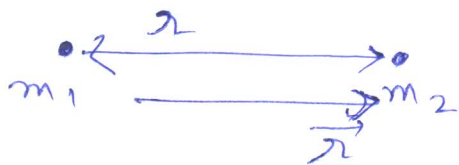


16/7/07

General Relativity

Find a theory of gravity.

Newton's theory: \rightarrow (we already have this theory)



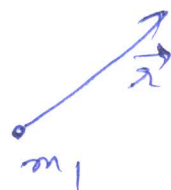
$$\vec{F} = - \frac{G m_1 m_2}{r^2} \hat{r}$$

(Divide the problem in 2 parts) \rightarrow

Introduce Gravitational potential $\phi(\vec{r})$

A point mass m_1 produces:

$$\phi = - \frac{Gm}{r}$$



(then we say that) \rightarrow

Force of a point mass m_2 at \vec{r}

$$\text{is } \vec{F} = -(\vec{\nabla}\phi) m_2$$

(what is wrong with Newtonian gravity?)

Theoretical reason :- Not compatible with special theory of relativity.

(no signal faster than c possible) (exp. reason :- mercury perihelion)

m_1

m_2

(to send a signal in Newtonian picture, just move m_1 a little bit - force in m_2 changes instantaneously - grav. field changes instantaneously)

within the framework of GR, it will be seen intrinsically that there can't be 2 kinds of charges as in EM - it can't be seen from Newtonian gravity.

The reverse case :-

$$\frac{\partial x}{\partial \theta} = a \cos \theta \cos \phi, \quad \frac{\partial x}{\partial \phi} = -a \sin \theta \sin \phi$$

$$\frac{\partial y}{\partial \theta} = a \cos \theta \sin \phi, \quad \frac{\partial y}{\partial \phi} = a \sin \theta \cos \phi$$

$$\begin{aligned} \tilde{g}_{\theta\theta} &= g_{xx} \frac{\partial x}{\partial \theta} \frac{\partial x}{\partial \theta} + g_{yy} \frac{\partial y}{\partial \theta} \frac{\partial y}{\partial \theta} + 2g_{xy} \frac{\partial x}{\partial \theta} \frac{\partial y}{\partial \theta} \\ &= \left(1 + \frac{x^2}{a^2 - x^2 - y^2}\right) a^2 \cos^2 \theta \cos^2 \phi \\ &\quad + \left(1 + \frac{y^2}{a^2 - x^2 - y^2}\right) a^2 \cos^2 \theta \sin^2 \phi \\ &\quad + 2 \frac{xy}{a^2 - x^2 - y^2} a^2 \cos^2 \theta \sin \phi \cos \phi \\ &= a^2 \cos^2 \theta + a^2 \cos^2 \theta \frac{x^2 \cos^2 \phi + y^2 \sin^2 \phi + 2xy \sin \phi \cos \phi}{a^2 - x^2 - y^2} \\ &= a^2 \cos^2 \theta \left[1 + \frac{(x \cos \phi + y \sin \phi)^2}{a^2 - x^2 - y^2} \right] \end{aligned}$$

~~$$\tilde{g}_{\theta\theta} = a^2 \cos^2 \theta \left[1 + \frac{a^2 \sin^2 \theta (1 - \cos^2 \phi)}{a^2 - a^2 \sin^2 \theta} \right]$$~~

$$\begin{aligned} \tilde{g}_{\theta\theta} &= a^2 \cos^2 \theta \left[1 + \frac{(a \sin \theta \cos \phi + a \sin \theta \sin \phi)^2}{a^2 - a^2 \sin^2 \theta} \right] \\ &= a^2 \cos^2 \theta \left[1 + \frac{a^2 \sin^2 \theta}{a^2 \cos^2 \theta} \right] \\ &= a^2 \cos^2 \theta + a^2 \sin^2 \theta = a^2 \\ \Rightarrow \tilde{g}_{\theta\theta} &= a^2 \end{aligned}$$

Again,

$$\begin{aligned} \tilde{g}_{\phi\phi} &= \left(1 + \frac{x^2}{a^2 - x^2 - y^2}\right) a^2 \sin^2 \theta \sin^2 \phi \\ &\quad + \left(1 + \frac{y^2}{a^2 - x^2 - y^2}\right) a^2 \sin^2 \theta \cos^2 \phi \\ &\quad - \frac{2xy}{a^2 - x^2 - y^2} a^2 \sin^2 \theta \sin \phi \cos \phi \\ &= a^2 \sin^2 \theta + \frac{a^2 \sin^2 \theta}{a^2 - x^2 - y^2} (x^2 \sin^2 \phi + y^2 \cos^2 \phi - 2xy \sin \phi \cos \phi) \\ &= a^2 \sin^2 \theta \left[1 + \frac{(x \sin \phi - y \cos \phi)^2}{a^2 \cos^2 \theta} \right] \\ &= a^2 \sin^2 \theta \left[1 + \frac{a^2 (\sin^2 \phi - \cos^2 \phi)^2}{a^2 \cos^2 \theta} \right] \end{aligned}$$

