We showed that $\Theta = \frac{\delta T}{\bar{T}} = -\frac{2}{5} R$ for superhorizon perturbation in $E=0, B=0$ gauge.

$T = \bar{T} + \delta T$ is measured in locally inertial frame.

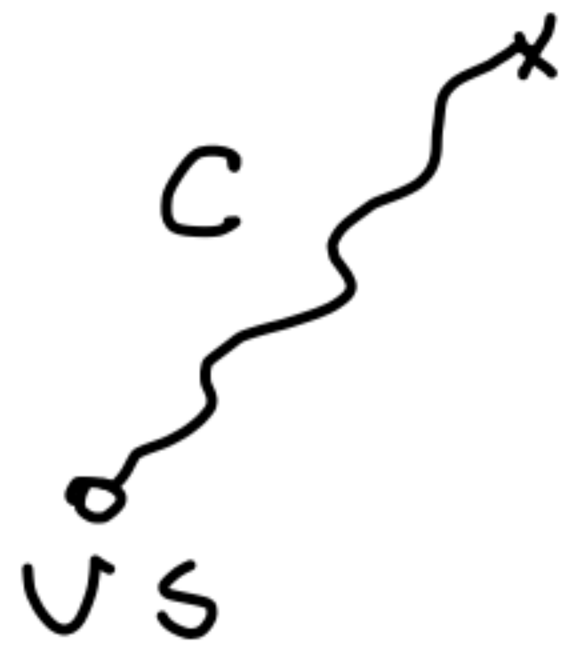
$$ds^2 = -dt^2(1+2\Phi) + d\vec{x}^2(1-2\Psi) \cdot a(t)^2$$

At $(\vec{x}, t)$ we choose an inertial frame in which $\frac{d\vec{x}}{dt} = 0$ at that instant.

$ds^2 = -dt^2(1+2\Phi)$. $\quad d\tau = \sqrt{-ds^2} = dt(1+\Phi)$

$\bar{T} + \delta T$ is measured by this observer.

$\Delta\tau = -(1+\Phi)\Delta t$, Proper temp $= T(1+\Theta)$



Travels along null geodesic.

$$ds^2 = 0 \Rightarrow dt^2(1+2\Phi)$$
$$= \lambda(t)^2(1-2\Psi)|d\vec{x}|$$

$$\frac{dt}{\lambda(t)} = |d\vec{x}|(1-\Psi-\Phi)$$

$$\int_{t_{ls}}^{t_0} \frac{dt}{\lambda(t)} = \int_c |d\vec{x}|(1-\Psi-\Phi)$$

Assume for now that Ψ, Φ are time indep.
Then one can show that c is time independent.

$$\int_{t_{ls}}^{t_0} \frac{dt}{\lambda(t)} = \int_c |d\vec{x}|\,(1-\Psi-\Phi)$$

$$\int_{t_{ls}+\Delta t_{ls}}^{t_0+\Delta t_0} \frac{dt}{\lambda(t)} = \int_c |d\vec{x}|\,(1-\Psi-\Phi)$$

<u>subtract:</u>

$$\frac{\Delta t_0}{\underbrace{\lambda(t_0)}_{1}} = \frac{\Delta t_{ls}}{\lambda(t_{ls})} \Rightarrow \Delta t_0 = \frac{\Delta t_{ls}}{\lambda(t_{ls})} = \frac{\Delta\tau\,(1-\Phi)}{\lambda(t_{ls})}$$

$$\Rightarrow (\bar{T}+\delta T)\big|_{obs} = (\bar{T}+\delta T)\big|_{ls} \times \frac{\lambda(t_{ls})}{1-\Phi} = \bar{T}_{ls}\,\lambda(t_{ls})\,(1+\Theta+\Phi)$$

$$(T+\delta T)\big|_{ls} = \bar{T}_{ls}(1+\Theta)$$

$$(\bar{T} + \delta T)_{obs} = \bar{T}_{ls} \cdot \lambda(t_{ls})\,(1+\Theta+\Phi)$$

$$\bar{T}_{obs} = \bar{T}_{ls}\, \lambda(t_{ls})$$

$$\Rightarrow \quad \frac{\delta T}{\bar{T}}\bigg|_{obs} = (\underbrace{\Theta}_{= -\frac{2}{5}R} + \underbrace{\Phi}_{= \frac{3}{5}R}) = \frac{1}{5} R$$

$$\Delta t_{obs} = (\quad)\, \Delta t_{ls}$$

$$\nu_{obs} = (\quad)^{-1}\, \nu_{ls}$$

$$\Rightarrow \quad T_{obs} = (\quad)^{-1}\, T_{ls}$$

Planck distn. formula $f\left(\frac{\nu}{T}\right)$

Path of light:

$$ds^2 = -dt^2(1+2\Phi) + d\vec{x}^2(1-2\Psi)\,\lambda(t)^2$$

Define a new time coordinate σ s.t.

$$d\sigma = dt/\lambda(t)$$

$$ds^2 = -\lambda(t)^2(1+2\Phi)\,d\sigma^2 + \lambda(t)^2(1-2\Psi)\,d\vec{x}^2$$

$$= \lambda(t)^2(1+2\Phi)\left\{-d\sigma^2 + \frac{1-2\Psi}{1+2\Phi}\,d\vec{x}^2\right\}$$

t independence of $\Psi, \Phi \Rightarrow \sigma$ independence of Ψ, Φ.

$$-d\sigma^2 + g_{ij}(\vec{x})\, dx^i dx^j$$

Ex. Null geodesics of this metric span geodesics of the spatial metric $g_{ij}(\vec{x})$.

$\Rightarrow$ Path in the spatial slice is time independent.

Unfortunately time independence of Ψ and Φ is not good assumption.

In actual computation of CMB fluctuation we have to take into account the time dependence of Ψ, Φ.

Final effect: Fluctuation of the last scattering surface δt_{LS}.

t_{LS} depends on various data like the photon temp., density of e^-, density of p etc.

$\lambda(t)T(t) = \text{const}$

$\Rightarrow \frac{\dot{\lambda}}{\lambda} + \frac{\dot{T}}{T} = 0$

→ these could fluctuate.

$$(\bar{T} + \delta T)\Big|_{obs} = T_{LS}\, \lambda(t_{LS})\, (1 + \Theta + \Phi)$$

$\therefore$ Cont. to $$1 + \frac{\delta T}{\bar{T}}\Big|_{obs} = 1 + \Theta + \Phi + \cancel{\frac{\dot{\lambda}(t_{LS})}{\lambda(t_{LS})}}\delta t_{LS} + \cancel{\frac{\dot{T}}{T}\Big|_{LS}}\delta t_{LS}$$

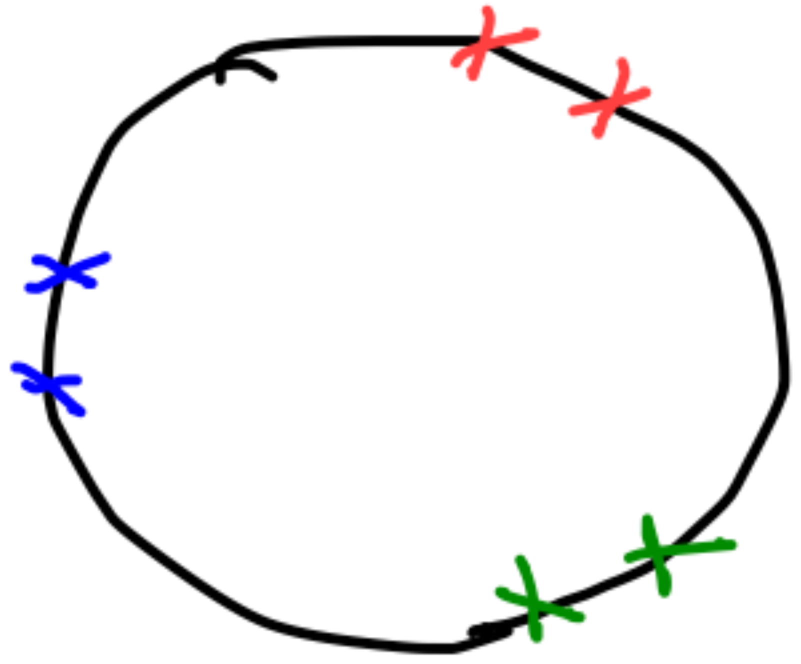
Cosmic variance

→ an intrinsic uncertainty in measuring CMB fluctuations.

$\langle \quad \rangle$ → average over many copies of the universe.

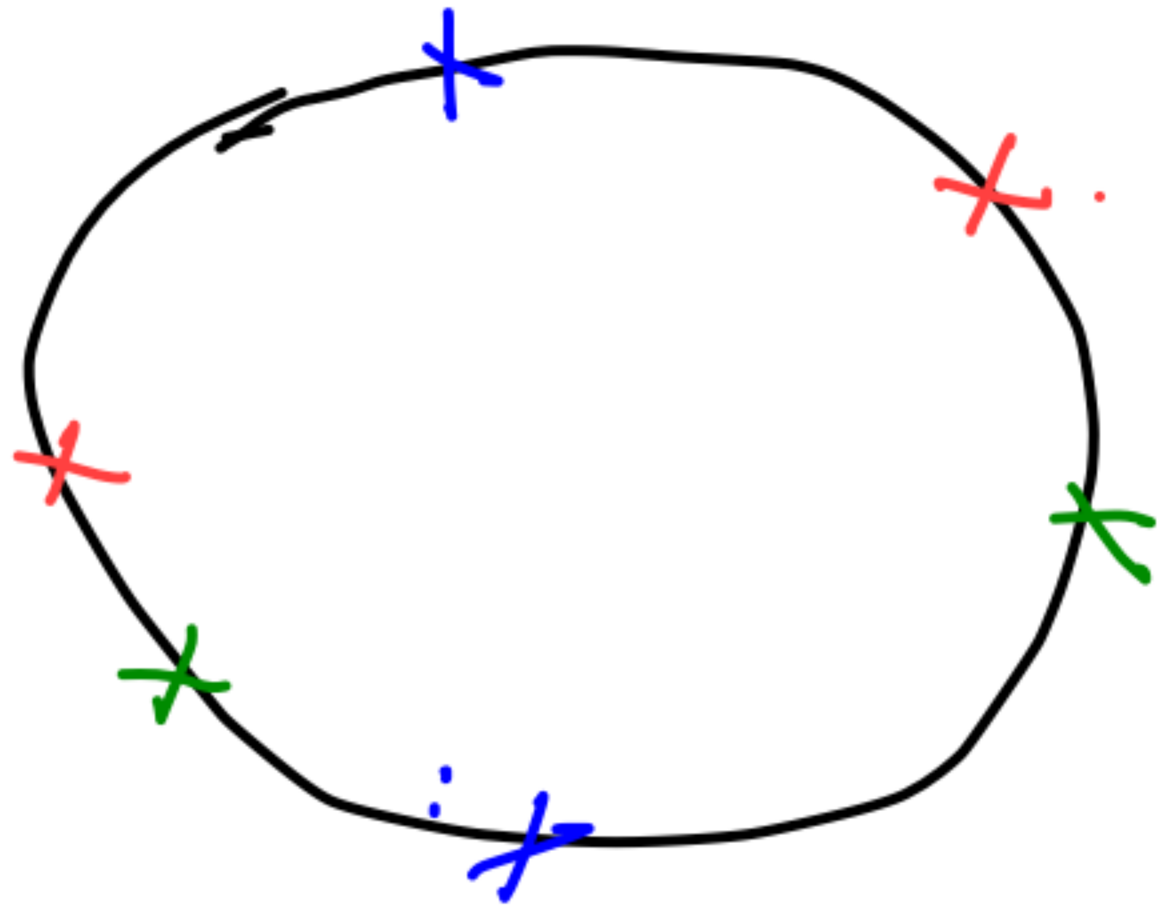
↓

Angular average (average over different choices of coordinate frames.

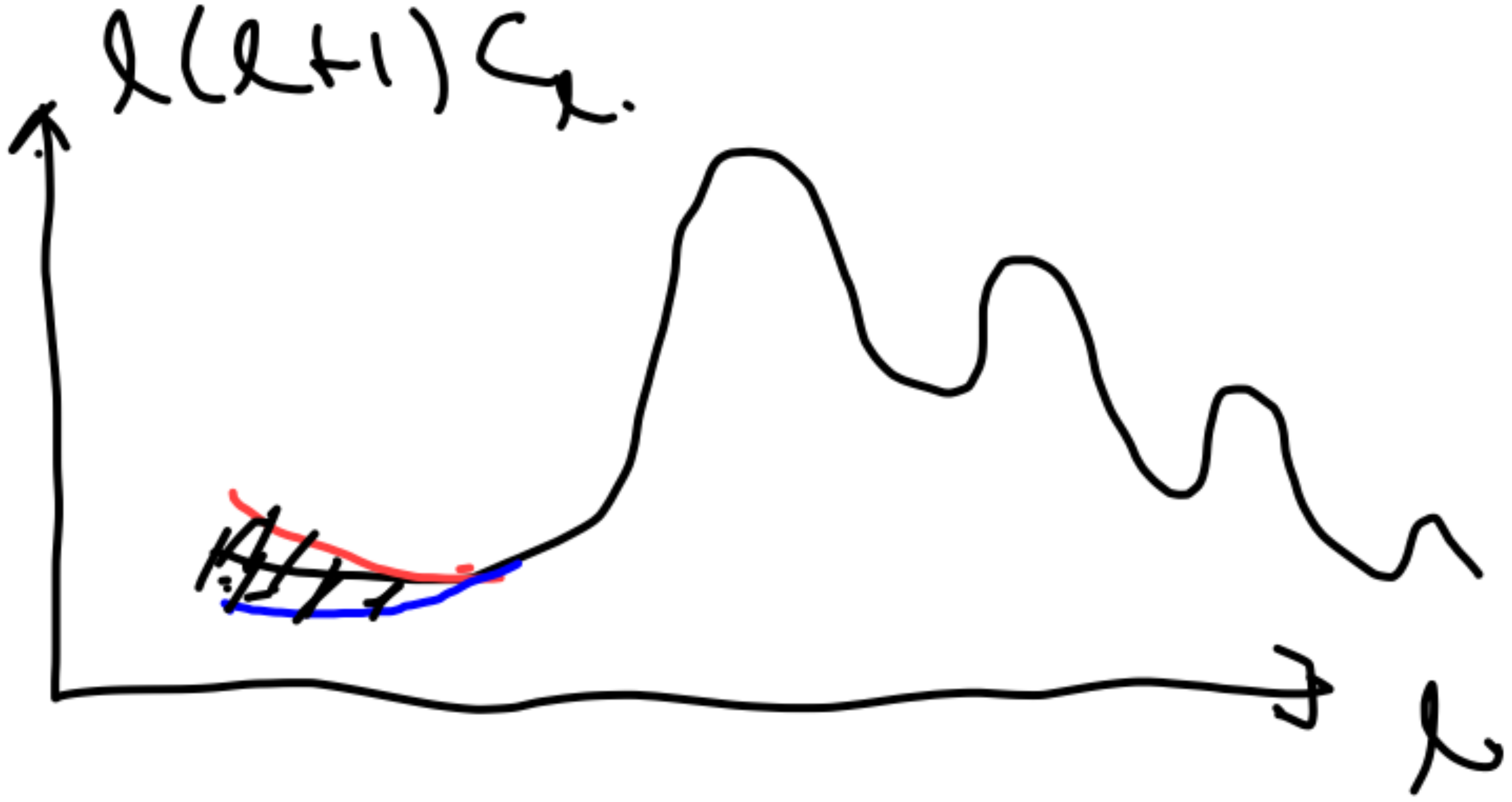


Limitation for low ℓ.

$\Leftrightarrow$ large angular separation



$\rightarrow$ not independent data.



Cosmic variance