

15/11/06

$$S_{\text{total}} = S + S_{\text{g.f.}} + S_{\text{ghost}}$$

is invariant under a BRST symmetry

Ex. If we take the most general action Lagrangian density with dimension ≤ 4 terms & require that it is invariant under BRST transformation, we arrive at the action given.

Example \rightarrow Take the gauge-fixing term as

$$S_{\text{g.f.}} = - \int d^4x \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 \rightarrow \text{Lorentz invariant}$$

Consider a general ^{total} Lagrangian ^{density} containing all possible Lorentz invariant terms with arbitrary coefficients.

Then require $\delta S_{\text{tot}} = 0$ under BRST t.o.s.

Take cubic & quartic coupling constants for A^μ to be diff.

$\xrightarrow{\text{this gives us}}$ relations among the coefficients appearing in S .

set coeff. of each operator equal to zero

Ex. Show that these relations lead to

$$S_{\text{tot}} = S + S_{\text{g.f.}} + S_{\text{ghost}}$$

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) \\
& - \frac{g}{2} (\partial_\mu A_\nu^c - \partial_\nu A_\mu^c) f^{abc} A^{\mu a} A^{\nu b} \\
& - \frac{1}{4} g^2 f^{abc} f^{a'b'c'} A_\mu^a A_\nu^b A^{\mu a'} A^{\nu b'} \\
& - \frac{1}{2\alpha} \partial^\mu A_\mu^c \partial^\nu A_\nu^c \\
& + b_a(x) \partial^\mu \partial_\mu c_a(x) \\
& + g f^{bca} \partial^\mu b_a(x) A_\mu^c(x) c_b(x) \\
& + \bar{\Psi}_k (i\gamma^\mu \partial_\mu - m) \Psi_k \\
& + g \bar{\Psi}_k \gamma^\mu (R(T^a))_{kl} \Psi_l A_\mu^a
\end{aligned}$$

We haven't added scalars
We have just added fermions in a particular repr.

one could add scalars & fermions in diff. reprs & couplings betw. scalars & fermions

(We will not directly use BRST sym. here to prove renormalizability)

$$\begin{aligned}
A_\nu^a &= \tilde{Z}_A^{1/2} A_{\nu R}^a, \quad b_a = \tilde{Z}_{ghost}^{1/2} b_{aR}, \\
c_a &= \tilde{Z}_{ghost}^{1/2} c_{aR}, \quad \Psi = \tilde{Z}_\Psi^{1/2} \Psi_R
\end{aligned}$$

$$g = Z_g g_R \mu^{\epsilon}, \quad \alpha = Z_\alpha \alpha_R, \quad m = Z_m m_R$$

We have manifestly made const. for b_a & c_a equal bco they appear together

Ex. Check ~~that~~ mass dimensions of g, α, m in $4-\epsilon$ dim.

(g_R, α_R are dimensionless & m_R has dim. of mass)

