

27/7/07

[We derived
Eqns of motion of a charged particle in presence
of a background metric & a background EM
- Now we address the reverse problem...]

Generalise Maxwell's eqns in
presence of gravity :-

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \rightarrow \text{Bianchi identities}$$

These are consequences of $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

In the component notation, they corr. to

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \& \quad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$F_i = -F_{0i} \quad \text{where } i = 1, 2, 3$$

$$F_{ij} = \epsilon_{ijk} B_k \quad \text{where } i, j, k = 1, 2, 3$$

In absence
of gravity

These eqns
are used
to solve for
the vector
potential

$$\partial_\mu F_{\nu\rho}(x) = -\partial_\nu J^\rho(x)$$

where $J^0(x) = \rho(\vec{x}, t)$ → charge density

$$\& \quad J^i = j^i(\vec{x}, t)$$

↓
current

In the presence of gravity, (Bianchi identities generalise to)

$$D_\mu F_{\nu\rho} + D_\nu F_{\rho\mu} + D_\rho F_{\mu\nu} = 0$$

Ex. Show that this is equivalent to:

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0$$

[T's cancel]
even though it doesn't explicitly look covariant,
it is indeed covariant

Also, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \overline{D}_\mu A_\nu - \overline{D}_\nu A_\mu$
→ T's cancel

[We have to know the tr. prop. of $J^\mu(x)$,
which isn't an indep. quantity, to get a
cov. eqn. from $\eta^{\mu\nu} \partial_\mu F_{\nu\sigma} = -\eta_{\sigma\kappa} J^\kappa$]

~~What is J^μ in flat space-time?~~

What is J^μ in flat space-time?

→ Suppose we have ^{several} particles with
charges e_1, e_2, \dots with trajectories

$\vec{x}_1(t), \vec{x}_2(t), \dots$

The charge density is given by

$$\rho(\vec{x}, t) = \sum_n e_n \delta^{(3)}(\vec{x} - \vec{x}_n(t))$$

$$\therefore J^i(\vec{x}, t) = \sum_n e_n \frac{d\vec{x}_n^i}{dt} \delta^{(3)}(\vec{x} - \vec{x}_n(t))$$

Manifestly Lorentz inv. form

Represent the trajectory as

$$x^\mu = X^\mu(u)$$

parameter
labelling the trajectory

e.g. → if $u = x^0$, $x^0 = u^0$ → first eqn.

