Last scattering surface: $\lambda = \lambda_L \simeq 10^{-3}$

For $\lambda > \lambda_L$, CMB stops interacting with matter.

- Also known as recombination since e^- & p bind to form neutral H.

$\Rightarrow$ CMB photons we observed today shows us an image of the universe at $\lambda \sim \lambda_L \sim 10^{-3}$

Observation: The radiation as seen by us is highly isotropic: $T = T_0(1 + O(10^{-5}))$

$\Rightarrow$ the universe at $\lambda = \lambda_L$ was homogeneous to a great degree.

Inhomogeneities grow to form structure.

Look for Harrison-Zeldovich spectrum.

Q.1. Why was the universe so uniform?

Q.2. What is the origin of the inhomogeneities?

Q1. First guess: If the universe was in thermal equilibrium, then that would make it homogeneous.

→ This cannot be the justification.

We need to calculate ① r_H at $\lambda = \lambda_L$

② r_L = comoving distance of the last scattering surface from us today.

Note that $\lambda_L \sim 10^{-3}$ $\lambda_{eq} \sim 10^{-4}$

$\Rightarrow$ We are well into matter dominated era by the time of last scattering.

$$t_L = H_0^{-1} \int_0^{\lambda_L} du \left(\Omega_m u^{-1} \right)^{-1/2} = H_0^{-1} \Omega_m^{-1/2} \frac{2}{3} \lambda_L^{3/2}$$

$$\sim H_0^{-1} \times \left(10^{-3}\right)^{3/2} = 10^{10} \text{ years} \times 10^{-9/2}$$

$$\Downarrow$$

$$\sim \frac{1}{3} \times 10^{6} \text{ Years} \quad 10^{-4} \times \frac{1}{3}$$

$$\sim 400000 \text{ years.}$$

$$r_H\big|_{\lambda_L} = H_0^{-1} \int_0^{\lambda_L} du\, u^{-1} \left(\Omega_m u^{-1}\right)^{-1/2}$$

$$= H_0^{-1}\, \Omega_m^{-1/2}\, 2\, \lambda_L^{1/2}$$

$$r_L = r_H\big|_{\lambda=1} - r_H\big|_{\lambda_L} = 3 H_0^{-1} - H_0^{-1}\, \cancel{\Omega_m^{-1/2}}\, 2\, \lambda_L^{1/2}$$

$$\frac{r_L}{r_H} = \frac{3}{2\, \Omega_m^{-1/2}\, \lambda_L^{1/2}} \sim 10^{3/2} \sim 30$$

$$2 r_L$$

$$r_H = \frac{1}{60} \times 2 r_L$$

$$A \times \cdots \xrightarrow{r_L} \underset{\text{US}}{\otimes} \xleftarrow{r_L} \cdots \times B$$

$\rightarrow$ Horizon problem.

If we take a distance r_H on the last scattering surface, then the angle that it subtends on us:

$$\frac{r_H}{2\pi r_L} \times 2\pi \text{ radians} = \frac{r_H}{\cancel{2}\pi r_L} \times \cancel{2}\cancel{\pi} \times \frac{180}{\cancel{\pi}}$$

$$\simeq 180 \times \frac{1}{30 \times \pi} \sim 2^\circ$$

Proposed (speculative) solution $\Rightarrow$ Inflation.

What we need to solve this problem is to somehow increase $r_H|_{x_L}$.

Contribution to r_H from radiation dominated era is small.

We know physics up to $k_B T \sim 100\,GeV$.

$\lambda = 1$ at $k_B T = 8.64 \times 10^{-5}\, eV/K \times 2.73\,K \sim 10^{-4}\, eV$.

At $k_B T = 100\,GeV = 100 \times 10^{9}\, eV$

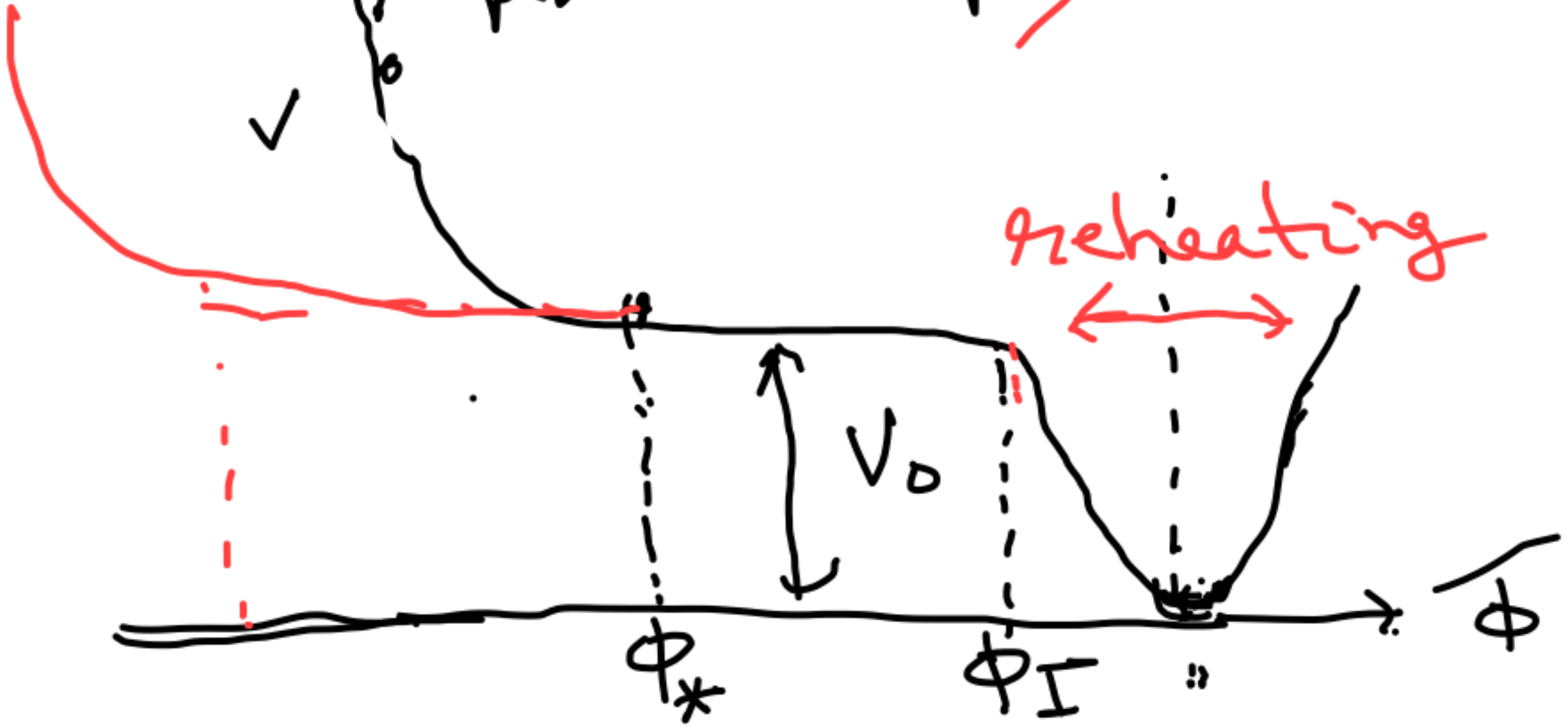
$$\lambda = \frac{10^{-4}}{10^{11}} \sim 10^{-15}$$

Inflation:

Consider a scalar field with action:

$$S = \int d^4x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \sqrt{-\det g}$$

$$T_{\mu\nu} = \cancel{\partial_\mu \phi} \partial_\nu \phi - \left[\frac{1}{2} \cancel{g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi} + V(\phi) \right] g_{\mu\nu}$$



Scalar field will slowly increase

→ gently roll down the slope.

$$\ddot{\phi} + V'(\phi) + 3H\dot{\phi} = 0$$

$$H = \frac{\dot{a}}{a}$$

We'll show that if inflation lasts long enough, then we can solve the horizon problem.

λ_* : Value of λ at the beginning of inflation

λ_I : Value of λ at the end of inflation.

During inflation: $\left(\frac{\dot{\lambda}}{\lambda}\right)^2 \simeq \frac{8\pi G}{3} V_0$ $\Rightarrow$ $\lambda = \lambda_0 \exp\left[\sqrt{\frac{8\pi G V_0}{3}}\, t\right]$

$$r_H\Big|_{\lambda_I} - r_H\Big|_{\lambda_*} = \int_{\lambda_*}^{\lambda_I} du\, u^{-1} \left(\frac{8\pi G}{3} V_0 u^2\right)^{-1/2}$$

$$= \left(\frac{8\pi G V_0}{3}\right)^{-1/2} \left(\frac{1}{\lambda_*} - \frac{1}{\lambda_I}\right) \rightarrow$$ can be made large by taking λ_* small.

We need $\left(\frac{8\pi G}{3} V_0\right)^{-1/2} \frac{1}{\lambda_*} > 2 r_L$

$r_H\Big|_{\lambda_L} < 2 r_L \quad \Rightarrow r_H\Big|_{\lambda_L} > 2 r_L$

solving the horizon problem.

$t_0 = t_0\Big|_{\text{calculated}}$ + contribution from the inflationary era.

After reheating energy density is converted to radiation.

Assume for simplicity that reheating is fast so that V_0 = radiation energy density.

T_I: temp. of radiation then

$$V_0 = K T_I^4 \qquad \lambda = \lambda_I$$

K: universal constant.

End of inflation $\Rightarrow$ Initial condition for standard big bang theory.

Once we fix T_I, then λ_I is fixed

since $\lambda_I = \frac{T_0}{T_I}$, V_0 is fixed

$= K T_I^4$

$k_B T_I > 100$ GeV.

Define $N = \ln \frac{\lambda_I}{\lambda_*}$ $\Rightarrow$ large number since λ_* should be small.

For solving the horizon problem we need $N > N_0$ (some no.) fixed by T_I.

Ex. Show that for $k_B T_I \sim 10^{16}$ GeV,

$$N_0 \simeq 60$$

$\Rightarrow$ we need $\dfrac{\lambda_I}{\lambda_*} > e^{60}$ for solving the horizon problem.

Typically we can take $\dfrac{\lambda_I}{\lambda_*} \gg e^{60}$.

$N = \ln \dfrac{\lambda_I}{\lambda_*}$ = Number of e-folding during inflation.

Physical understanding:

Exponential expansion blows up small regions into huge regions.

A pair of points that look far away and could not be within each others horizon in standard big bang theory, were actually much closer before inflation and could have been within each others horizon.

$\boxed{V'', \; V', \; V}$

$$\boxed{V = \frac{1}{2} m^2 \phi^2}$$

$m << M_{pl}$.

$\boxed{V', V'' \text{ small}, \; V \text{ large}}$

Inflation erases every past information.

$$\frac{\lambda_I}{\lambda_*} > e^{60} \Rightarrow \frac{T_{\text{end of inflation}}}{T_*} \sim e^{-60}$$

If some matter were present, then inflation dilutes the density by

$$\left(\frac{\lambda_*}{\lambda_I}\right)^3 \sim e^{-180}$$

Inflation removes past inhomogeneities

→ Quantum fluctuation during inflation can produce these fluctuations.