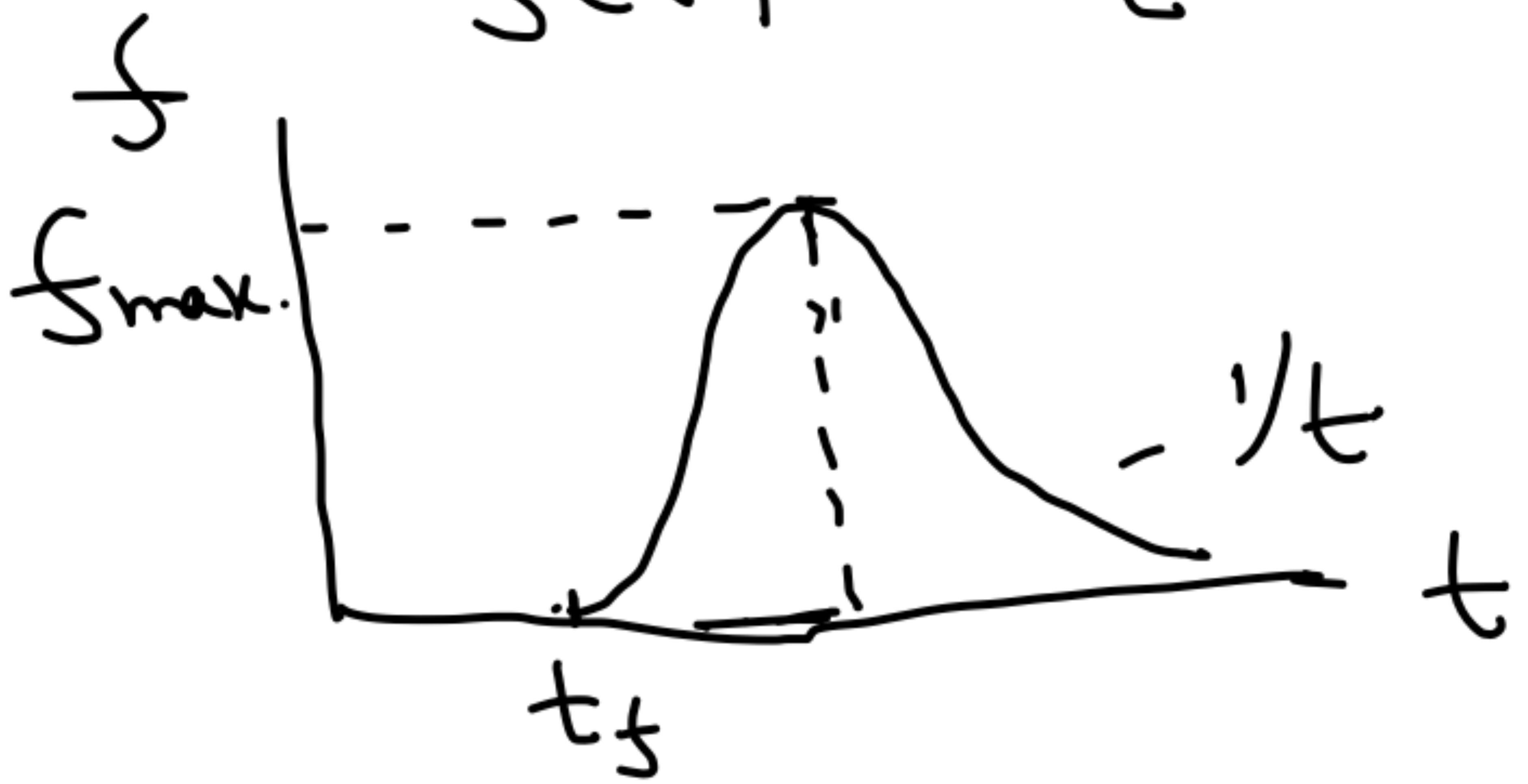
$$\rho_R(t) \simeq \Gamma \quad f(t).$$

$$= \rho_f \lambda_f^3 \lambda(t)^{-4} \int_{t_f}^{t} \lambda(t')\, dt'.$$

$f(t_f) = 0$

For large t, $\lambda(t) \sim t^{2/3}$ $\qquad \int_{t_t}^{t} \lambda(t')\, dt' \propto t^{5/3}$

$$f(t) \sim t^{-8/3}\, t^{5/3} \sim t^{-1}$$



f_{max} is Γ independent

$\rho_R(t)\big|_{max} \propto \Gamma. \rightarrow$ Low Γ.

$\Downarrow$

inefficient reheating.

Origin of fluctuation

Inflation makes the universe very smooth.
How do fluctuations arise?
~ could be thermal fluctuations after inflation.
Thermal particles could carry energy densities from one region to another

Such fluctuations can possibly explain fluctuations over horizon size scale

↳ computed after inflation

$\Rightarrow$ 2° angle on the last scattering surface.

$\Rightarrow$ If we average T over distances larger than the 2° angle on last scattering surface, we should get uniform result.

Observed fluctuations occur at all scales.

→ scale invariant spectrum

- Harrison - Zeldovich spectrum.

✓

We know this to be true by direct observation.

→ Predicted even before discovery of CMB fluctuations — from the study of large scale structure in the Universe.

Puzzle: How to generate fluctuations over large scales?

FRW metric: $ds^2 = -dt^2 + a(t)^2 \left(dr^2 + r^2 d\Omega^2\right)$

$$a(t) = a_0 \lambda(t)$$

$a_0 r \to r$.

$$ds^2 = -dt^2 + \lambda(t)^2 \left(dr^2 + r^2 d\Omega^2\right).$$

r measures distance in today's metric (comoving distance).

Consider some given epoch:

$$H(t) = \frac{\dot{\lambda}}{\lambda}.$$

The universe "doubles" its size over time scale $H(t)^{-1}$.

Comoving distance travelled during this time:

$$dr = \frac{dt}{\lambda(t)}$$

$$\Delta r = \frac{1}{\lambda(t)}\frac{1}{H(t)} = \lambda^{-1}H^{-1}.$$

(1) During inflation:

$$\lambda \simeq A e^{ct} \qquad A, c \text{ constants}$$

$$H = \frac{\dot{\lambda}}{\lambda} = c \qquad H^{-1} \lambda^{-1} = c^{-1} A^{-1} e^{-ct}$$

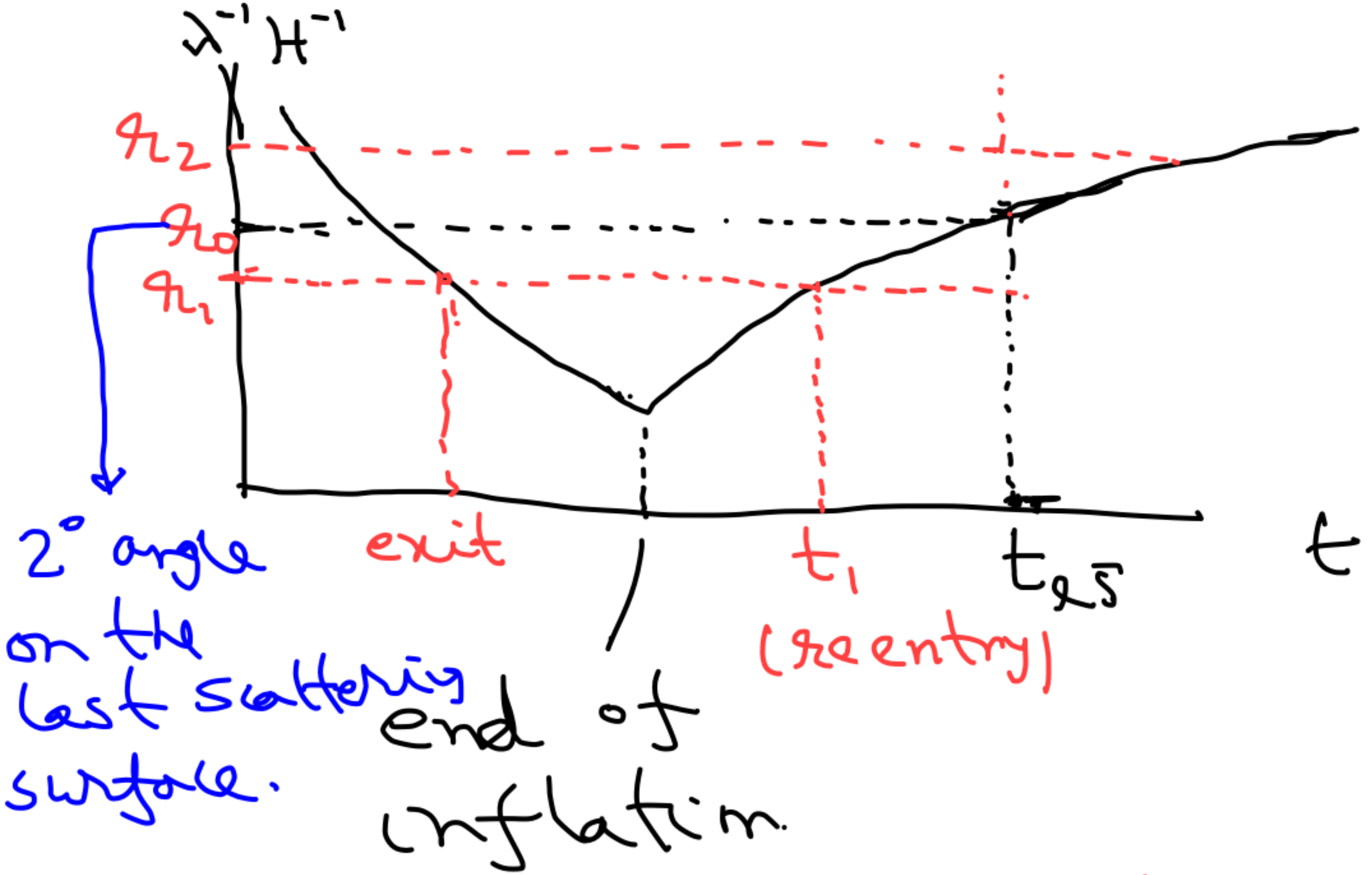
$\rightarrow$ falls exponentially.

(2) Radiation dominated:

$$\lambda \simeq B t^{1/2} \qquad H = \frac{\dot{\lambda}}{\lambda} = \frac{1}{2t} \Rightarrow \lambda^{-1} H^{-1} = \frac{1}{2B} t^{1/2}$$

(3) Matter dominated $\longrightarrow \lambda^{-1} H^{-1} = \frac{3}{2} D^{-1} t^{1/3}$

$$\lambda = D t^{2/3}$$



Microscopic physics between t_1 and t_{ls} can affect fluctuations over scale λ_1.

Fluctuations at scale λ_2 can only be affected by physics during inflation.