

24/7/07

Suppose we have  $R(x)$   $\tilde{R}(y)$

Is there  $y = f(x)$  such that

Many of possible choices  $f(x)$  → it doesn't fix  $f(x)$  uniquely

$$\tilde{R}(f(x)) = R(x)$$

If  $R$  is a const., we can't do this, as in the case of sphere.

so look at other quantities :-

$$R_{ij}(x), R^{ij}(x), \tilde{R}_{ij}(y), \tilde{R}^{ij}(y)$$

We must have  $R^{ij}(x) R_{ij}(x) = \tilde{R}^{ij}(f(x)) \tilde{R}_{ij}(f(x))$

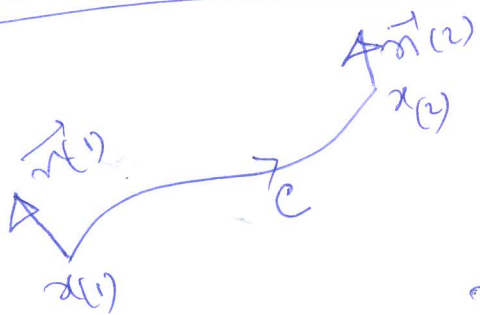
It gives another condn on  $f(x)$

and 2-D it fixes the cond. too.

Look at a 3rd scalar :-

If  $R_{ijkl}(x) R^{ijkl}(x) = \tilde{R}_{ijkl}(f(x)) \tilde{R}^{ijkl}(f(x))$

Now it's a consistency check & we have no freedom in choosing  $f(x)$



$$\frac{dx^i(u)}{du} + \Gamma_{jk}^i(x(u)) \frac{dx^j}{du} x^k(u) = 0$$

$$x^i(u=0) = x^i(1)$$

$$x^i(u=1) = x^i(2)$$

$\{x^i(u)\}$  : Path connecting  $x(1)$  to  $x(2)$   
 $0 \leq u \leq 1$

$$x(2) = M(x(1), x(2), C) x(1)$$

② Rules for // transport are reversible (clearly, if  $\tilde{m} \parallel \tilde{n}$ ,  $\tilde{m} \parallel \tilde{n}$  in the local coord. sys.)

changing  $u$  to  $(1-u)$

$$\frac{dn^i(1-u)}{du} + \Gamma_{lk}^i(x(1-u)) \frac{dx^k}{du} n^l(1-u) = 0$$

[follows from  $\frac{dn^i(u)}{du} + \Gamma_{lk}^i(x(u)) \frac{dx^k}{du} n^l(u) = 0$ ]

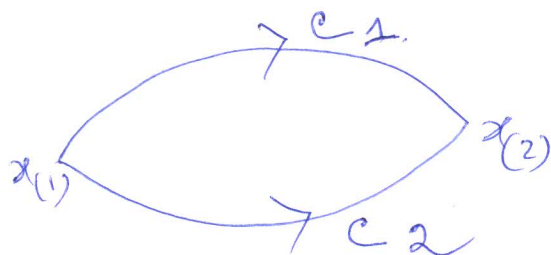
[The opp. happens in Newtonian mech. in presence of friction —  $\frac{d^2x}{dt^2}$  &  $\frac{dx}{dt}$  pick up opp. sign on ~~time~~ reversal]

$$\left. \begin{aligned} x^i(1-u) &= x^i_{(2)} & \text{at } u=0 \\ &= x^i_{(1)} & \text{at } u=1 \end{aligned} \right\} \rightarrow \text{B.C.}$$

$\rightarrow$  paths reversed

$$M(x_{(2)}, x_{(1)}; -c) M(x_{(1)}, x_{(2)}; c) = \mathbb{1}$$

$$M(x_{(2)}, x_{(1)}; -c) = (M(x_{(1)}, x_{(2)}, c))^{-1}$$



$$\begin{aligned} &M(x_{(1)}, x_{(2)}, c_1) \\ &M(x_{(1)}, x_{(2)}, c_2) \end{aligned}$$

(Under what conditions can these be equal?)

if they are equal, then  $\rightarrow$

$$M(x_{(1)}, x_{(2)}, c_2)^{-1} (M(x_{(1)}, x_{(2)}, c_1)) = \mathbb{1}$$

$$\Rightarrow M(x_{(1)}, x_{(2)}, -c_2) M(x_{(2)}, x_{(1)}, +c_1) = \mathbb{1}$$

So equivalent question: Under what condition // transport along the closed curve  $c_1 - c_2$  brings a vector to the same vector?

We'll study this in the context of small loops.

