Some general comments:

① The description in terms of quantum error correction is qualitative.
→ does not take into account the dynamics of CFT.

② There are no small parameters like g_{YM}, $1/N$ $\Rightarrow$ all scales are of the same order

$R \sim l_s \sim l_p$

→ each bulk point represents a model for a region of AdS of size R → sub-AdS.
locality not visible.

Within these limitations we'll try to describe a detailed version of bulk-boundary correspondence.

More details in Harlow: 1802.01040

Pastawski, Yoshida, Harlow, Preskill.

Basic building blocks: 5 qubit code.

→ encodes one qubit into 5 qubits.

2-d → qubit

→ $2^5 = 32$ dim.

$\langle i_1 \cdots i_5 | \tilde{j} \rangle$ $\quad T_{i_1 \cdots i_5, j} = 2 \langle i_1 \cdots i_5 | \tilde{j} \rangle$

Code subspace is a 2-D subspace of the 32-dimensional space that is invariant under:

$$S_1 = \sigma_x \otimes \sigma_z \otimes \sigma_z \otimes \sigma_x \otimes I$$
$$S_2 = I \otimes \sigma_x \otimes \sigma_z \otimes \sigma_z \otimes \sigma_x$$
$$S_3 = \sigma_x \otimes I \otimes \sigma_x \otimes \sigma_z \otimes \sigma_z$$
$$S_4 = \sigma_z \otimes \sigma_x \otimes I \otimes \sigma_x \otimes \sigma_z$$

$S_i^2 = 1$

$[S_i, S_j] = 0$

S_i's are permuted under cyclic perm. of the qubits.

In this 2D subspace choose an orthonormal basis and call one of them $|\tilde{0}\rangle$ and the other $|\tilde{1}\rangle$.

$T_{i_1 \cdots i_5, j} \equiv 2 \langle i_1 \cdots i_5 | \tilde{j} \rangle$ is a perfect tensor.

$M_{j i_1 i_2, i_3 i_4 i_5} \equiv T_{i_1 \cdots i_5, j}$

$M^\dagger M = I \rightarrow 2^3 \times 2^3$ unitary matrix.

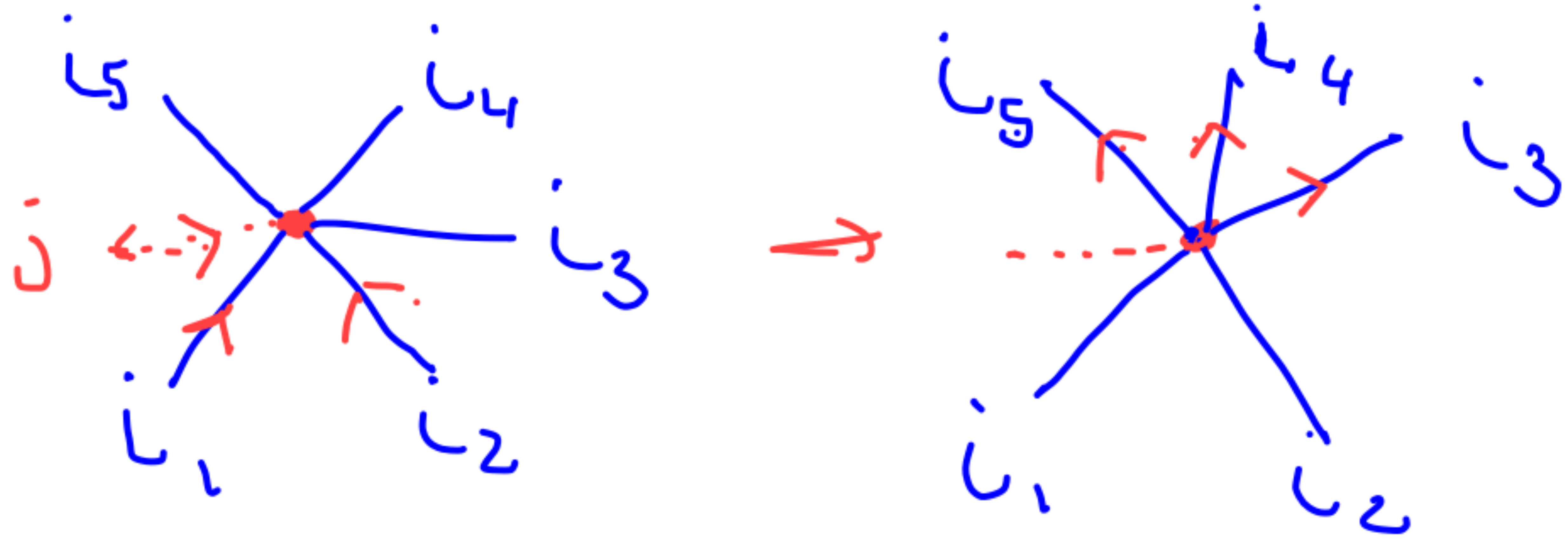
A property of T:

Take any $2^3 \times 2^3$ operator G.

$$G_{i_1 i_2 i_3, j_1 j_2 j_3} M_{j_1 j_2 j_3, k_1 k_2 k_3} = M_{i_1 i_2 i_3, j_1 j_2 j_3} G'_{j_1 j_2 j_3, k_1 k_2 k_3}$$

$G' = M^\dagger G M.$

$T_{i_1 i_2 i_3 i_4 i_5, j}$



Recall AdS_3 geometry:

$$ds^2 = -(r^2+R^2)\,dt^2 + \frac{dr^2}{1+r^2/R^2} + r^2\,d\theta^2$$

$t=0$ slice: $\quad ds_2^2 = dr^2\left(1+\frac{r^2}{R^2}\right)^{-1} + r^2\,d\theta^2$

→ 2-D Hyperbolic space.

→ Admits pentagon tiling.

Embedding coords:

$t=0 \Rightarrow T_2 = 0$

$T_1^2 - X_1^2 - X_2^2 = R^2$

$ds^2 = -dT_1^2 + dX_1^2 + dX_2^2$

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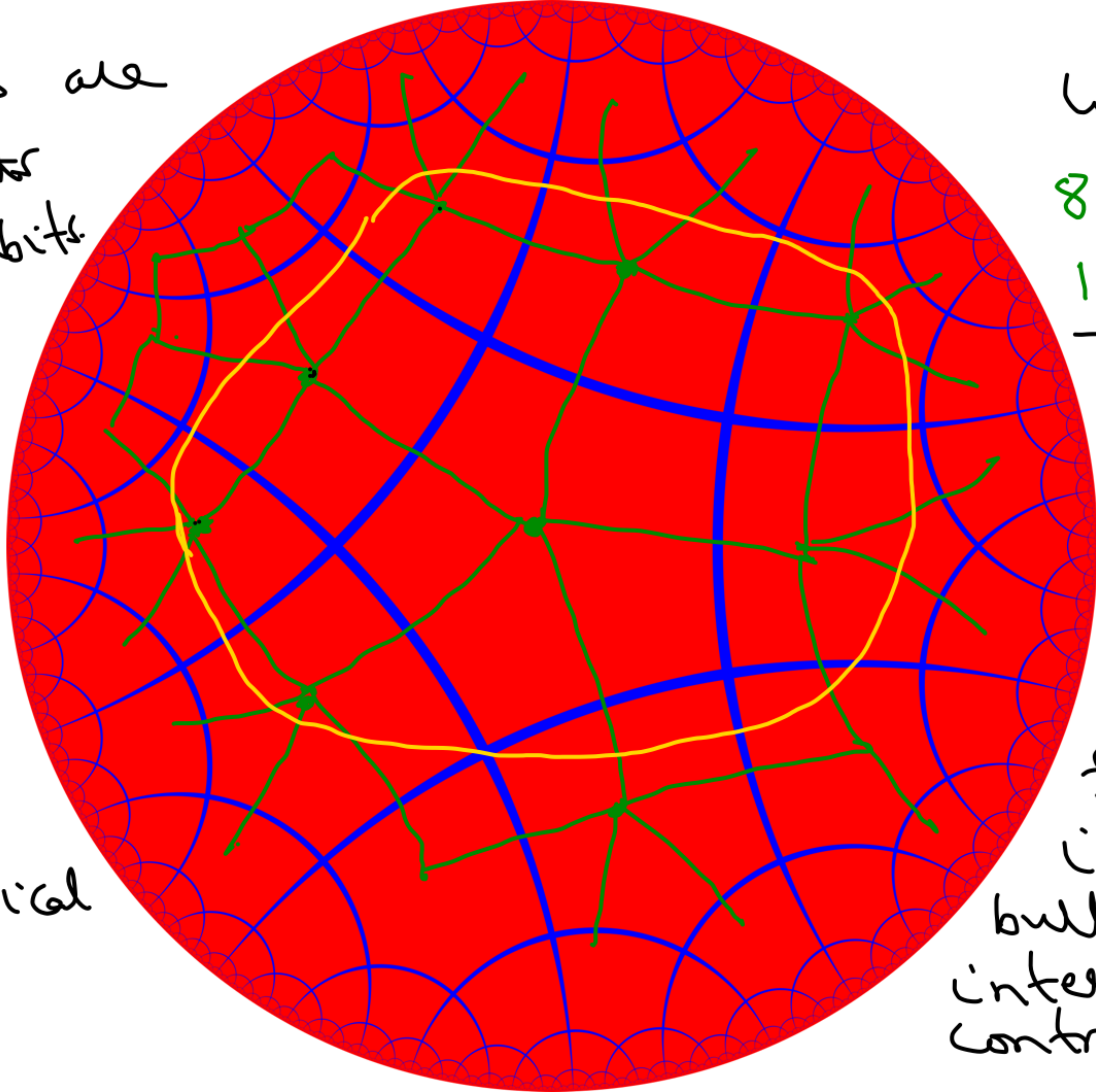
① Vertices are bulk points → logical qubits

② Choose an arbitrary boundary + intersection with the links are boundary points – physical qubits.



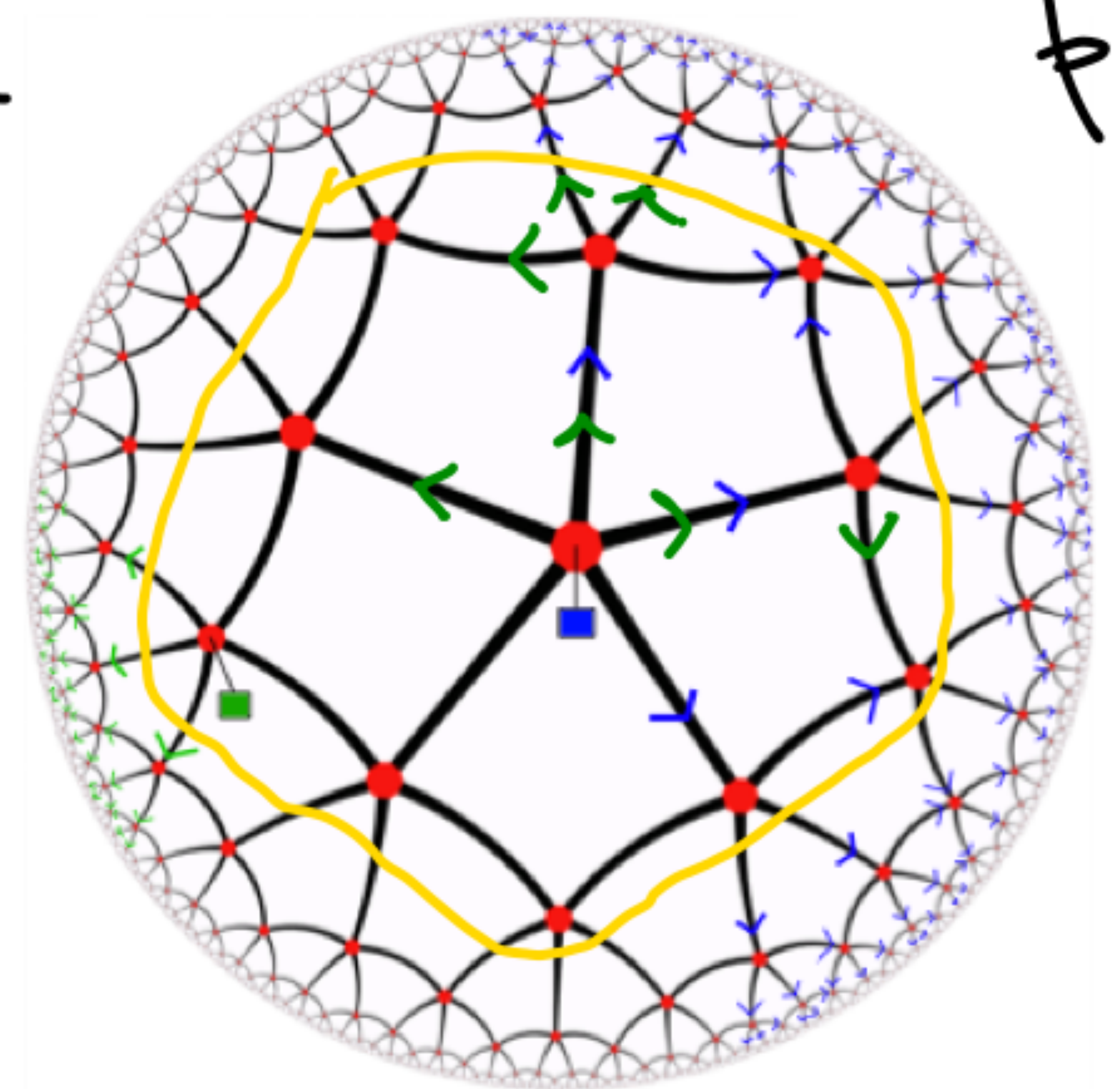
Wikipedia.

8 logical.

19. physical.

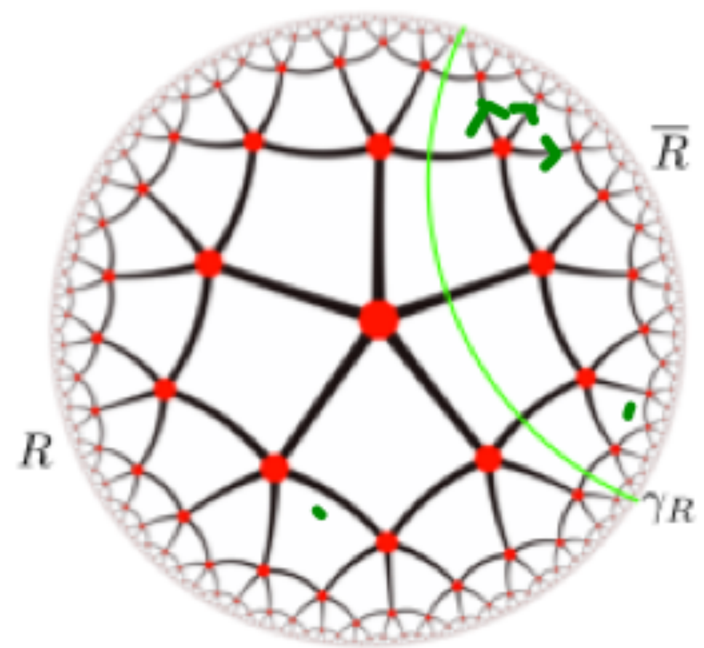
$\langle i_1 \cdots i_n | \underbrace{}_{\vdots} j_1 \cdots j_k \rangle$ = Product of $T_{i_1' \cdots i_s', j'}$ for each interior bulk pts. with internal indices contracted.

We want to show that this code can be used to push the action of any operator on the logical qubits to an operator on the physical qubits.



$OM = MO'$

bulk operator $\rightarrow$ boundary operator.



$\overline{R}$
R
γ_R