Bulk reconstruction in AdS/CFT:

Given a boundary CFT data, can we reconstruct the (approximate) local physics in the bulk.

$$\left\langle \prod_i O_i(X_i) \right\rangle_{CFT} \xrightarrow{?} \left\langle \prod_i \phi_i(x_i) \right\rangle_{AdS}$$

$\uparrow$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\downarrow$

$$\lim_{r_i \to \infty} \prod_i r_i^{\Delta_i} \left\langle \prod_i \phi_i(x_i) \right\rangle$$

$$x_i = (r_i, \underbrace{t_i, \theta_i}_{X_i})$$

is not expected to be well defined in quantum gravity.

First explore a direct approach based on solution to differential eq.

Q. Given the boundary value of ϕ as $r \to \infty$, can we find $\phi(x_i)$?

— analogous to initial value problem except that the evolution is along the radial direction instead of time.

We use space-like Green's function.

Green's fn. $G(x,x')$:

$$\left(\Box'_x - m^2\right) G(x,x') = \delta^{(d+1)}(x,x')$$

$\hookrightarrow$ includes $\frac{1}{\sqrt{-\det g(x')}}$

Look for $G(x,x')$ which vanishes when x, x' are time-like separated.

Note: opposite of retarded or advanced Green's fn. which vanish for space-like separation.

Construction in AdS_{d+1} (1201-3664).

① $G(x,x')$ is not symmetric under $x \leftrightarrow x'$.

② As $r' \to \infty$

$$G(x,x') \to \frac{1}{2\Delta - d}\left[r'^{-\Delta} L(x,X') + \overset{\text{dominant}}{r'^{-(d-\Delta)}} K(x,X') \right]$$

$$\Delta = \frac{d}{2} + \frac{1}{2}\sqrt{d^2+4m^2R^2}, \qquad d-\Delta = \frac{d}{2} - \frac{1}{2}\sqrt{d^2+4m^2R^2}$$

$$\phi(x) = \int d^{d+1}x' \sqrt{-\det g(x')} \; \phi(x') \left(\Box'_x - m^2\right) G(x,x')$$

$$= \int d^{d+1}x' \sqrt{-\det g(x')} \Big[D'_\mu \{ \phi(x') D'^\mu G(x,x')$$

$$- (D'^\mu \phi(x'))\, G(x,x') \} + \underbrace{(\Box' - m^2)\, \phi(x')}_{}\, G(x,x') \Big]$$

Vanish if ϕ is free field.

$$= \int d^{d+1}x' \, \partial'_\mu \Big[\sqrt{-\det g(x')} \{ \phi(x') D'^\mu G(x,x')$$

$$- (D'^\mu \phi(x'))\, G(x,x') \} \Big]$$

$G(x,x')$ vanishes outside the cylindrical region

Normal to the boundary is along r'.

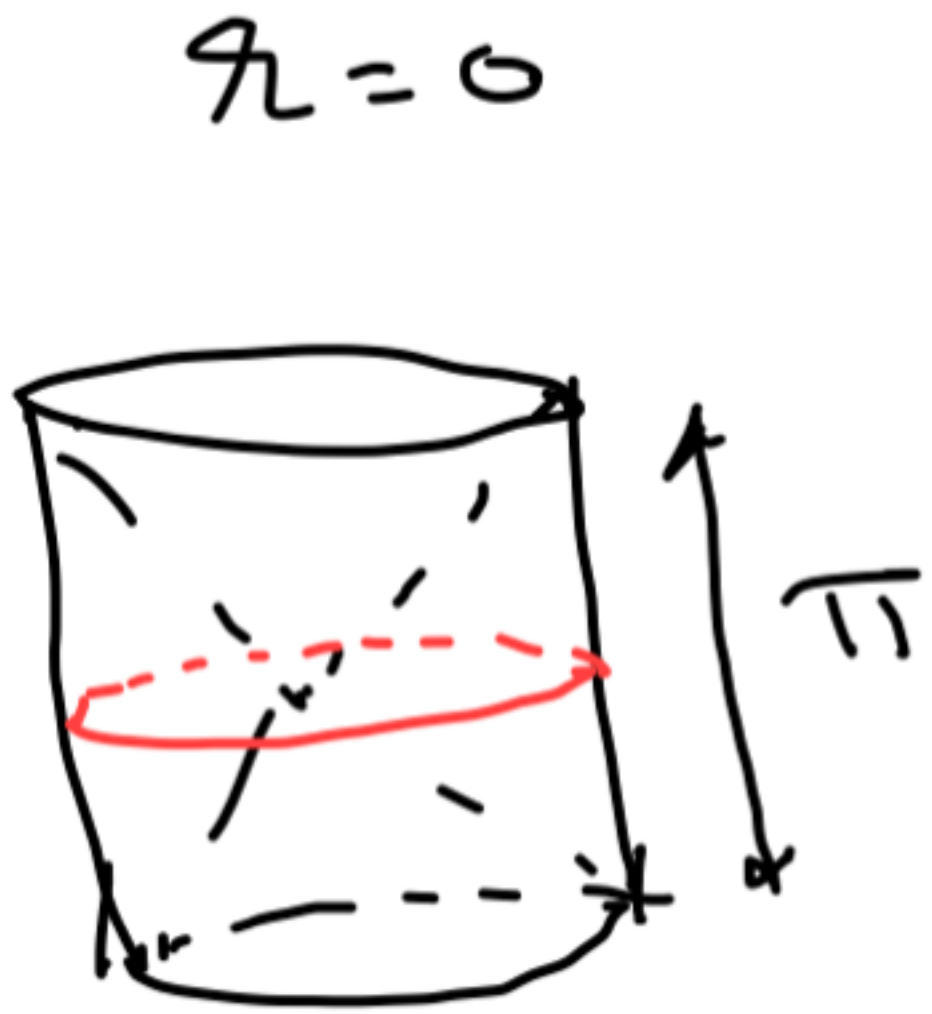
$$\boxed{ds^2 = -(R^2+r^2)\,dt^2 + \frac{dr^2}{1+r^2/R^2} + r^2\,d\Omega_{d-1}^2}$$

Boundary term.

$$= \underbrace{\sqrt{-\det g(r')}}_{r' \cdot \frac{R}{r'}\, r'^{d-1}\, \vartheta_{d-1}} \Big(\phi(r',x')\, g^{r'r'}\, \partial_{r'} G(x,x') - \partial_{r'}\phi(r',x')\, g^{r'r'}\, G(x,x') \Big)$$

$\phi \sim r'^{-\Delta}\, \mathcal{O}(x')$, $\quad g^{r'r'} \sim \left(\frac{r'}{R}\right)^2$, $\quad G(x,x') \sim \frac{1}{2\Delta-d}\, r'^{\Delta-d}\, K(x,x')$

Ex. $\phi(x) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dt' \, d\Omega'_{d-1} \, K(x, x') \, \mathcal{O}(x')$



If the boundary theory is described by a QFT with some Hamiltonian that generates time evolution, then $\mathcal{O}(x)$ can be expressed in terms of $\mathcal{O}(t=0, \vec{\theta})$.

Interacting theory.

Suppose we have $-\frac{\lambda}{3}\int d^{d+1}x \; \phi^3$ term in S.

$$(\Box - m^2)\phi = \lambda \phi^2.$$

$$\int G(x,x') \; \lambda \phi(x') \; \phi(x') \rightarrow \text{extra term.}$$

$$\int dx'' \, K(x', x'') G(x'') \int dx''' \, K(x', x''') G(x''') + O(\lambda^2)$$

Note: what we have done so far shows that general bulk reconstruction requires data on the full $t=0$ slice on the boundary.

$\phi(x)$ is given in terms of $O(X)$ over the entire S^{d-1} at $t=0$.

What we'll show next is that there is another reconstruction that needs boundary data on half of S^{d-1}.

Introduction to Rindler coordinates

Minkowski space: $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$

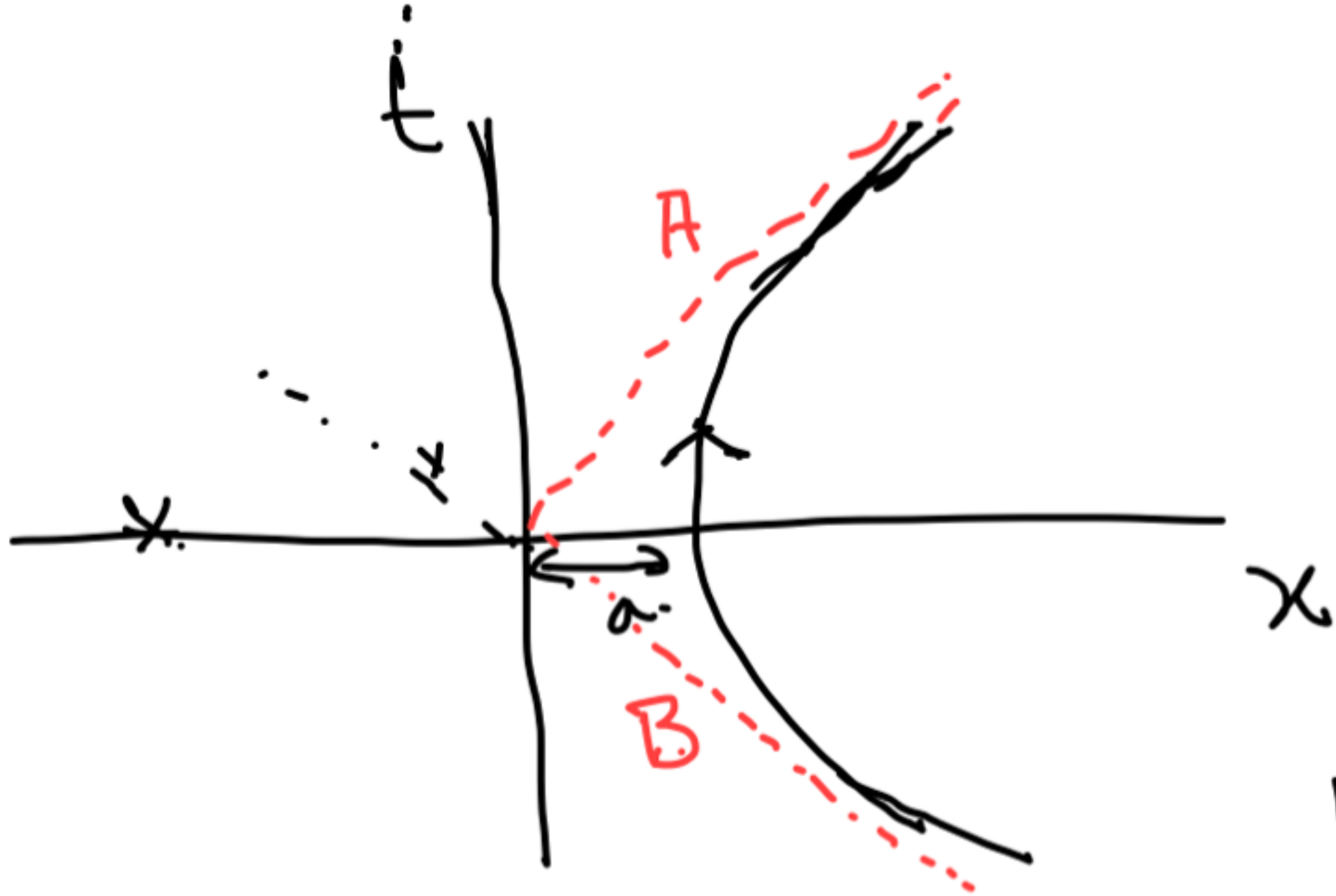
Consider an observer moving along x with constant acceleration.

Ex. $x = a\cosh\frac{\tau}{a}$, $t = a\sinh\frac{\tau}{a}$ → parameter labelling the trajectory

$y = 0$, $z = 0$.

$ds^2 = -d\tau^2 \Rightarrow \tau$ is the proper time.

$x^2 - t^2 = a^2 \rightarrow$ hyperbola.



The observer cannot receive signal from beyond A.
Cannot send signal beyond B

A, B act as horizons – Rindler horizons.
The observer can do an experiment only on the right wedge $|x| > |t|$
Rindler wedge.

Change coordinates $(a=1)$

$x = \sqrt{\rho}\cosh\tau$, $t = \sqrt{\rho}\sinh\tau$

$$ds^2 \underset{\text{ex.}}{=} -2\rho\, d\tau^2 + \frac{d\rho^2}{2\rho} + dy^2 + dz^2$$

$\rho > 0$
$-\infty < \tau < \infty$
$x > 0$
$|x| > |t|$

Rindler wedge.

$\rho = 0 \Rightarrow$ Horizon.

$\rho = x^2 - t^2$. $\quad \rho = 0 \Rightarrow x = \pm t$.

Rindler observer sits at fixed ρ.
τ is the proper time of the observer.

AdS_{d+1} Rindler coordinates!

Recall: $T_1^2 + T_2^2 - \sum_{i=1}^{d} X_i^2 = R^2$, $ds^2 = -dT_1^2 - dT_2^2 + \sum_{i=1}^{d} dX_i^2$

$T_1 = \rho \cosh \chi$

$T_2 = \sqrt{\rho^2 - R^2} \sinh \tau$

$X_d = \sqrt{\rho^2 - R^2} \cosh \tau$

$\underbrace{X_1^2 + \cdots + X_{d-1}^2 = \rho^2 \sinh^2 \chi}_{(d-2) \text{ angles.}}$

$X_d \geq 0, \; T_1 \geq 0$

$\rightarrow$ covers part of global AdS_{d+1}

$\rho = R$

$\rightarrow$ horizon.

Ex. $T_1^2 + T_2^2 - \sum_{i=1}^{d} X_i^2 = R^2$

$\rho > R, \quad -\infty < \chi < \infty$

$-\infty < \tau < \infty$.

$$ds^2 = -(\rho^2 - R^2)\, d\tau^2 + R^2 \frac{d\rho^2}{\rho^2 - R^2} + \rho^2\, dH_{d-1}^2$$

$$dH_{d-1}^2 = d\chi^2 + \sinh^2 \chi \, d\Omega_{d-2}^2$$