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Spontaneous symmetry breaking

Suppose we have a continuous symmetry group G.

Suppose we have a set of scalar fields

$$\vec{\Phi} = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$$ is a representation R of G

ϕ can be taken to be real or complex → take twice...

For $g \in G$, $\vec{\Phi} \to R(g)\vec{\Phi}$

$R(g)$ → representation matrix

the repn need not be irreducible

Suppose $V(\vec{\Phi})$ is the potential

$V(R(g)\vec{\Phi}) = V(\vec{\Phi})$ since G is the symmetry group of $V(\vec{\Phi})$.

Suppose $V(\vec{\Phi})$ is a minimum at $\vec{\Phi}_0$.

Then $R(g)\vec{\Phi}_0$ is also a minimum of $V(\vec{\Phi})$.

In general $R(g)\vec{\Phi}_0 \neq \vec{\Phi}_0$

\# $H \subset G$: Collection of elements of G such that $R(h)\vec{\Phi}_0 = \vec{\Phi}_0$ if and only if $h \in H$.

look at a subset of elements which leave Φ_0 unchanged

① $I \in H$

② If $h_1, h_2 \in H$

$\Rightarrow R(h_1)\vec{\Phi}_0 = \vec{\Phi}_0$, $R(h_2)\vec{\Phi}_0 = \vec{\Phi}_0$

$\Rightarrow R(h_1 h_2)\Phi_0 = R(h_1)R(h_2)\vec{\Phi}_0$

$\Rightarrow R(h_1)\, R(h_2)\, \vec{\Phi}_0 = R(h_1)\, \vec{\Phi}_0 = \vec{\Phi}_0$

$\Rightarrow h_1 h_2 \in \mathcal{H}$

3) $R(h)\, \vec{\Phi}_0 = \vec{\Phi}_0 \Rightarrow \vec{\Phi}_0 = R(h)^{-1}\, \vec{\Phi}_0 = R(h^{-1})\, \vec{\Phi}_0$

$\Rightarrow h^{-1} \in \mathcal{H}$ if $h \in \mathcal{H}$

$\xRightarrow[\text{prop. show that}]{\text{these}}$ $\mathcal{H}$ is a subgroup of G

Infinitesimal transformations

$$g = \mathbb{1} + i \sum_{a=1}^{n_G} \epsilon^a T^a$$

[$T^1, \ldots, T^{n_G}$ are generators of G]

We take $T^1, \ldots, T^{n_{\mathcal{H}}}$ to be generators of $\mathcal{H}$.

what linear combs of generators we choose is upto us

$$\therefore\ h = \mathbb{1} + i \sum_{a=1}^{n_{\mathcal{H}}} \epsilon^a T^a$$

$$V\Big(\underbrace{R\big(\mathbb{1} + i \sum_{a=1}^{n_G} \epsilon^a T^a\big)}_{\mathbb{1} + i \sum_{a=1}^{n_G} \epsilon^a R(T^a)} \vec{\Phi}\Big) = V(\vec{\Phi})$$

$$\Rightarrow V\Big(\vec{\Phi} + i \sum_{a=1}^{n_G} \epsilon^a R(T^a)\, \vec{\Phi}\Big) = V(\vec{\Phi})$$

$$\Rightarrow V\Big(\vec{\Phi}_0 + i \sum_{a=1}^{n_G} \epsilon^a R(T^a)\, \vec{\Phi}_0\Big) = V(\vec{\Phi}_0)$$

min. of pot. has certain flat dirns along these deviations