Main remaining task in scalar pert.

→ find the precise relation between $\delta T/T$ and $\mathcal{R}_I$.

↓

Calculate the fn. $f(\vec{k},t)$ that relates $\delta T/T$ to $\mathcal{R}_I$.

$\sim$ 2 lectures ⇒ Postpone to next week.

Today: How to detect tensor perturbation

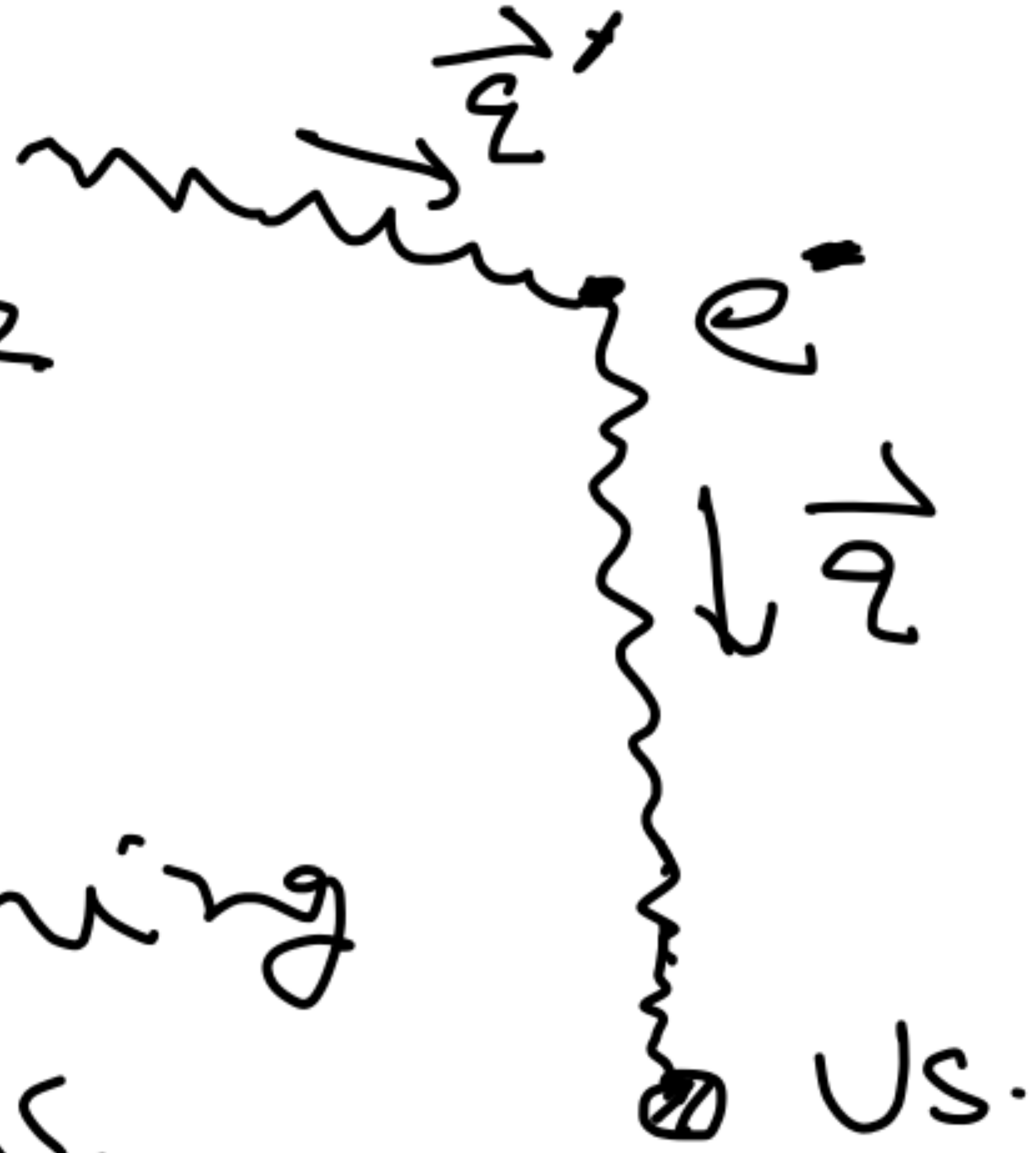
- only give the result (Dodelson, Modern Cosmology)

(Could be project topic).

Main tool: Polarization of CMB.

How is CMB polarized?

Last scattering:

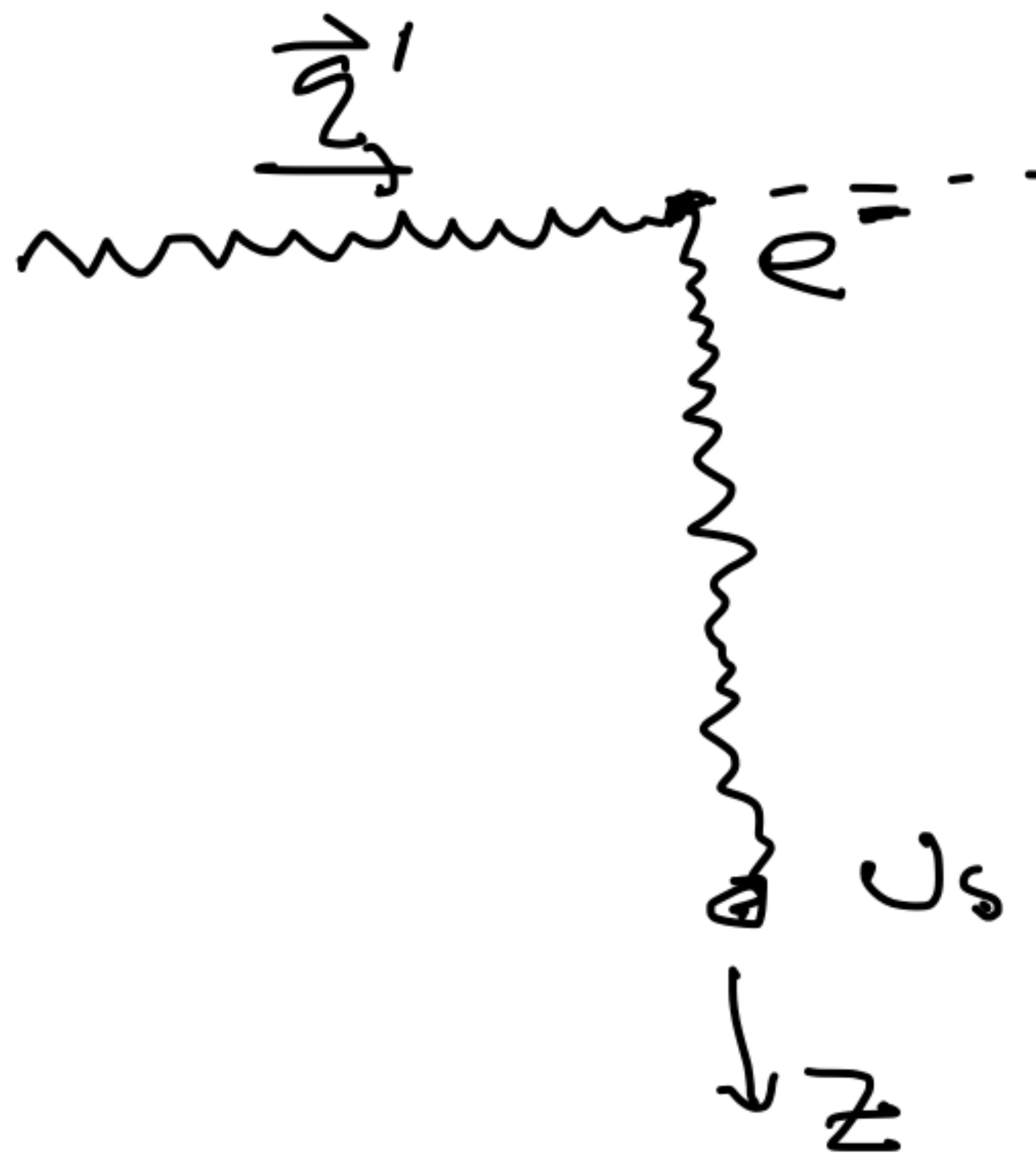


X-Section $\propto |\vec{\epsilon}\cdot\vec{\epsilon}\,'|^2$

polarizations of incoming and outgoing photons.

For incoming γ coming from a specific direction will give polarized outgoing γ even if the incoming γ is not polarized.

Extreme case:



$$\vec{\epsilon}\,' = (0, \epsilon'_y, \epsilon'_z)$$

$$\vec{\epsilon} = (\epsilon_x, \epsilon_y, 0)$$

$$\vec{\epsilon}\cdot\vec{\epsilon}\,' = \epsilon_y \epsilon'_y$$

$$\Downarrow$$

No outgoing γ with ϵ_x polarization.

Even if the incoming photon has equal probability of y and z polarization, the outgoing γ always has y polarization.

Ex. If the incoming radiation is unpolarized and isotropic, then the outgoing γ is not polarized.

However due to fluctuations, the incoming radiation may have small anisotropy.

$\Rightarrow$ causes polarization in CMB.

In general, both scalar and tensor modes can cause this anisotropy.

We'll describe, without proof, a way to isolate tensor mode effect.

Define intensity tensor:

$$I_{ij} \propto \langle E_i E_j \rangle$$

E_i → electric field component

$\langle\ \rangle$ → average over radiation coming from a given direction (not angular average)

$\vec{E}$ is $\perp$ the direction of travel.

For every direction, introduce a pair of orthonormal basis vectors $\hat{e}_1, \hat{e}_2$

$\Rightarrow I_{ij}$: 2x2 matrix.

Define:

$$Q = \frac{1}{4}(I_{11} - I_{22}), \quad U = \frac{1}{2} I_{12}$$

$$T = \frac{1}{4}(I_{11} + I_{22})$$

Ex. Under a ratation of basis vectors:

$$\begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \end{pmatrix} \to \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \end{pmatrix}$$

$\Rightarrow$ T is invariant.

$$(Q \pm iU) = e^{\mp 2i\alpha}(Q \pm iU)$$

Now we consider radiation from different direction

$\rightarrow T(\theta,\phi)$, $Q(\theta,\phi)$, $U(\theta,\phi)$

$$T(\theta,\phi) = \sum_{l,m} Y_{lm}(\theta,\phi)\, T_{lm}.$$

$$Q \pm iU = \sum_{l,m} {}_{\pm 2}Y_{lm}(\theta,\phi)\, a_{\pm 2 lm}$$

$${}_{\pm 2}Y_{lm}(\theta,\phi) = \epsilon^i_{\pm} \epsilon^j_{\pm} \partial_i \partial_j Y_{lm}(\theta,\phi)$$

$(\hat{\epsilon}_1 \pm i\hat{\epsilon}^2)$

Define:

$$a_{E,\ell m} = -\frac{1}{2}\left(a_{2,\ell m} + a_{-2,\ell m}\right)$$

$$a_{B,\ell m} = -\frac{1}{2i}\left(a_{2,\ell m} - a_{-2,\ell m}\right).$$

Result: $a_{E,\ell m}$ is a linear combination of δ_I and $h^{(s)}$.

$a_{B,\ell m}$ is a linear combination of $h^{(s)}$ only

$\Rightarrow \langle a_{B,\ell m}\, a_{B,\ell' m'} \rangle$ contains information on $\rho_T \propto \Delta_T^2$

Current bound:

$$r = \frac{\Delta_T^2}{\Delta_S^2} < .2$$

Observation of $r \Rightarrow$ existence of tensor mode fluctuations

$\Rightarrow$ quantum gravity fluctuation during inflation.

Possible project topics.

① B-mode $\leftrightarrow$ tensor perturbation

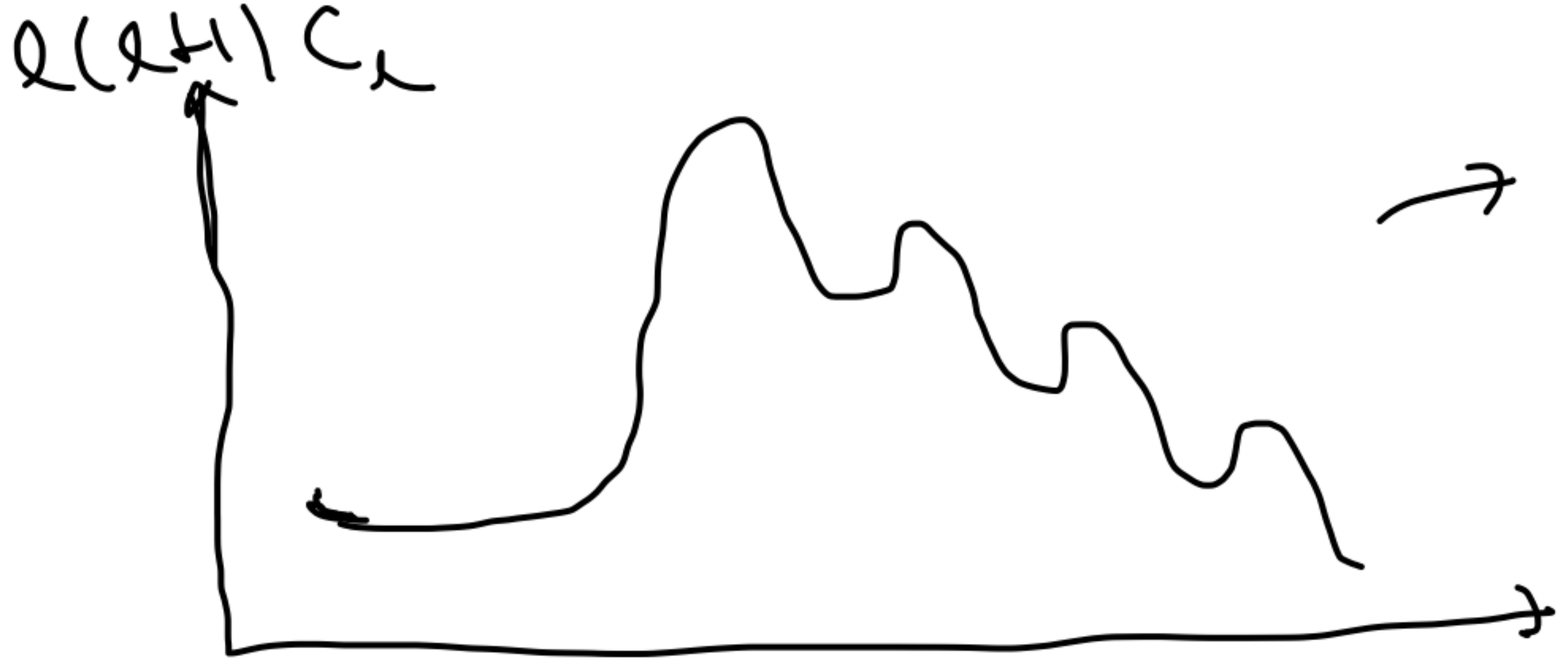
② Hubble tension:

Tension between two sets of measurements

of H_0 !

ⓐ Observe "nearby" objects and find distance-redshift relation.

ⓑ Cosmological observation (CMB)



$H_0 \approx 67$ Km/s/Mpc

Near object measurement $\Rightarrow H_0 \simeq 74$ km/s/Mpc

Assume some values of cosmological parameters : $\Omega_m, \Omega_\Lambda, H_0, \Omega_B, \cdots$

Predict the $\ell(\ell+1)C_\ell$ vs. ℓ curve.

& adjust the parameters to find the best fit value.

③ Effective field theory of inflation.

→ requires some knowledge of QFT.

④ Axionic dark matter:

production mechanism.

⑨ Structure formations from Cosmological pert.

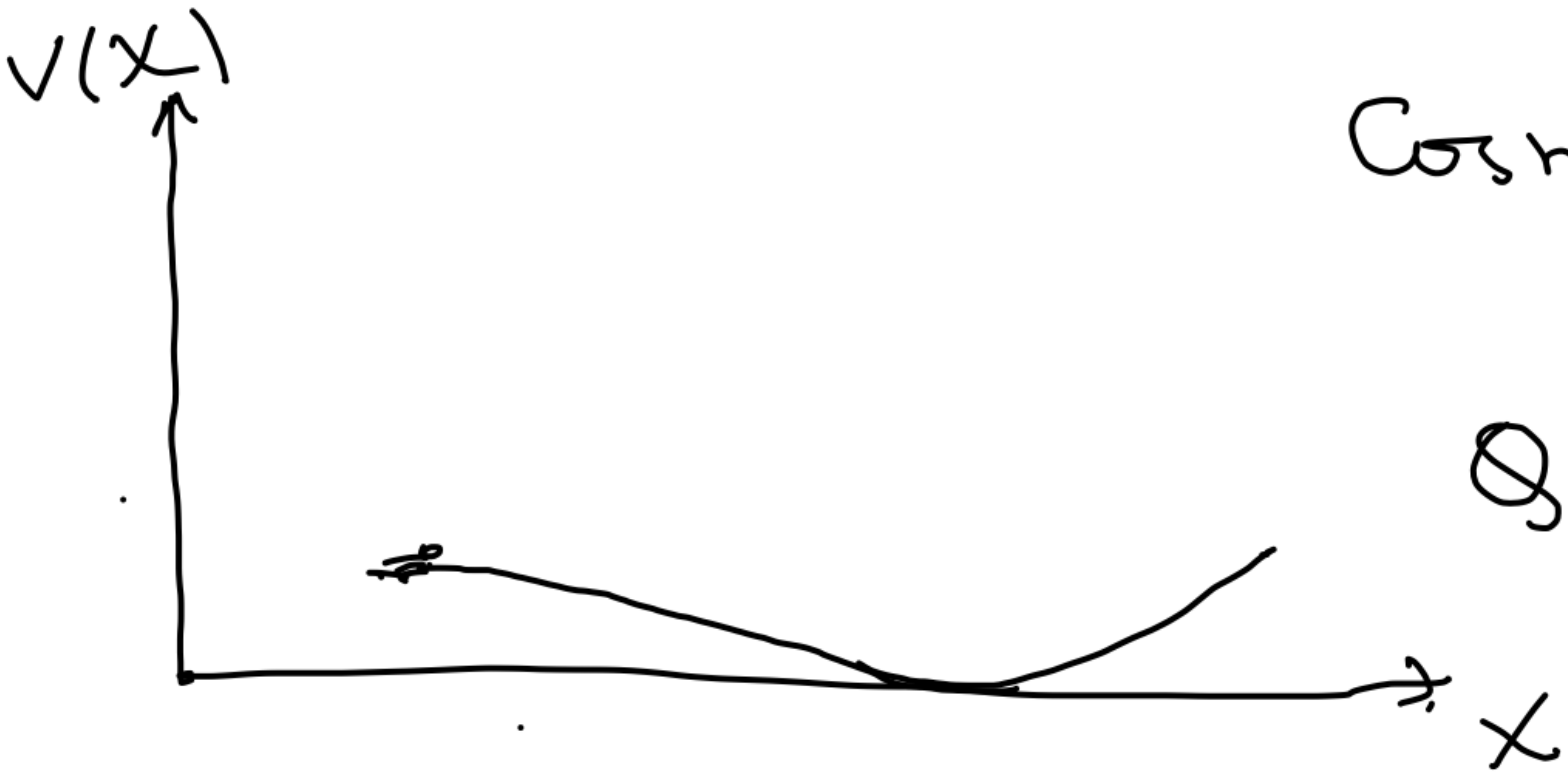
⑩ Cosmological const. problem.

⑤ Constraints on neutrino masses from Cosmology.

⑪ Other models of Cosmology - $\boxed{\text{Ekpyrotic universe}}$

⑥ Reheating. ⑦ Galaxy formation.

⑧ Neutrino oscillation in curved space-time.



Cosmo. const. today

V(X)

Quintessence:

Cosmological const. problem:

0 is not a natural value.