Lecture 15 01.03.2011

Δ_+ satisfies the field eqn.

and Δ_F satisfies the green's function,

$$\Delta_+(x,y) = \langle 0|\phi(x)\phi(y)|0\rangle$$

$$= \frac{1}{(2\pi)^3}\int d^4k\, \delta(-k^2-m^2)\,\theta(k_0)\, e^{ik\cdot(x-x')}$$

$$= \frac{1}{(2\pi)^3}\int \frac{d^3k}{2\omega_k}\, e^{-i\omega_{\vec{k}}(x^0-x'^0)+i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$\Delta_+(x,x) = \langle 0|\phi(x)^2|0\rangle$$

we have to set $x^0 = x'^0$ & $\vec{x} = \vec{x}'$

$$\Delta_+(x,x) = \langle 0|\phi(x)^2|0\rangle$$

$$= \frac{1}{(2\pi)^3}\int \frac{d^3k}{2\omega_{\vec{k}}} = \frac{1}{(2\pi)^3}\int\frac{d^3k}{2\sqrt{k^2+m^2}}$$

however

$$\omega_{\vec{k}} = \sqrt{k^2+m^2}$$

$$\Delta_+(x,x) = \frac{1}{(2\pi)^3}\int\frac{4\pi k^2\, dk}{2\sqrt{k^2+m^2}} \longrightarrow \infty.$$

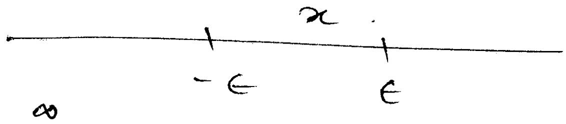
If x & y is not same

then also $\displaystyle\int\frac{d^3k}{2\omega_k}\, e^{-\omega_{\vec{k}}(x^0-x'^0)+i\vec{k}\cdot(\vec{x}-\vec{x}')}$

$\hookrightarrow$ diverging $\hookrightarrow$ highly oscillatory.

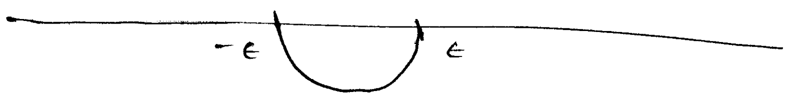
$\longrightarrow$ converging.

$\frac{1}{x}$



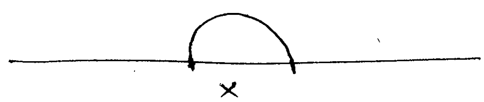
$$\int_{-\infty}^{-\epsilon} \frac{1}{x}\,dx + \int_{\epsilon}^{\infty} \frac{1}{x}\,dx$$

we can also calculate it



for

$$\frac{1}{x+i\epsilon}$$



put this pole on the top.

$$\int f(x)\cdot\frac{1}{x-i\epsilon}\,dx =$$

Over the semicircle we can approximate the value of func.

$$= -i\pi f(0).$$

Thus $\mathrm{Im.}\ \frac{1}{x+i\epsilon} = \pi f(0)$

Thus $\mathrm{Im}\ \frac{1}{x+i\epsilon}$ behaves as $\delta(x)$

A_F has $\dfrac{1}{-k^2 - m^2 - i\epsilon}$.

so $\delta(-k^2 - m^2)$ is the imaginary part of the Feynmann integral,

As long as x & y are not same

$A_+(x,y)$ - finite!

This divergence is same as what we got in the energy. These divergence come from large $\vec{k}$. we can think each $\phi(\vec{x})$ to be can be considered as infinite no. of momentum. This divergences are called ultraviolet / short distance divergence.

short distance $x \to x$. comes when x & y dist. goes to zero.

$\langle 0| \phi(x)^2 |0\rangle$ - divergent.

$\langle 0| \phi(x)|0\rangle = 0$.

but $\langle 0| \phi(x)\, \phi(x)\, |0\rangle$

$\downarrow$

$|n\rangle\langle n|$

$\hookrightarrow$ one particle states

~~there are~~ ∞ no. of states

same pts. phase nullify.

$$\langle 0 | a(k)\, a^{+}(k') | 0 \rangle = \delta^3 (k - k')$$

$\phi(x)$ - creating superposition of one particle state.

$$\langle x | x' \rangle = \delta^3 (x - x')$$

$$\phi(x) = \int \tilde{\phi}(k)\, e^{ikx}\, d^3k$$

$$= \int \hat{\phi}(\vec{k})\, e^{i\vec{k}x}\, d^3k$$

$$= \cancel{\int \delta \left[a(k) + i \right.}$$

In the first quantized formulation there is no ~~$\phi(x)$~~ operator $\phi(x)$.

$$\langle 0 | T\, (\phi(x_1) \ldots \phi(x_n)\ \phi(x)^2) | 0 \rangle \longrightarrow \infty$$

so $\phi(x)^2$ is not a good operator.

<u>Normal Ordered operator</u> :-

$$: \phi^2(x) : = \lim_{y \to x} \{ \phi(x)\, \phi(y) - \Delta_+ (x, y) \}$$

$\langle 0 | : \phi^2(x) : | 0 \rangle$

$= \langle 0 | \lim_{y \to x} \{ \phi(x)\, \phi(y) - \Delta_+(x,y) \} | 0 \rangle$

$\longrightarrow \lim_{y \to x} (\phi(x)\phi(y)) - \Delta_F(x,y).$

$= 0.$

~~lim~~ $\langle 0 | \phi(x)\, \phi(y) | 0 \rangle$

$\longrightarrow \cancel{\delta^3(x-y)}.\ \Delta_+(x,y)$

$\langle 0 | : \phi^2(x) : | 0 \rangle = 0.$

Any comb. of operator is also an operator.

$\langle 0 |$ 1) First express ϕ^2 in terms of a, a^+.

2) Then simply by hand moving all the a's to the right of a^+'s.

$\phi(x)\, \phi(y) \longrightarrow aa,\ aa^+,\ a^+a,\ a^+a^+.$

$\longrightarrow a^+a$ by subtracting it by $\delta[a^+, a]$.

subtracting $\Delta_+ \Rightarrow aa^+ \longrightarrow a^+a.$

$aa^+ = a^+a + [a, a^+].$

$[a^-, a^+] \longrightarrow \Delta_+$

Thus remaining $\Delta_+ \Rightarrow$ we are remaining $[a, a^+]$

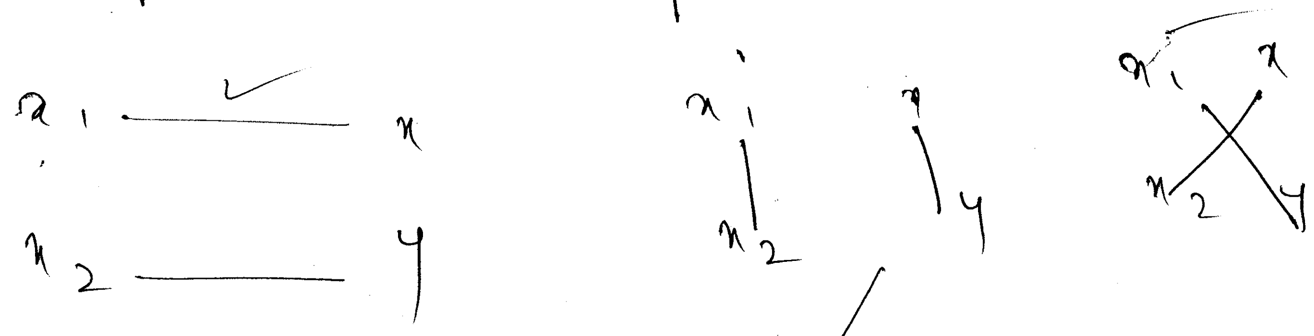
for the term which has $a a^+$

we will switch to $a^+ a$, thus for any a^+ in right we will bring it to left.

$$\langle 0|T(\phi(x_1)\phi(x_2) : \phi^2(x) :)|0\rangle$$

$$= \lim_{y\to x} \Big[\langle 0|T(\phi(x_1)\phi(x_2)\phi(x)\phi(y))|0\rangle - \langle 0|T(\phi(x_1)\phi(x_2)\Delta_F(x,y))|0\rangle$$

$\rightarrow$ products of 4 ϕ.



(3 term)

same

$\longrightarrow \Delta_F(x_1, x_2)\,\Delta_F(x, y)$.

Thus the rule is we connect

$x_1 \longrightarrow x$, $x_2 \longrightarrow y$

& x_1 to y & x_2 to x,

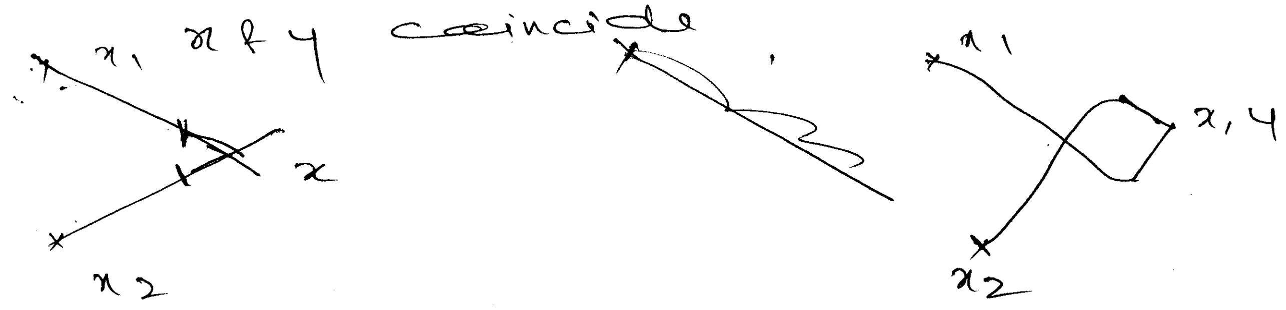
$x' \; y$

$:\phi^2(x):$

x, y → two lines come out

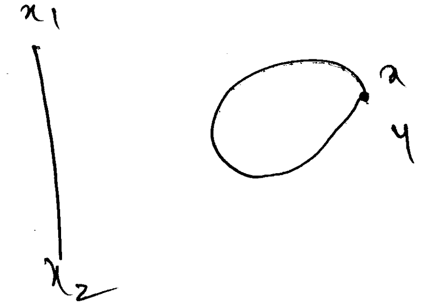
$x_2 \; y$

~~thus~~ two lines coming out of same ~~...~~ pt. should not contract with each other.



$2\,\Delta_F(x_1, x)\,\Delta_F(x_2, x)$

As ~~there~~ are two lines coming from x (So 2 is multiplied). if it is not normal order ~~the~~ is allowed.



$\phi(x)^2 \Rightarrow$ two lines coming out

Generalise $\langle 0| :\phi(x)^2: \; :\phi(y)^2: |0\rangle$.

~~$:\phi^4(x):$~~

$2\left[\Delta_F(x, y)\right]^2$.

$\tau(x_1, x_2)\,\tau(x, y)$

we can not contract two lines coming from the same pt.

$:\phi^4(x):$

$$= \lim_{y\to x}\lim_{w\to x}\lim_{z\to x} T\Big(\phi(x)\phi(y)\phi(w)$$

$$\phi(z) - \Delta_F(x,y)\,\phi(w)\,\phi(z)\Big)$$

$$-\Delta_F(y,w)\,\phi(x)\,\phi(z)$$

$$-\Delta_F\Big($$

$$:\phi^4(x): \;=\lim_{y\to x}\lim_{w\to x}\lim_{z\to x} T\big(\phi(x)\phi(y)\phi(w)\phi(z)\big)$$

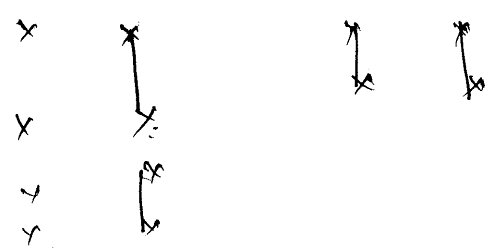
$$-\Delta_F(x,y)\{\phi(w)\phi(z)-\Delta_F(w,z)\}$$

$$-\Delta_F(x,w)\{\phi(y)\phi(z)-\Delta_F(y,z)\}$$

$$\vdots$$

$$-\Delta_F(x,y)\,\Delta_F(w,z)\cdots\}$$

any thing that could possibly contract two lines through a same pt. should be removed.

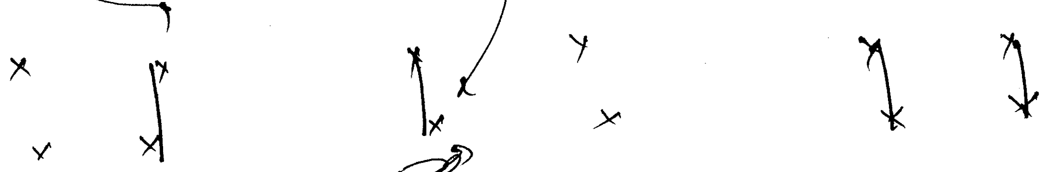


$$\lim_{y\to x}\lim_{w\to x}\lim_{z\to x} T\big(\varphi(x)\,\varphi(y)\,\varphi(w)\,\varphi(z)$$

$$-\Delta_F(x,y)\,\{\varphi(w)\,\varphi(z)-\Delta_F(w,z)\}$$

$$-\Delta_F(w,z)\,\{\varphi(x)\,\varphi(y)-\Delta_F(x,y)\}$$

$$-\Delta_F(x,y)\,\Delta_F(w,z)\cdots\big).$$



In the operator formulation in any term being the annihilation op. to the right of creation op:

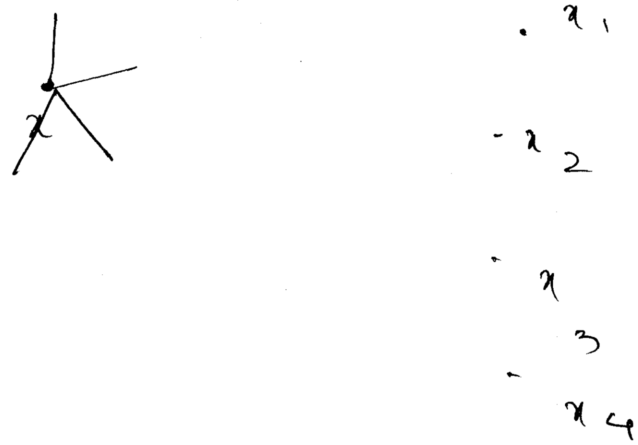
$$a\,a\,a^+\,a^+ = a^+\,a^+\,a\,a.$$

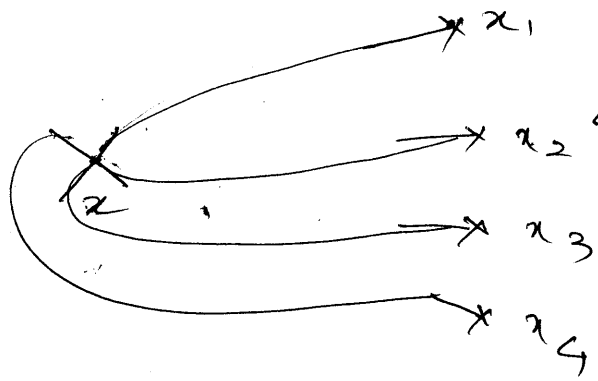
$$\langle 0|\,a\,a^+\,a^+\,a^+\,|0\rangle = 0$$

But it is not sufficient so we have to

$$\langle 0|\,a\,a^+\,a^+\,a^+\,a\,|0\rangle.$$

$\langle 0| :\phi^4(x): \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)|0\rangle$





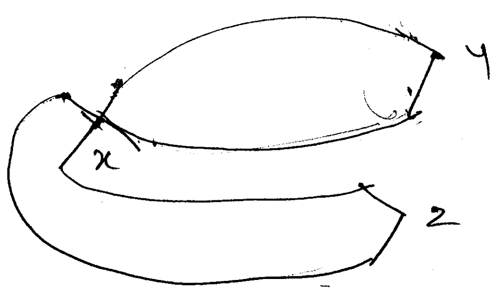
$$4!\,\Delta_F(x,x_1)\,\Delta_F(x,x_2)\,\Delta_F(x,x_3)\,\Delta_F(x,x_4)$$

x_1 - can be contracted to any of the 4.

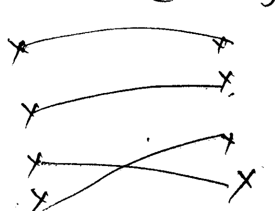
x_2 - " " " " 3

& so on.

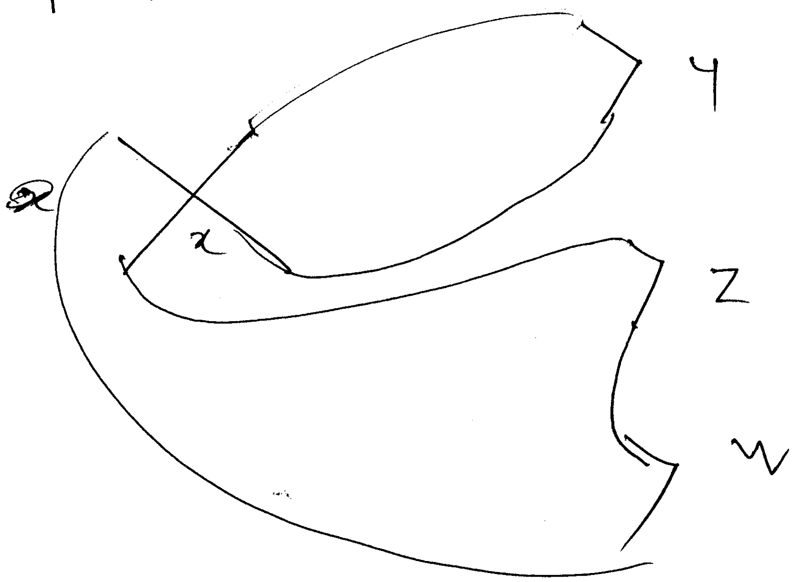
$\langle 0|:\phi^4(x)::\phi^2(y)::\phi^2(z):|0\rangle$

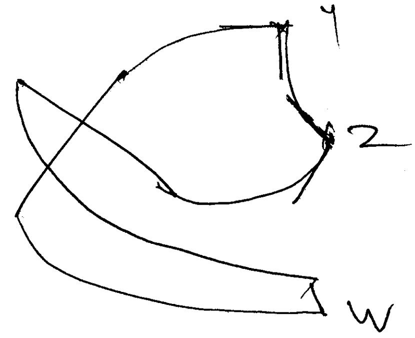


$$4!\,\Delta_F(x,y)^2\,\Delta_F(x,z)^2$$



$\langle 0 | :\phi^4(x): \; :\phi^2(y): :\phi^2(z): :\phi^2(w): | 0 \rangle$

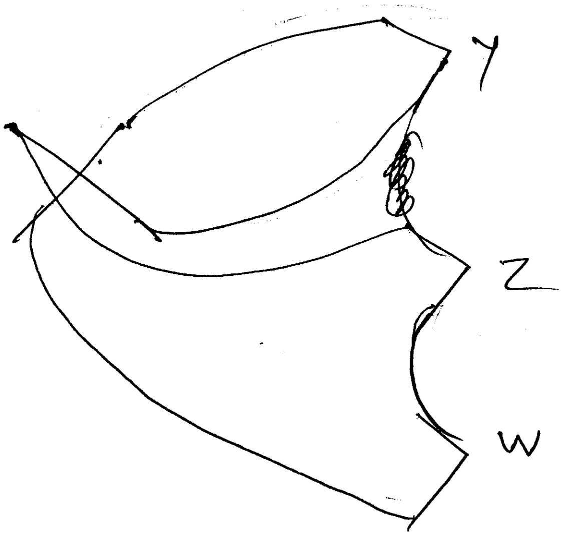




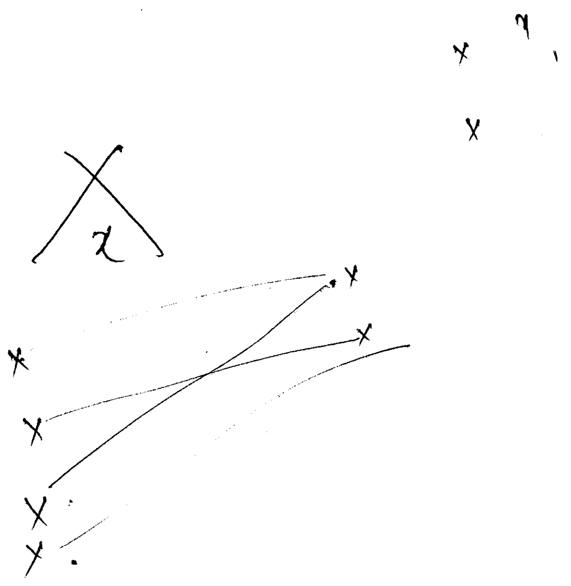
$4! \; \Delta_F(x,y)\; \Delta_F(y,z)\; \Delta_F(x,z)\; \Delta_F^2(x,w)$

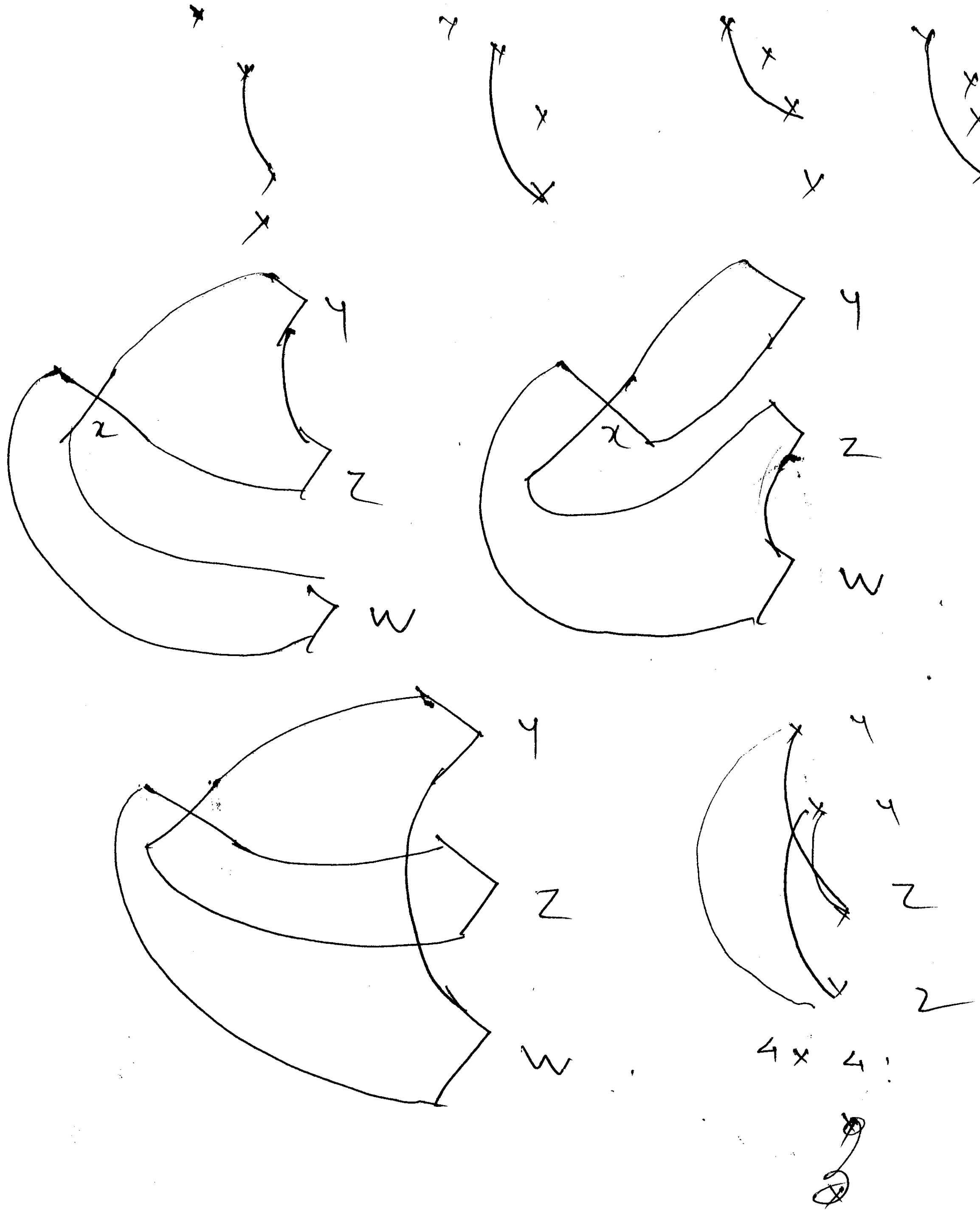
$4! \times 2 \times 2 \;\; \Delta_F(y,z)\; \Delta_F(x,z)\; \Delta_F(x,z)$

$\Delta_F^2(x,w)$



$4! \times 2 \times 2 \;\; \Delta_F$





Free field theory :-

$$H = H_{free} + \frac{\lambda}{4!} \int :\phi(x)^4: \, d^3x .$$

we will change the Hamiltonian

→ corr. to $\mathcal{L} - \frac{\lambda}{4!} \int :\phi(x)^4:$

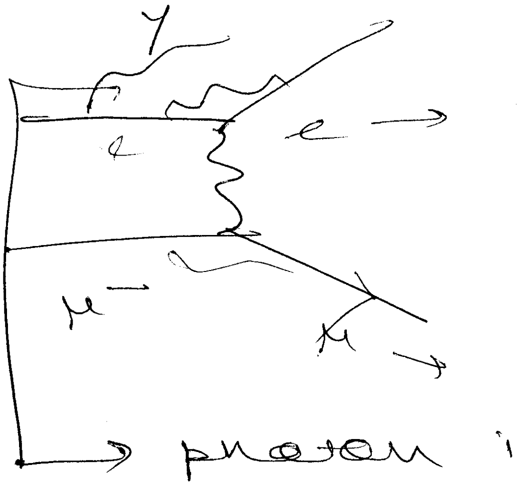
~~to~~ to put meaning into the H
we are putting normal order.

H free $\rightarrow$ $m^2 \varphi^2$ $\rightarrow$ ~~at~~ is in
normal order
as we have removed
the const. term.

$\varphi(x)^4 - : \varphi(x)^4 :$
$=$ const. term $+ : \varphi^2(x) :$

$m^2 : \varphi^2(x) :$
Subtraction of $: \varphi^2(x) :$ from ^{4}
redefinition of m^2.
without using ~~$\varphi(x$~~ normal
order then the corr. coeff.
for φ^2 is ∞.

If we don't use normal order.
then the parameters are infinite.



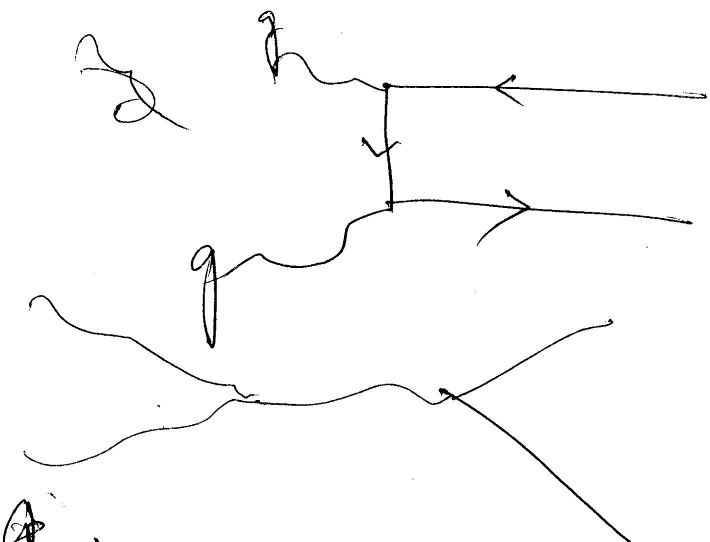
$A_{tree} \sim \alpha$

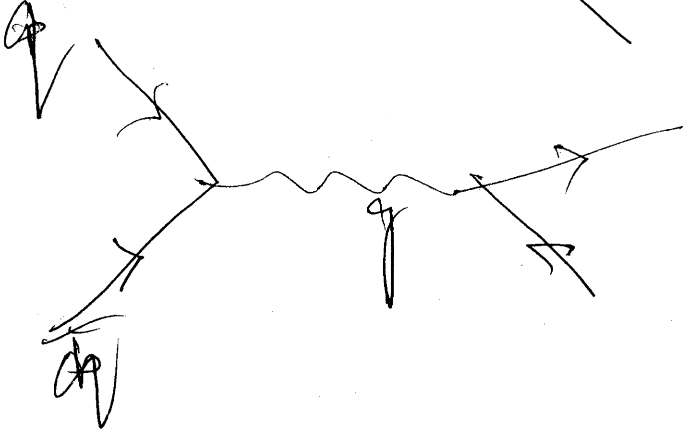
$A_{1l} \sim \alpha^2$

~~when~~ we consider higher order ~~coll~~ corrections, then divergence goes away.

→ Higher order effect.

Top quark :- $pp \to t\bar{t}$





wavefn. is functional of ϕ.
role of the space coordinate
is played by $\phi(x)$.

$T\left(\phi(x_1), \phi(x_2) \dots \phi(x_n)\right)$
as long as fields are local we
can always use Time order product.

---

we will add to Hamiltonian

$$\mathcal{H} = H_0$$
$\hookrightarrow$ free Hamiltonian.

$$\int d^3x \, \frac{1}{2}\left(\pi^2 + (\nabla\phi)^2 + m^2\phi^2\right).$$

and add to it

$$\frac{\lambda}{4!}\int d^3x \, :\phi^4: \longrightarrow H_{int}.$$

$$:\phi^2: = \phi^2 + c \longrightarrow \text{infinite}.$$

$$:\phi^4: = \phi^4 + c_1\phi^2 + c_0.$$

Thus we can keep ϕ^4, only

if $m^2 \longrightarrow$ infinite.

Thus we have to take this const.
to infinity.

parameters that appear in the

Hamiltonian - ∞.

Just normal ordering ϕ^4 ~~this~~ won't remove all the ~~∞~~ infinity.

If the theory doesn't have the property, that by adjusting the parameter we don't get finite result — then the theory is not sensible.

Let just proceed without putting the normal ordering factor.

$|\Omega\rangle$ — ground state of H.

1) First goal :- compute

$$\langle \Omega | T\{\phi(x_1) \ldots \phi(x_n)\} | \Omega \rangle.$$

2) Relate these to physically measurable quantities

We will calculate it by perturbation.

λ - small

so that the theory is convergent, when higher order corrections are small w.r.t lower order

$$H = H_0 + H_{int}.$$

Doesn't matter how we split
just to ~~make~~ in useful :-
Ho - should be quadratic
$(\pi^2 + \phi^2)$
but it is not necessary for
Ho to contain all quadratic term
then can have ϕ^2,
we can have to redefine Ho
to include ϕ^2.
~~Take part of~~ we will consider
Ho to have

ϕ^2 & π^2.

H $\longrightarrow$ time independent

H free + H int ;
both are not time independent

Define :-
$H_0 = H_{free}(t_0)$

$\phi_I(\vec{x}, t) = e^{iH_0(t-t_0)}\phi(\vec{x}, t_0)e^{-iH_0(t-t_0)}$

$\pi_I(\vec{x}, t) = e^{iH_0(t-t_0)}\pi(\vec{x}, t_0)e^{-iH_0(t-t_0)}$

Interaction picture

ϕ_I & π_I satisfy usual commutation relation

$[\phi_I(\vec{x}, t), \phi_I(\vec{x}', t)] = 0.$

$[\pi_I(\vec{x}, t), \pi_I(\vec{x}', t)] = 0.$

$[\phi_I(\vec{x}, t), \pi_I(\vec{x}', t)] = \delta^3(\vec{x}-\vec{x}').$

e.g.:-

$$\left[\phi_I(\vec{x},t)\ \pi_I(\vec{y},t)\right] = i\,\delta^3(\vec{x}-\vec{y})$$

$$\cancel{\left[e^{iH_0(t-t_0)}\phi(x,t_0)e^{-iH_0(t-t_0)}\right.},$$

$$\cancel{\pi_I}\ (y$$

$$\phi(\vec{x},t) = e^{iH(t-t_0)}\ \phi(\vec{x},t_0)\,e^{-iH(t-t_0)}$$

$$\phi(\vec{x},t) = e^{iH(t-t_0)}\ \phi(\vec{x},t_0)\,e^{-iH(t-t_0)}$$

$$\pi(\vec{x},t) = \pi(\vec{x},t_0)$$

$$\phi_I(\vec{x},t) = U(t)\ \phi(\vec{x},t)\ U(t)^{-1}$$

$$U(t) = \left[e^{iH_0(t-t_0)},\ e^{-iH(t-t_0)}\right]$$

$$U(t)\cdot U(t)^{-1}$$

$$H_0 = \frac{1}{2}\int d^3x\ \left[\pi(\vec{x},t_0)^2 + (\nabla\phi(\vec{x},t_0))^2 + m^2\phi(\vec{x},t_0)^2\right]$$

$$H_0 = e^{iH_0(t-t_0)}\,H_0\,e^{-iH_0(t-t_0)}$$

$$= \frac{1}{2}\int d^3x\ \left\{(\pi_I(\vec{x},t_0))^2 + (\nabla\phi_I(\vec{x},t))^2 + m^2\phi_I(\vec{x},t)^2\right\},$$

$$\frac{\partial\pi_I}{\partial t} = i\left[H_0,\pi_I\right].$$

$$\frac{\partial\phi_I}{\partial t} = i\left[H_0,\phi_I\right].$$

Thus Φ_I & Π_I evolve acc. to free field theory.

$$\frac{\partial \Pi_I}{\partial t} = i\,[H_0, \Pi_I]$$

$$\frac{\partial \Phi_I}{\partial t} = i\,[H_0, \Phi_I]$$

free field eqns of motion

So,

$$H_0 = H_{free}\left(\Phi_I(\vec{x},t), \Pi_I(\vec{x},t)\right).$$

suppose $|0\rangle$ is the ground state of H_0.

Then

$$\langle 0|\, T\left(\Phi_I(\vec{x},t), \Phi_I(\vec{x}_2,t_2) \ldots \Phi_n(\vec{x}_n,t_n)\right)|0\rangle$$

$\Phi_I \to$ satisfies free field eqn.

$\to$ given by free field function.

$\to \Delta_F(\vec{x}_1, \vec{x}_2)\, \Delta_F(\vec{x}_3, \vec{x}_4) \ldots$

+ other pairings

To express $\langle \Omega|\, T\left(\Phi(x_1) \ldots \Phi(x_N)\right)|0\rangle$

$$\Phi_I(\vec{x},t) = U(t)\,\phi(\vec{x},t)\,U(t)^{-1}.$$

$$U(t) = e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)}.$$

$$\langle \Omega|\, T\left(\phi(x_1) \ldots \phi(x_n)\right)|\Omega\rangle.$$

$$U(t)\,U(t')^{-1} = e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)}\, e^{iH(t'-t_0)}\, e^{-iH_0(t'-t_0)}$$

$= \langle \Omega | \phi(x_{i_1}) \phi(x_{i_2}) \ldots \phi(x_{in}) | \Omega \rangle$

$x^0_{i_1} > x^0_{i_2} > \ldots x^0_{2n}$,

$= \langle \Omega | U^{-1}(t_{i_1}) \phi_I(x_1) U(t_{i_1}) U^{-1}(t_{i_2}) \phi_I(x_{i_2}) U(t_{i_2}) \ldots U^{-1}(t_{in}) \phi_I(x_{in}) U(t_{in}) | \Omega \rangle$,

$\boxed{t = x^0}$

$U(t, t')$
$= U(t) U^{-1}(t')$

$\langle \Omega | U^{-1}(t_{i_1}) \phi_I(x_1) U(t_{i_1}, t_{i_2}) \phi_I(x_{i_2}) U(t_{i_2}, t_{i_3}) \phi_I(x_{i_3}) \ldots | \Omega \rangle$

Express U in terms of ϕ_I's & π_I's.

<u>claim</u> (to be proven).

$$U(t, t') = T \exp\left(-i \int_{t'}^{t} d\tau\, H_I(\tau)\right)$$

$$H_I(\tau) = H_{int} \Big|_{\substack{\phi \to \phi_I(\tau) \\ \pi \to \pi_I(\tau)}}$$

$$T \exp\left(-i \int_{t'}^{t} d\tau\, H_I(\tau)\right)$$

$\longrightarrow$ expand it in the power series

$$= T\left[1 - i \int_{t'}^{t} d\tau\, H_I(\tau) + \frac{(-i)^2}{2!} \int_{t'}^{t} d\tau_1 \int_{t'}^{t} d\tau_2\, H_I(\tau_1) H_I(\tau_2)\right.$$

$$= \left[1 - i \int_{t'}^{t} d\tau\, H_I(\tau) + \frac{(-i)^2}{2!} \int_{t'}^{t} d\tau_1 \int_{t'}^{t} d\tau_2\, T\left(H_I(\tau_1) H_I(\tau_2)\right) \right.$$

$$\left. + \cdots \frac{(-i)^k}{k!} \int_{t'}^{t} d\tau_1 \int_{t'} d\tau_2 \cdots \int_{t'} d\tau_k\, T\left(H_I(\tau_1) \cdots H_I(\tau_k)\right) \right].$$

large time arguement on the left

→ we can remove the 2! by implicity restrict $\tau_2 < \tau_1$,

(2)

$$U(t,t') = \left[1 - i \int_{t'}^{t} d\tau\, H_I(\tau) + \frac{(-i)^2}{\cancel{2}} \int_{t'}^{t} d\tau_1 \int_{t'}^{\tau_1} d\tau_2\, \left(H_I(\tau_1) H_I(\tau_2)\right), \right.$$

$$+ \cdots + \frac{(-i)^k}{\cancel{k!}} \int_{t'}^{t} d\tau_1 \int_{t'}^{\tau_1} d\tau_2 \int_{t'}^{\tau_2} d\tau_3 \cdots \int_{t'}^{\tau_{k-1}} d\tau_k\, H_I(\tau_1) \cdots H_I(\tau_k).$$

$$\cdots + \cdots$$

→ claim:

$$U(t,t') = U(t) \cdot U(t')^{-1}.$$

$$U(t) = e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)}$$

→ we can think U to be soln. of 1st order diff. eqn. and if we specify it to a ~~be~~ boundary cond. → then its unique.

we will show $U(t,t')$ ~~obeys~~ satisfying the same eqn. under same B.C.

$$\frac{\partial U(t,t')}{\partial t} = e^{i(H_0(t-t_0))}$$

$$\frac{\partial U(t)}{\partial t} = e^{iH_0(t-t_0)}\left(iH_0 - iH\right)e^{-iH(t-t_0)}$$

$$= e^{iH_0(t-t_0)}\, -iH_{int}(t_0)\, e^{-iH(t-t_0)}$$

$$e^{iH_0(t-t_0)}\, \phi(\vec{x},t_0)^4\, e^{-iH_0(t-t_0)}$$

$$= e^{iH_0(t-t_0)}\, \phi(\vec{x},t_0)^4\, e^{-iH_0(t-t_0)}\, e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)}$$

$$= \phi_I^4(\vec{x},t_0)\, U(t).$$

$$\frac{\partial U}{\partial t} = -iH_I(t)\,U(t).$$

t' - at a particular time (const.)

$t \longrightarrow$ variable.

$$\frac{\partial U(t) U^{-1}(t')}{\partial t} = -i H_I(t) U^{-1}(t')$$

$$\Rightarrow \frac{\partial}{\partial t} U(t, t') = -i H_I(t) U(t').$$

Boundary condi-

$U(t, t') = 1$ at $t = t'$.

(2.) $U(t, t')$ - ~~satisfies~~ the B.C.

$$\frac{\partial U(t, t')}{\partial t} = 0 - i H_I(t) + (-i)^2 \int_{t'}^{t'} d\tau_2 H_I(t) H_I(\tau_2)$$

rename $\tau_2 \to \tau_1$

$$+ (-i)^k \int_{t'}^{t} d\tau_2 \cdots \int_{t'}^{\tau_{k-1}} d\tau_k \, H_I(t) H_I(\tau_2) \cdots H_I(\tau_k)$$

$$\frac{\partial U(t, t')}{\partial t} = -i H_I(t) U(t, t')$$

Now to convert $|\Omega\rangle \longrightarrow |0\rangle$.

nonlinear rule from now linear term $\Rightarrow$ that represents interaction.

ϕ^4 - theory represents mutual interaction of particles

Lecture 17 04/02/2011

If H becomes time independent - time evolution becomes time order product.

$U(t)$ - in some sense evolution operator but it converts from ϕ_H to ϕ_I.

H_I - is not a physical quantity.

Review:

H = Hamiltonian of our interest

$= H_{free} + H_{int}$.

H_{free} ↳ quadratic part

H_{int}: $\frac{\lambda}{4!}\phi^4 + (\text{some } c\phi^2)$.

H_{free} - free Hamiltonian.

$H_0 = H_{free}(t_0)$

$$U(t) = \exp\left(iH_0(t-t_0)\right)\exp\left(-iH(t-t_0)\right)$$

$$\phi_I(\vec{x},t) = U(t)\,\phi(\vec{x},t_0)\,U(t)^{-1}$$

$$U(t,t') = U(t)\,U(t')^{-1}.$$

$$T \exp\left(-i\int_{t'}^{t} H_I(\tau)\,d\tau\right)$$

$H_I(\tau) \longrightarrow H_{Int} \mid \phi \to \phi_I$

$1-3$
$3-5$
$5-7$
$7-9$
$10-12$
$12-2$

$$\langle \Omega | T(\phi(x_1) \cdots \phi(x_n)) | \Omega \rangle$$

Ω → vacuum of H

$$\langle \Omega | U(t_i)^{-1} \phi_I(x_{i_1})\, U(t_{i_1}, t_{i_2})\, \phi_I(x_{i_2}) \cdots U(t_{i_{n-1}}, t_{i_n})\, \phi_I(x_n)\, U(t_{i_n}) | \Omega \rangle.$$

$$\langle 0 | \phi_I(x_{i_1}) \cdots \phi_I(x_{i_n}) | 0 \rangle$$

$$\longrightarrow \Delta_F(x_{i_1}, x_{i_2})\, \Delta_F(x \ldots) + \ldots$$

$\langle \Omega | \rightarrow$ vacuum of H.

→ we want to express Ω in terms of H^0.

$|0\rangle$ - ground state of H^0.

$$H_0 |0\rangle = 0,$$

$$|0\rangle = \sum_n |n\rangle\langle n|0\rangle.$$

→ complete set of basis states

$|n\rangle$ - eigenstate of H with eigenvalue E_n

$$H|n\rangle = E_n|n\rangle$$

$$e^{-iH(T(1-i\epsilon)+t_0)}|0\rangle$$

$$e^{-H(T(1-i\epsilon)+t_0)}|0\rangle \; |n\rangle$$

$$= \sum_n e^{-iH(T(1-i\epsilon)+t_0)}|n\rangle\langle n|0\rangle$$

$$= \sum_n e^{-iE_n(T(1-i\epsilon)+t_0)}|n\rangle\langle n|0\rangle,$$

if $T \longrightarrow \infty$. $= \sum_n e^{-iE_n(T - i\epsilon T + t_0)} |n\rangle\langle n|0\rangle$

Keep ϵ fix and first take $\lim T \to \infty$.

$$= \sum_n e^{-iE_\Omega(T - i\epsilon T + t_0)}|\Omega\rangle\langle \Omega|0\rangle$$

only contribution from Ω ground term ~~takes~~ takes place (E_n - lowest that is ground state of H - $|\Omega\rangle$,)

oscillatory but multiplied by the highly damped term.

$$e^{-\epsilon E_n T} , e^{-\epsilon E_m T}$$

$E_n < E_m$

$$e^{-\epsilon (E_n - E_m) T}$$

$$= \left| e^{-\epsilon \underbrace{(E_m - E_n)}_{+\text{ve (always)}} T} \right.$$

$$e^{-iH(T(1-i\epsilon)+t_0)} |0\rangle$$

$H_0 = H^{\dagger}$

$$= N_1 |\Omega\rangle \langle \Omega | 0 \rangle .$$

$$\rightarrow e^{-iH(T(1-i\epsilon)+t_0)} e^{iH_0 (T(1-i\epsilon)+t_0)} |0\rangle .$$

$$= U(-T(1-i\epsilon))^{-1} |0\rangle$$

$$|\Omega\rangle = N_1^{-1} U(-T(1-i\epsilon))^{-1} |0\rangle .$$

$$N_1 = \langle \Omega | 0 \rangle e^{-iE_\Omega (T(1-i\epsilon)+t_0)} .$$

Ex:- ~~check that~~

$$\langle \Omega | = N_2^{-1} \langle 0 | U(T(1-i\epsilon)) .$$

$$\cancel{\langle 0 | U (} \langle 0 | e^{-iH_0 (T(1+i\epsilon) + t_0)} e^{iH(T(1+i\epsilon)+t_0)}$$

$$= U$$

$$|\Omega\rangle = (N)^{-1}\, U\left(T(1-i\epsilon)\right)^{-1} |0\rangle.$$

$$\cancel{\langle\Omega| = \left(N_1^{-1}\right)^{*}}$$

$$\cancel{\langle 0|\, U\,(T}$$

$$U\left(-T(1-i\epsilon)\right)^{-1}$$

$$= e^{-iH\left(T(1-i\epsilon)+t_0\right)}\, e^{iH_0\left(T(1-i\epsilon)+t_0\right)} |0\rangle$$

$$pq|0\rangle \longrightarrow \cancel{(0 q^+ p^+}$$

$$\cancel{\langle 0| e} \qquad \langle 0| q^+ p^+$$

$$\langle 0|\, e^{-iH_0\left(T(1+i\epsilon)+t_0\right)}\, e^{iH\left(T(1+i\epsilon)+t_0\right)}$$

$$= \langle 0|\, e^{-iH_0\left(T+iT\epsilon+t_0\right)}\, e^{iH\left(T+iT\epsilon+t_0\right)}$$

change $T \to -T$, $T\epsilon \to T\epsilon$ | $\epsilon \to \epsilon$

$$= \langle 0|\, e^{-iH_0\left(-T+iT\epsilon+t_0\right)}\, e^{iH\left(-T+iT\epsilon+t_0\right)}$$

I need $T\epsilon$ = true

$$\langle\Omega|\, \phi(x_1) \cdots \phi(x_n) |\Omega\rangle$$

$$= \langle 0|\, U\left(T(1-i\epsilon), t_{i_1}\right) \phi_I(x_{i_1})\, U(t_{i_1}, t_{i_2})\, \phi_I(x_{i_2}) \cdots \phi_I(x_{i_n})\, U\left(t_{i_n}, -T(1-i\epsilon)\right)|0\rangle$$

$$U(t, t') = T \exp\left(-i \int_{t'}^{t} H_I(\tau)\, d\tau\right).$$

$$\langle \Omega | \phi(x_1) \cdots \phi(x_n) | \Omega \rangle.$$

$$= \langle 0 | T \left(\phi_I(x_1) \cdots \phi_I(x_n) \exp\left(-i \int_{-T(1-i\epsilon)}^{T(1-i\epsilon)} H_I(\tau)\, d\tau\right)\right) | 0 \rangle.$$

$N_1^{-1} N_2^{-1}$

inside the time ordering we can always consider it to be commuting.

$$T\left(e^A . e^B\right) = T\left(e^{A+B}\right) = T\left(e^{BA}\right).$$

Time order product is always unique it depends upon the fact how are we ordering the time.

$$= \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \langle 0 | T\left(\phi_I(x_1), \ldots, \phi_I(x_2)\right.$$

$$= \sum_{k=0}^{\infty} \frac{(-i)^k}{k!} \int_{-T(1-i\epsilon)}^{T(1-i\epsilon)} d\Gamma_1 \cdots \int_{-T(1-i\epsilon)}^{T(1-i\epsilon)} d\Gamma_k$$

$$\langle 0 | T (\phi_I (x_1) \, \phi_I (x_2)$$

$$\cdots \phi_I (x_n) \, H_I (\Gamma_1)$$

$$\cdots H_I (\Gamma_k) \, | 0 \rangle \, N_1^{-1} N_2^{-1}$$

$$H_I (T) = \int d^3x \, \frac{\lambda}{4!} \phi^4_I (x) \longmapsto \mathcal{L}_I .$$

$$= \sum_{k=0}^{\infty} \frac{(-i)^k}{k!} \int d^4y_1 \cdots \int d^4y_k$$

time arguement goes from $-T(1-i\epsilon)$ to $T(1-i\epsilon)$

$$\langle 0 | T (\phi_I (x_1) \cdots \phi_I (x_n) \, (-1)^k \, \mathcal{L}_I (y_1) \cdots \mathcal{L}_I (y_k) | 0 \rangle \, N_1^{-1} N_2^{-1}$$

each $\mathcal{L}_I$ contains a factor of λ

So if λ is smaller,

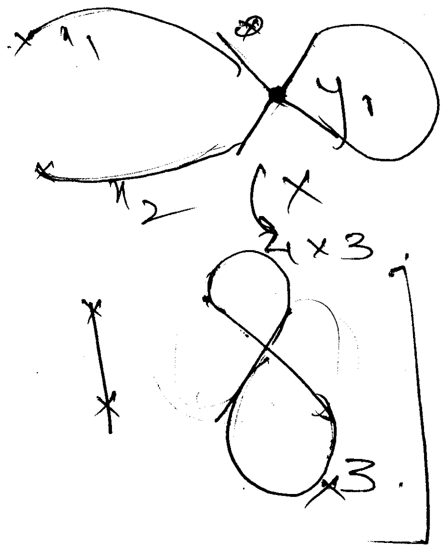
series expansion of λ.

we have a final no. of terms.

→ actually they can calculated by feynmann rule of free field theory'

$\langle \Omega | \phi(x_1) \, \phi(x_2) | \Omega \rangle$

$= N_1^{-1} N_2^{-1} \Big[\; x_1 \updownarrow x_2 \qquad (i)(-i)\frac{\lambda}{4!}$



lowest term $\to \Delta_F$

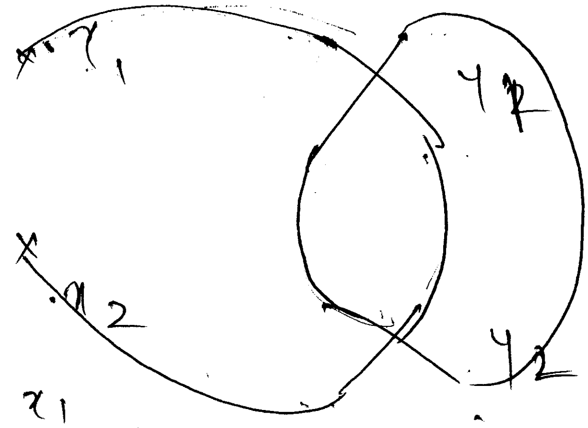
$\phi_I(x_1) \, \phi_I(x_2)$

$\Delta_F(x_1, x_2)$

second term

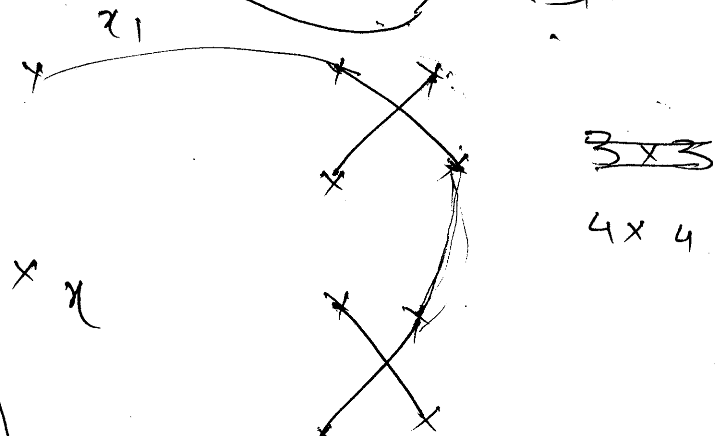
(i)

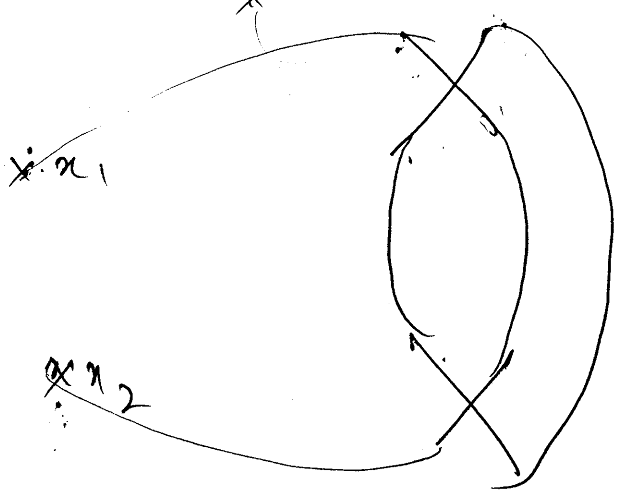
$+$



$-i\frac{\lambda}{4!}\Big[12 \int d^4y \, \Delta_F(x_1, y) \, \Delta_F(x_2, y) \, \Delta_F(y, y) \Big\}$

$+ 3 \, \Delta_F(x_1, x_2) \int d^4y \, \big(\Delta_F(y, y)\big)^2 \Big] \, N_1^{-1}$

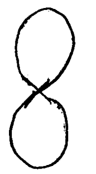




How to calculate $N_1^{-1} N_2^{-1}$

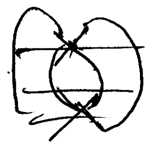
$$\langle \Omega | \Omega \rangle = 1$$

$$= N_1^{-1} N_2^{-1} \Big\{ 1 - i \frac{\lambda}{4}$$



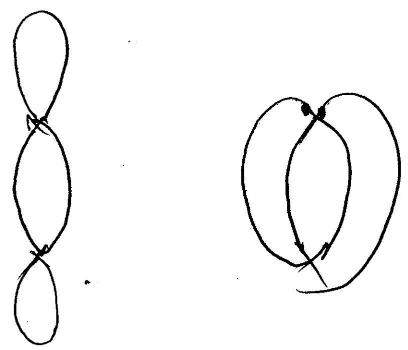
$\times 3$

$+$



$\Big\}$

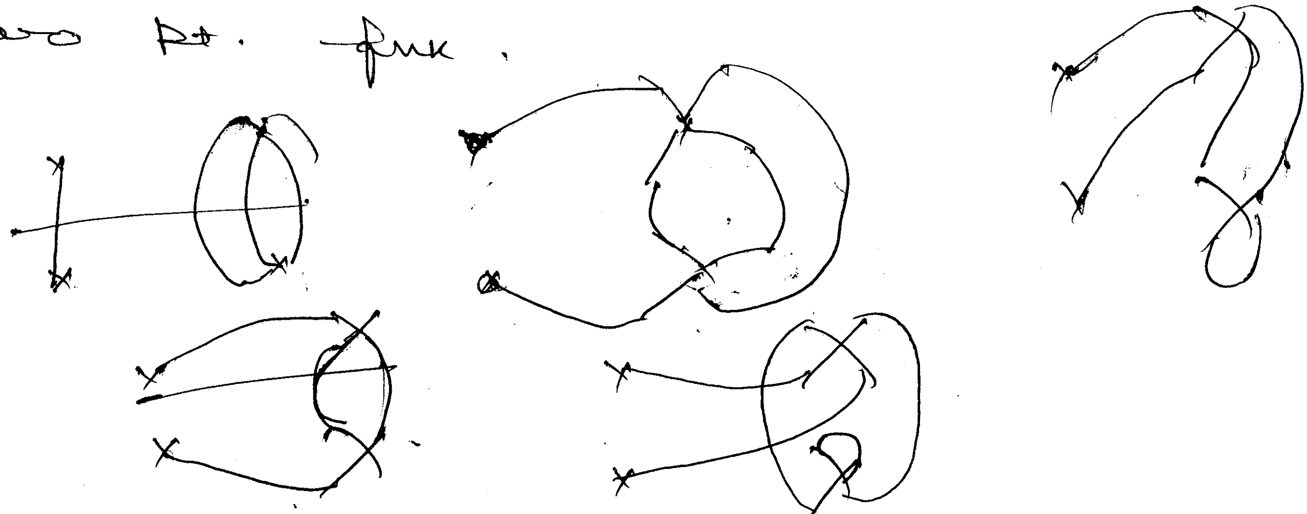
$N_1 N_2 = 1 +$ sum of bubbles (no external leg)

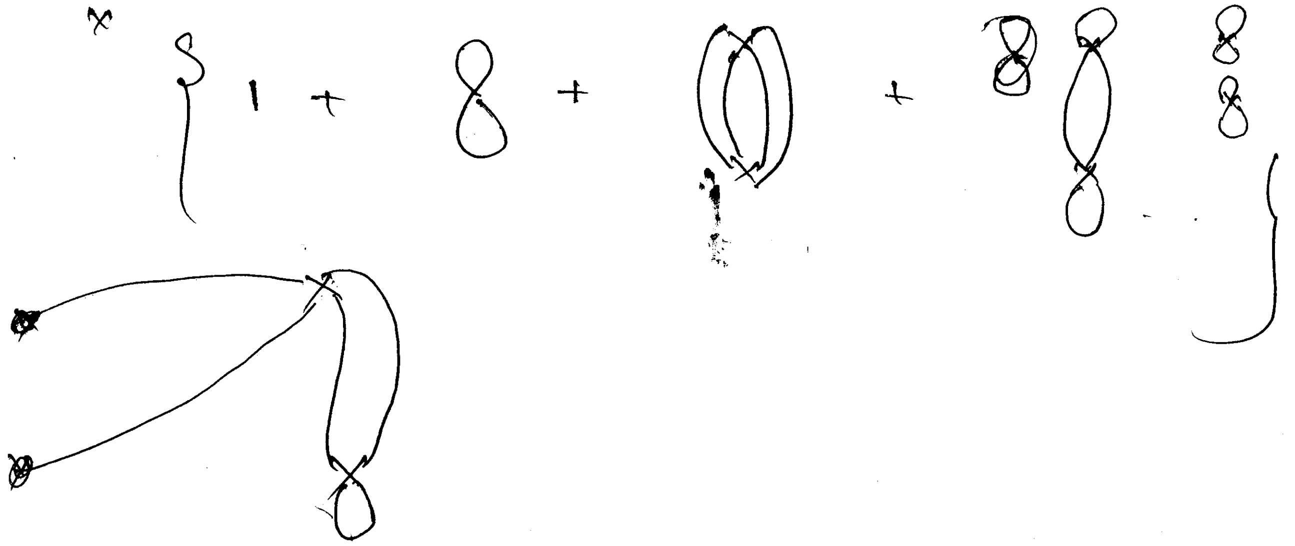


$$\langle \Omega | T(\phi(x_1) \dots \phi(x_n) | \Omega \rangle$$

$= N_1^{-1} N_2^{-1}$ [Sum over graphs without bubbles] × [1 + sum over bubbles].

Let us consider ~~$\langle \Omega | \phi(x_1) \phi(x_2) \rangle$~~ two pt. func.





Full result for $\langle \Omega | T(\phi(x_1) \cdots \phi(x_N)) | \Omega \rangle$ is given by sum of Feynmann graphs without ~~[illegible]~~:

From the rules,

$\langle \Omega | \Omega \rangle = 1.$

Lecture 18 15/03/2011

$$\langle\Omega|T\left(\phi(x_1)\cdots\phi(x_n)\right)|\Omega\rangle$$

$$= \frac{\langle 0|T\left(\phi_I(x_1)\cdots\phi_I(x_n)\exp\left(-i\int_{-T(1-i\epsilon)}^{T(1-i\epsilon)} \mathcal{H}_I(t')\,dt'\right)\right)|0\rangle}{\langle 0|T\left(\exp\left(-i\int_{-T(1-i\epsilon)}^{T(1-i\epsilon)} \mathcal{H}_I(t')\,dt'\right)\right)|0\rangle}$$

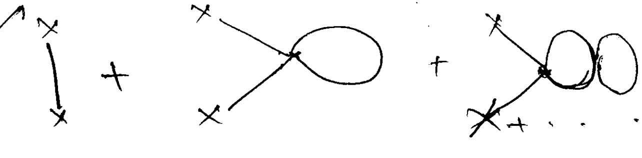
$$\mathcal{H}_I = \frac{\lambda}{4!}\int \phi_I(x)^4\, d^3x ,$$

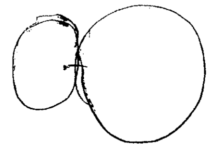
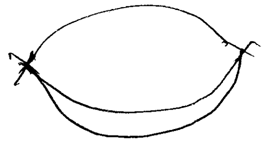
we should calculate the matrix element in the free field theory.

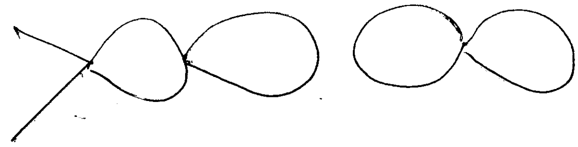
Numerator :- Sum over all Feynman diagrams

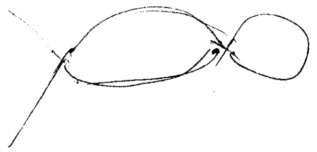
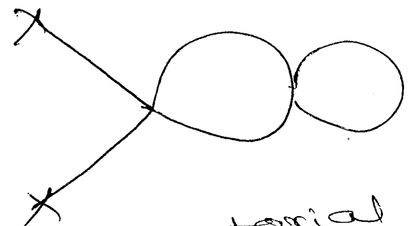
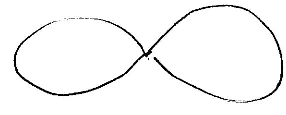
Den :- 1 + Sum over bubbles' Feynman diagram with no external legs.

Numerator :- Sum over Feynman diagrams without bubbles $\times$ (1 + Sum over bubbles).

2-point func :- ( + ...)

$\times \Big(1 +$  $+$  $+ \ldots \Big)$

Diagrams with 

 $\stackrel{?}{=}$  $\times$ 

if the combinatorial factor is same.

Then

$$\frac{\text{Num}}{\text{Den}} = \text{Sum } \cancel{\text{of}} \text{ over Feynman diagrams without bubble.}$$

suppose a diagram has $m+n$ vertices of which m are part of the bubble and the rest are part of the diagrams connected to external pts.

The order of the perturbation that gives us the diagram is λ^{m+n}.

then the factor as it is exponential $= \frac{1}{\lambda^2} \frac{1}{(m+n)!}$

In the connected part

$\frac{1}{n!}$ $\frac{1}{m!}$

if we think it as 1 diagram
then the factor $= \frac{1}{(m+n)!}$

But when we consider
the other diagram
then ~~so~~ $\frac{1}{n!} \times \frac{1}{m!}$.

$(m+n)$ vertices where m
are parts of the bubble &
n belong to the rest.

no. of ways of choosing
m vertices in the bubble
from $(m+n)$ vertices $= {}^{m+n}C_m$.

rest of the bubble comes
from either permuting legs
inside the bubble or external
leg but that is independent
of the existence of the
other diagram. Thus it
is same in both the cases.

So the part that given. cancel

$$\frac{1}{(m+n)!} \left({}^{m+n}C_m\right) = \frac{1}{n!} \times \frac{1}{m!},$$

thus we will consider diagrams without bubbles.

$i\epsilon$ prescription:

we have,

$$e^{-iH\{T(1-i\epsilon)+t_0\}}\,|n\rangle$$

$$= e^{-iE_n\{T(1-i\epsilon)+t_0\}}\,|n\rangle$$

$$= e^{-\epsilon E_n T + \text{Imaginary}}$$

~~dominant~~

if we ~~take~~ ~~the~~ limit $T \to \infty$, ~~then~~ dominant term comes from ~~the~~ least energy state.

→ gets dominant contribution ~~from~~ $|\Omega\rangle$.

In principle we can achieve the same goal if instead of using $i\epsilon$ ~~with~~ ~~H~~ we can introduce it with ~~the~~ energy

m_p = physical mass → valid in case of interacting particle $m \to$ gets change

$$m_p^2 \longrightarrow m_p^2 - i\epsilon$$

vacuum $|\Omega\rangle$ has energy $= 0$.

1-particle states have energy

$$= \sqrt{\vec{p}^2 + m_p^2 - i\epsilon} = \sqrt{\vec{p}^2 + m_p^2} - \frac{i\epsilon}{2\sqrt{\vec{p}^2+m^2}} + O(\epsilon^2)$$

Multiparticle states:

$$E = \sum_i \sqrt{p_i^2 + m_i^2 - i\epsilon} = \sum_i \left\{\sqrt{p_i^2+m_i^2} - \frac{i\epsilon}{2\sqrt{p_i^2+m_i^2}}\right.$$

if we replace $m^2 \to m^2 - i\epsilon$

then we get negative part of energy in all states.

Now let us consider

$$e^{-iH(T+t_0)}|n\rangle$$

$$= e^{-i(E_n - i\epsilon_n)(T+t_0)}|n\rangle$$

ϵ_n = small no.

= depends on what state we are considering.

$$\xrightarrow[T \to \infty]{} e^{-iE_n} e^{-iE_0} |\Omega\rangle$$

~~same~~ we can proceed by ~~taking~~ T real and taking physical mass^2 ~~square~~ complex as $m^2 - i\epsilon$

$$e^{-\epsilon_n(T+t_0)} + \text{imaginary}.$$

→ only term that survives

Maximum Term = 0.

$\epsilon_n = 0$

In path I.F → we express the ~~action~~ propagator [illegible] path by replacing $m^2 \to m^2 - i\epsilon$

effect is to make the oscillatory func. into damped func.

In order to make the path integral well defined.

In the final results whether we replace $T \to T - i\epsilon$

or $m^2 \to m^2 - i\epsilon$.

is same for finite T all possible intermediate state contributes. & expt. difficult

Lagrangian has a parameter $= m^2$

$\to$ in free field theory $= m^2$, $=$ physical mass^2.

But in $\lambda\phi^4$ interacting theory that's not physical mass.

But in lowest order, leading order to $m_p^2 = m_p^2 (1 - i\epsilon)$ is same as $m^2 \to$ ~~$m^2(1 - i\epsilon)$~~ $m^2 - i\epsilon$

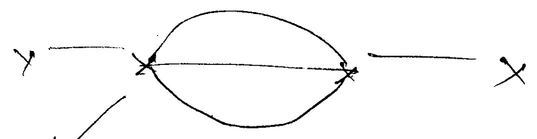
practically we will replace

$m^2 \to m^2 - i\epsilon$,

$m^2 =$ parameter in the Lagrangian.

In expt. case - the scale at which we do the expt. is infinite when we consider n pt function, ~~then~~ we have to considering divergence of diagram

Normal ordering diagram



$$\int \Delta_F^3(x,y)\, d^4x\, d^4y$$

In doing the 4 dimn. integral we will consider the pt. where y comes to close to x.

Then $\Delta_F(x,y)$ - infinite.

Relating Matrix element of time order product of operator to experiment.

In the theory, if there are particle like states what is the mass of this particle.

if $\lambda = 0$.

then m^2 = mass

And in the presence of λ

m^2 is not the mass.

We will see that

$$\langle \Omega | T(\phi(x)\,\phi(x')) |\Omega\rangle$$

contains information about the mass of the particle.

Let us start with

$$\langle \Omega | \phi(x)\,\phi(x') |\Omega\rangle$$

$$= \sum_n \langle \Omega | \phi(x)\,\phi(x$$

$$= \sum_n \langle \Omega | \phi(x) | n\rangle \langle n | \phi(x') |\Omega\rangle$$

Let us chose $|n\rangle$ - eigenstates of energy & momentum

$$H|n\rangle = E(n)|n\rangle \longrightarrow H = \int p^{00} d^3x$$

$$P_i|n\rangle = p^i(n)|n\rangle \quad , \quad P_i = \int p^{i0} d^3x$$

$$\left.\begin{aligned} H|\Omega\rangle &= 0 \\ P_i|\Omega\rangle &= 0 \end{aligned}\right\} \quad \begin{aligned} [\phi(x), H] &= +i\frac{\partial}{\partial x^0}\phi(x) \\ [\phi(x), P_i] &= -i\frac{\partial}{\partial x^i}\phi(x) \end{aligned}$$

$$\langle \Omega | [P_i, \phi(x)] | n\rangle$$

$$= i\frac{\partial}{\partial x_i}\langle \Omega | \phi(x) | n\rangle$$

$\longrightarrow$ state doesn't have explicite x dependence

$[P_i \phi(x) - \phi(x) P_i]$

$$\langle \Omega | P_i \phi(x) - \phi(x) P_i | n \rangle$$

$$= -P_n^i \langle \Omega | \phi(x) | n \rangle .$$

$$i \frac{\partial}{\partial x_i} \langle \Omega | \phi(x) | n \rangle$$

$$= -P_n^i \langle \Omega | \phi(x) | n \rangle$$

$$\langle \Omega | [H, \phi(x)] | n \rangle = -i \frac{\partial}{\partial x^0} \langle \Omega | \phi(x) | n \rangle .$$

$$-\langle \Omega | \phi(x) | n \rangle = -i \frac{\partial}{\partial x^0} \langle \Omega | \phi(x) | n \rangle .$$

$$\langle \Omega | \phi(x) | n \rangle = \langle \Omega | \phi^{(0)} | n \rangle e^{i(\vec{P}_{(n)} \cdot \vec{x} - E_{(n)} t)} \to e^{i p_n \cdot x}$$

$$\langle \Omega | \phi(x) \phi(x') | \Omega \rangle$$

$$= \sum_n \langle \Omega | \phi(x) | n \rangle \langle n | \phi(x') | \Omega \rangle$$

$$= \sum_n \langle \Omega | \phi(0) | n \rangle \langle n | \phi(0) | \Omega \rangle$$

$$e^{i p_n \cdot (x - x')} .$$

we ~~have~~ not yet assumed
any reln. ~~between~~. Energy & mom.

introduce $\rightarrow \int \frac{d^4q}{(2\pi)^3} (2\pi)^3 \sum_n \delta^{(4)}(p(n)-q)$

$$= \int \frac{d^4q}{(2\pi)^3} (2\pi)^3 \sum_n \delta^4 (p(n) - q) .$$

$$e^{iq(x-x')} \langle n|\phi(0)|\Omega\rangle\langle\Omega|\phi(0)|n\rangle$$

$$\langle\Omega|\phi(x)\phi(x')|\Omega\rangle$$

$$= \int \frac{d^4q}{(2\pi)^3} P(q) \cancel{e^{iq\cdot(x-x')}} \cancel{|\langle\Omega|\phi(0)}|n\rangle|$$

$$P(q) = (2\pi)^3 \sum_n \delta^{(4)}(p_n - q) e^{iq\cdot(x-x')}$$

$$\langle\Omega|\phi(x)\phi(x')|\Omega\rangle$$

$$= \int \frac{d^4q}{(2\pi)^3} P(q) e^{iq\cdot(x-x')}$$

$$P(q) = (2\pi)^3 \sum_n \delta^{(4)}(p(n) - q) |\langle\Omega|\phi(0)|n\rangle|^2$$

$\longrightarrow$ spectral density.

2 properties of P :-

P is Lorentz invariant,

~~reduce~~ $P(\Lambda q) = P(q)$

$\longrightarrow \Lambda$ is the Lorentz transformation.

(2) $f(q)$ vanishes for spacelike momenta - $q^2 > 0$ & for $q^0 < 0$.

vacuum energy is 0.

unless q^0 is +ve

δ ~~fn.~~ is 0

~~There~~ are no state ~~where~~

$p_n^0 < 0$

when f vanishes for $q^2 > 0$

no state

~~then~~ $\Longrightarrow$ $p_n^2 > 0$.

$\Longrightarrow$ (Energy)^2 < (momentum)^2.

$E = \sqrt{p^2 + m^2}$

$p^\mu p_\mu = p^2 - E^2$.

As long as particle interpretat. exist, it implies f should vanish with the $p^2 > 0$.

(1)

Lorentz invariance of the vacuum.

Assumption :- ~~theory~~ is such ~~that~~ vacuum is invariant under $J^{\mu\nu}$

$$J^{\mu\nu} |\Omega\rangle = 0 .$$

~~Also~~ Show

Ex :- Start with

$$\langle \Omega | [J^{\mu\nu}, \phi(0)] | n \rangle .$$

$$= - \langle \Omega | \phi(0) J^{\mu\nu} | n \rangle .$$

Lorentz transformation of $\phi(0)$

$[J^{\mu\nu}, \phi(0)] \longrightarrow$ Lorentz transformation of $\phi(0)$

Lorentz transf of $\phi(x)$

$\phi(x) \longrightarrow \phi(\Lambda x)$.

Λ - Lorentz matrix.

$\phi(0) \longrightarrow \phi(0)$. origin ram. doesn't change with time.

$$[J^{\mu\nu}, \phi(0)] = 0 .$$

Prove $f(\Lambda q) = f(q)$

— using this ,

$J^{\mu\nu} |n\rangle$ will generate new state which have new momentum Λp.

We can use it to ~~form~~ find $\phi(q) \longrightarrow \phi(\Lambda q)$

$J^{\mu\nu}$ generates infinitesimal Lorentz transformation we started with lecture 19 16/03/2011

$\langle \Omega | \phi(x) \phi(x') | \Omega \rangle$

$\longrightarrow$ contains information about the physical mass

$$= \int \frac{d^4 q}{(2\pi)^3} \rho(q) \, e^{iq\cdot(x-x')}$$

$$\rho(q) = (2\pi)^3 \sum_n \delta^{(4)}\left(p^{(n)} - q\right) |\langle \Omega | \phi(0) | n \rangle|^2$$

(i) $\rho(q) = \rho(\Lambda q) \longrightarrow$ Lorentz ~~info~~ invariant

Λ = Lorentz matrix

$\Lambda \eta \Lambda^T = \eta$.

(ii) $\rho(q) = 0$ for $q^2 > 0$ and also for $q^0 < 0$.

When we insert the complete state, each state has the prop. that it is characterized by the energy.

$\rho(q)$ = Lorentz invariant & it vanishes for $q^0 \leq 0$.

then we ~~to~~ can rewrite

$$\rho(q) = f(-q^2)\, \theta(q^0)$$

where $f(\mu) = 0$ for $\mu < 0$.

q^2 - invariant.

$$f(q) = f(-q^2)\ \theta(q^0)$$

for fixed $q^2 \longrightarrow$ $q^0 = +ve$

$q^0 = -ve$

then

$\theta(q^0)$ - ensures that

for $q^0 = +ve \longrightarrow f(q) \neq 0$.

we have to calculate

$f(-q^2)$ using perturbation theory

There is a perturbation expansion for $f \Rightarrow$ as a result there will be perturbation expansion of f

$$\langle \Omega | \phi(x)\,\phi(x')\,| \Omega \rangle$$

$$= \int \frac{d^4q}{(2\pi)^3}\, f(-q^2)\,\theta(q^0)\,e^{iq\cdot(x-x')}$$

rewrite it as integral over M

$$= \int_0^\infty f(M)\,dM \int \frac{d^4q}{(2\pi)^3}\,\delta(-M-q^2)\,\theta(q^0)\,e^{iq(x-x')}$$

$f(M) = 0$ for < 0 so the integral is from $\int_0^\infty$

Bring ~~the~~ ~~int~~ $\int du$ inside $\int d^4q$

then use the delta fnc.

Consider Free K.G theory for a scaler field of mass m

$$\Delta_+(x,y;m) = \langle 0|\mathcal{T}(\phi(x)\cdot\phi(y))|0\rangle$$

in the free theory.

$$= \int \frac{d^4q}{(2\pi)^3}\,\delta(-m^2-q^2)\,\theta(q^0)\,e^{iq\cdot(x-y)}$$

$$\cancel{-m^2-q^2} \quad \int_0^\infty f(u)\,du\,\Delta_+(x,x';\sqrt{u})$$

we are identifying $m^2 = u$

Thus

~~$\Delta_+(x,y;m)$~~

$$\langle\Omega|\phi(x)\,\phi(x')|\Omega\rangle = \int_0^\infty f(u)\,\Delta_+(x,x';\sqrt{u})\,du.$$

if we calculate the two pt. fnc. ~~does $f(u)$ = two~~ $f(u)$ as follows

two pt. fnc. in the interacting theory = coeff. $f(u)$ × two pt. in the free theory

~~effective fnc.~~

$f(\mu)$ - weight that we put
on the propagator of mass $\sqrt{\mu}$.
for free field theory
$f(\mu) = \delta(\mu^2 - m^2)$,

μ = continuous parameter
: doesn't physically mean that
there are particles carrying varying
masses.

~~Only~~ Continuous superposition
of μ comes as there are
multiparticle states contributing.

Let us assume that theory
has certain particle of mass m.
we will ask what is the
contribution of single particle
state in $f(\mu)$.

$$f(q) = (2\pi)^3 \sum_n \delta^4(p(n) - q) \, |\langle \Omega | \phi(0) | n \rangle|^2$$

$$\| $$

$$f(-q^2)\,\theta(q^0).$$

Suppose the theory has
single particle states

of mass m_p.

$F_1(q)$ = contribution from the single particle states.

$f_1(q^2)$ = corresponding contribution

$q^2 = -m_p^2$ ~~single~~ particle state of mass m_p.

Three mom. is arbitrary, it fixes the 4th comp.

$F_1(q) \neq 0$ only if $q^2 = -m_p^2$.

$$f_1(\mu) = Z\delta(\mu - m_p^2)$$

$Z \to$ const.

unless $q^2 = -m_p^2$, $F(q) = 0$

& unless ~~$f(\mu)$~~ $\mu = -q^2$
$f(\mu) = 0$.

Thus

$$f_1(\mu) = Z\delta(\mu - m_p^2),$$

if we write

$$\langle\Omega| T(\phi(x)\phi(x'))|\Omega\rangle = \int^{\infty} f(\mu)\, d\mu\, \Delta_F(x, x', \sqrt{\mu}).$$

Contribution to

$$\langle \Omega | T(\phi(x)\,\phi(x')) | \Omega \rangle$$

from single particle states of mass m_p is

$$= \int_0^\infty \rho_1(\mu)\, d\mu\, \Delta_F(x, x', \sqrt{\mu})$$

$$= \int_0^\infty Z\, \delta(\mu - m_p^2)\, \Delta_F(x, x', \sqrt{\mu})$$

$$= Z\, \Delta_F(x, x', m_p).$$

m_p = fixed no. for that theory.

$m_p = f(m, \lambda)$.

$Z = f(m, \lambda)$

Given a ~~theory~~ we will have certain parameters m & λ.

m_p should be calculated in terms of m & λ.

Thus

~~Z (mp~~

Z, m_p = func. of m & λ.

parameters of that

~~theory~~ coeff ϕ^2.

m_p = mass of the single particle.

we know $\langle \Omega | T(\phi(x)\phi(x')) | \Omega \rangle$

& from this we will calculate m_p.

we will work in terms of its Fourier transform

→ first we will identify f,
then we will identify m_p.

As N (no. operator) doesn't exist
$[H, N] \neq 0$. as particle no. is not conserved

But we will assume single particle states, there are eigenstates of H
Assumption. (for stable particle).

→ A single (stable) particle will remain single, it won't decay

we assume → follows
$E_n^2 = p_n^2 + m_p^2$ for single particle state (N=0).

(as if $[H, N] \neq 0$, then it doesn't make sense).

for particle like (Muon)
single particle state is not eigenstate of Hamiltonian.

Define :-

$$G^{(2)}(q) = \int e^{-iq(x-x')} \langle \Omega | T(\phi(x)\phi(x')) | \Omega \rangle$$

So single particle contribution to $G^{(2)}$

$$Z \int e^{-iq(x-x')} \Delta_F (x, x'; m_p)$$

$\hookrightarrow$ fourier transform of $\Delta_F (x, x'; m_p)$

$$= \frac{Z i}{-q^2 - m_p^2 + i\epsilon}.$$

Thus

$$\underbrace{G^{(2)}(q)}_{\text{single particle contribution}} = \frac{Z i}{-q^2 - m_p^2 + i\epsilon}$$

$$G^{(2)}(q) = \frac{i Z}{-q^2 - m_p^2 + i\epsilon} + \underbrace{\cdots}_{\text{multiparticle contribution}},$$

Single particle states give rise to poles in $G^{(2)}$ on real q^2 axis

Conversely, we can identify the single particle states (mass)^2, can be found by examining the poles of $G^{(2)}$.

Invariant mass^2 is not fixed.
Multiparticle states p_μ^2 is not fixed

pole comes from δ func.
δ func. only where p^2 is fixed.

~~for~~ multiparticle states

$$G^{(2)}(q) = i\int \frac{Z \quad f(M)}{q^2 - M + i\epsilon}\, dM .$$

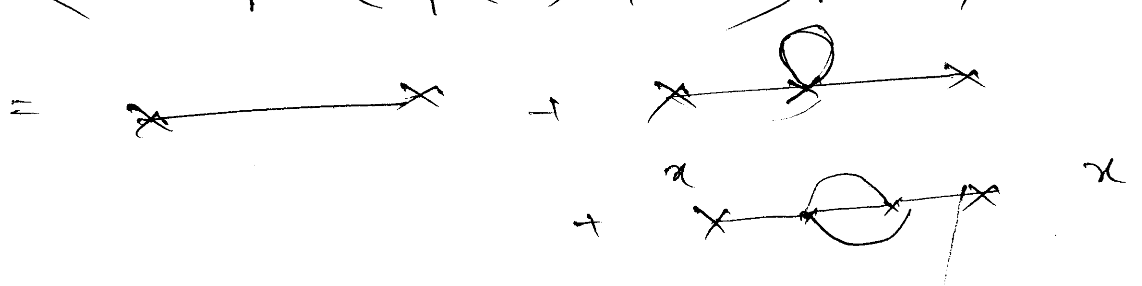
~~if M~~ we will not have branch cuts / other ~~poles~~ ~~but~~ singularity.

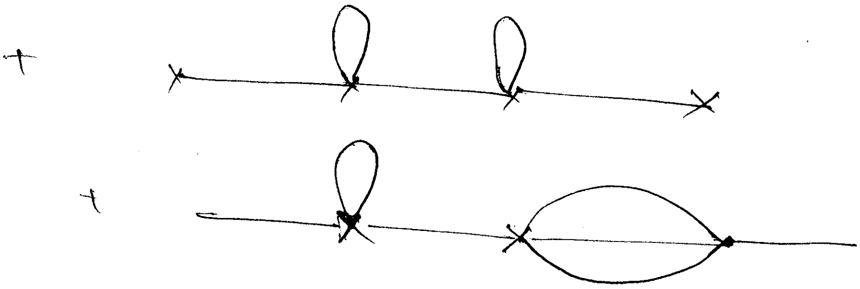
$$f(M) = \sum f_{\text{no. of particle}}(M)$$

The poles come only because of δ func. & i.e because of single particle state (m_p.)

How to extract pole from Feynman diagrams?

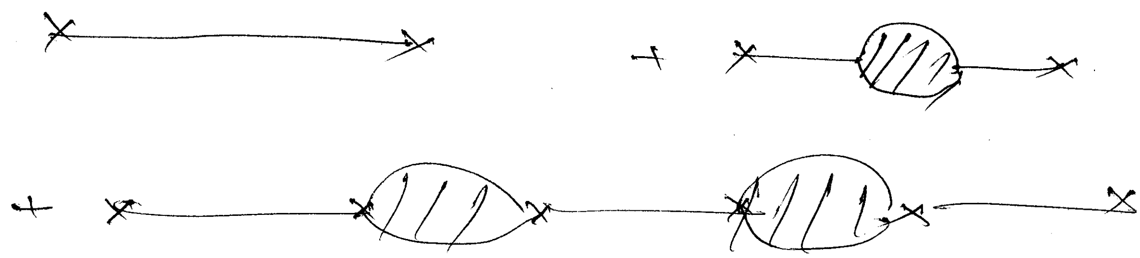
$$\langle \Omega | T(\phi(x)\phi(x')) | \Omega \rangle .$$

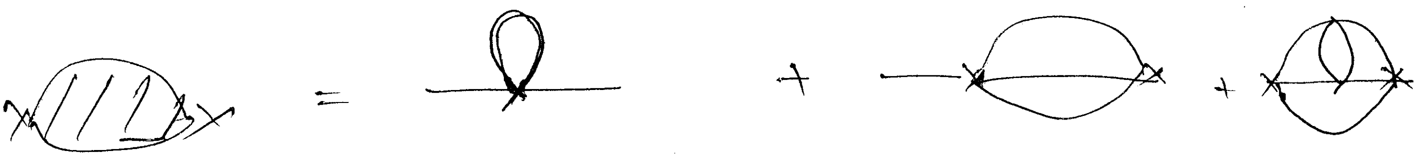




We are supposed to take fourier transform of Δ_F & look for poles. But we have to reorganize it

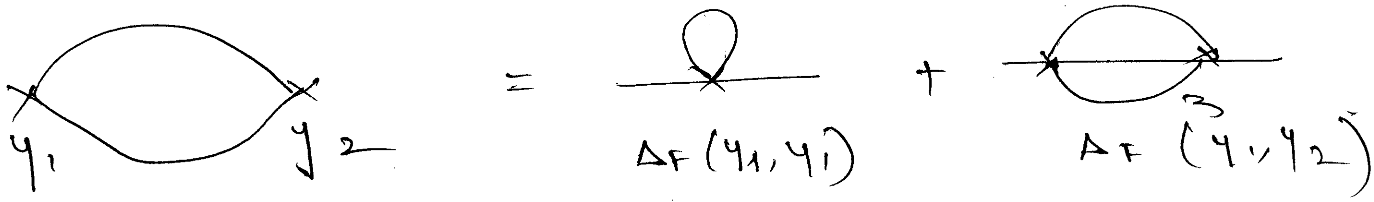
Reorganize the graphs as





One particle irreducible diagrams (Diagrams which do not get divided into 2 parts by cutting a single line) 1 (PI)

First we form all 1PI then we will form other diagrams by joining 1PI → 2 build the whole



$\to -i\Sigma(y_1, y_2)$ = Sum of all graphs.

$\delta^4(y_1 - y_2)$

$$\langle \Omega | T(\phi(x)\phi(y)) | \Omega \rangle$$

$$= \Delta_F(x, y, m) + \int \Delta_F(x, y_1)\left(-i\Sigma(y_1, y_2)\right) \Delta_F(y_2, y)\, d^4y_1\, d^4y_2$$

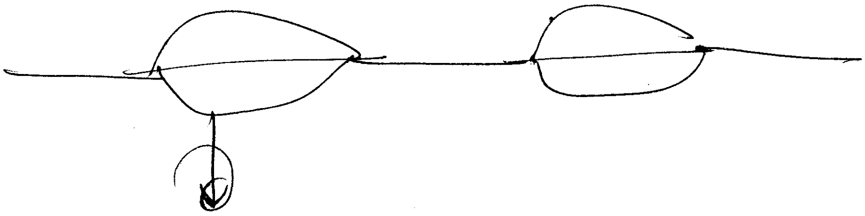
$$+ \int d^4y_1\, d^4y_2\, d^4y_3\, d^4y_4\, \Delta_F(x, y_1)\left(-i\Sigma(y_1, y_2)\right) \Delta_F(y_2, y_3) \left(-i\Sigma(y_3, y_4)\right) \Delta_F(y_4, y)$$

Check Combinatorial factor

$\to \frac{1}{N!}$ for $\Sigma \longrightarrow \frac{1}{n!}$

(when n is the no. of vertices in Σ).

In order to match the Combinatorial factor, we have to take in account how many ways we can chose the vertices in the total no. of vertices.



4 - vertices - $\frac{1}{4!}$

$4C_2 \times \frac{1}{4!} = \frac{1}{2!} \times \frac{1}{2!}$

$\frac{4}{2\times 2 \times 2 \times 2}$

Now

2 vertices - $\frac{1}{2!}$

2 vert - $\frac{1}{2!}$

$4C_2$

__Need Fourier transform:-__

$$\tilde{f}(k) = \int d^4x\, e^{-ik\cdot x} f(x)$$

$$f(x) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x} \tilde{f}(k)$$

If $h(x) = \int d^4w\, f(x-w)\, g(w)$

$\tilde{h}(k) = \tilde{f}(k)\, \tilde{g}(k).$

perturbation theory is used to do $\hat{\Sigma}(q)$
$\langle\hat{\Sigma}(q)$

$$\int e^{-ikx}\, h(x)\, d^4x.$$

$$= \int d^4w \int e^{-ikx}\, f(x-w)\, g(w)$$

$$= \int d^4w \int e^{-ik(x-w)}\, e^{-ikw}\, f(x-w)\, g(w)\, d^4x.$$

F.T of
$$\langle \Omega | T(\phi(x)\phi(y)) | \Omega\rangle.$$

$\hat{G}^{(2)}(q)$
$$= \frac{i}{-q^2-m^2+i\epsilon} + \frac{i}{-q^2-m^2+i\epsilon}\left(-i\hat{\Sigma}(q)\right)\frac{i}{-q^2-m^2+i\epsilon}$$

$+ \ldots$

$$\hat{G}^{(2)}(q) = \frac{i}{-q^2-m^2+i\epsilon}\left[1 + \frac{i\,(-i\hat{\Sigma}(q))}{-q^2-m^2+i\epsilon} + \left[\frac{\hat{\Sigma}(q)}{-q^2-m^2+i\epsilon}\right]^2\right]$$

$$= \frac{i}{-q^2-m^2+i\epsilon} \times \frac{1}{1-\frac{\hat{\Sigma}(q)}{-q^2-m^2+i\epsilon}}$$

$$= \frac{i}{-q^2-m^2+i\epsilon-\hat{\Sigma}(q)}$$

$\hat{\Sigma}(q)$ itself has ~~powers~~ of λ.

We have to identify o ~~of~~ $\hat{G}^2(q)$.

We can calculate $\tilde{\Sigma}(q)$.

Ex:- Check that Lorentz invariance implies

$\tilde{\Sigma}$ is a func. of q^2

$\tilde{\Sigma}(q) = F(-q^2)$ → can be calculated in perturbation theory.

How to determine m_p.

$$\frac{i}{-q^2 - m^2 - F(-q^2) + i\epsilon}$$

$m_p^2 + q^2_{m_p} + F(-q^2) - i\epsilon = 0$.

→ m_p can be determined from that eqn.

$m^2 - m_p^2 + F(m_p^2) = 0$

In perturbation theory m_p is determined m_p iteratively.

Zeroth order

$m_p^2 = m^2 + F(m_p^2)$.

Zeroth order

$m_p^2 = m^2$

$m_p^2 = m^2 + F(m^2)$, $\longrightarrow$ we keep upto λ

$m_p^2 = m^2 + F(m^2 + F(m^2))$ to order λ^2.

Thus we can calculate m_p, in terms of m & λ. systematically.

~~suppose~~ $F'(x) = A$ at $x = m_p^2$, $f'(x) = \frac{df}{dx}$,

Then near $-q^2 = m_p^2$

$$\cancel{\hat{G}^{(2)}(q) = \frac{i}{-q^2 - m^2 - F(-m_p^2) + i\epsilon}}$$

$$\hat{G}^{(2)}(q) = \frac{i}{-q^2 - m^2 - F(m_p^2) - (-q^2 - m_p^2)A}$$

$\longrightarrow$ Taylor expand, about $-q^2 = m_p^2$.

$$= \frac{i}{(-q^2 - m_p^2)(1 - A)}$$

$$Z = \frac{1}{1 - A}$$

~~To cal Algorithm theorem~~

for free field $A = 0$

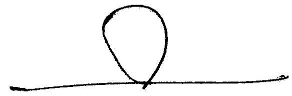
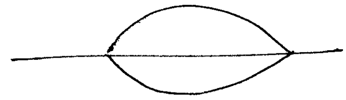
$Z = 1$.

Lecture 20

17|03|2011

$m_p = f(m, \lambda)$

$f(m, \lambda)$ is divergent.

$-i\Sigma =$  $+$  $+ \cdots$

→ already divergent.

$\Delta_F(y_1, y_1)$

we can avoid by normal ordering.

$$\int d^4y_1 \, d^4y_2 \, \Delta_F(y_1, y_2)^3 \, \Delta_F(x, y_1) \, \Delta_F(y_2, y)$$

→ it should be done over y_1 & y_2. when y_1 comes close to y_2. it blows up.

Not true in all field theories

From condensed matter system

F.T comes from lattice pt.

There is a limit of how... small

it can be. field comes from lattice pt.

There is summation over all lattice pt.

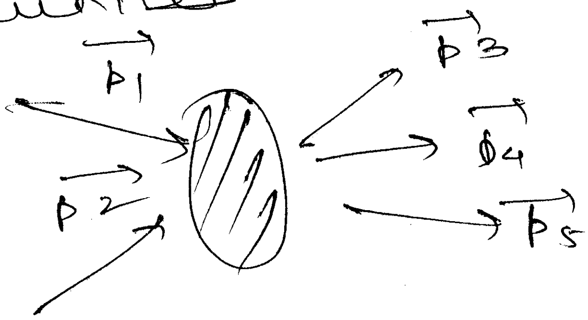
By appropriate regularization we can always throughout the short distance div. (coincident pt. are not imp.)

$\int d^3k \sqrt{\vec{k}^2 + m^2} \equiv$ there is a finite momentum cut off of

In particle physics

$m_p = f(m, \lambda)$ is divergent.

There are infinite no. of physical quantities we can calculate



gives us new measurable quantities.

All fncs. of m & λ

but ~~typically~~ divergent

we will pick two measurable quantities :- e.g. m_p

say A, B.

we can't measure m/λ

By doing experiment

→ we can't use the same treatment here

$$\int_{x=-\infty}^{y-\epsilon} \frac{1}{(x-y)^3} \sim \frac{1}{\epsilon^3}$$

$$A = f_A(m, \lambda) \quad \& \quad B = f_B(m, \lambda).$$

we measure scattering amplitude.

$f(p_1, \ldots, p_n)$,

Now pick another quantity C

$f_C(m, \lambda) \longrightarrow$ divergent.

we will invert f.

& rewrite m & λ as functions of A & B.

Now substitute it in $f_c(m, \lambda)$.

e.g.

A - can be mass of the particle

B - prob. of angular deflection of scattering.

= prob. amplitude

$= f(m, \lambda)$.

A & B are inputs. (that fix m & λ).

we will get m & $\lambda = f^{-1}(A \& B)$,

gives ∞ results of A & B as

f has infinity.

$f_c(m, \lambda) = F_C(A, B)$.

by eliminating m & λ using

$A = f_A(m, \lambda)$ & $B = f_B(m, \lambda)$.

if func.s are finite we can determine in $t\lambda$.

$$C = F_c(A, B)$$

In order that ~~the theory~~ is ~~sensible~~ C should be a finite func.' of A & B.

e.g.

$$A = 1 + \lambda K_1 + \lambda^2 K_2 + \cdots$$

$$= 1 + \lambda K_1(m) + \lambda^2 K_2(m)$$

$\longrightarrow$ inside it we infinity.

$$K_0 = A.$$

$$A - \lambda K_1 = K_0.$$

In perturbation theory even if the coeff. is infinite we always allow expansion in the power of λ.

scattering amplitude = finite,

sensible theory :- $C = F(A, B)$. true for any quantity C.

if this doesn't agree, then not a sensible theory

$$\lambda = g_\lambda(A, B)$$

$$m = g_m(A, B)$$

Let us consider only one variable

λ - parameter of Lagrangian

Γ - physically measurable quantity

infinite

$= a_0\lambda + a_1\lambda^2 + a_2\lambda^3$

$a_0, a_1, a_2 = \infty$

false

$$\lambda = \frac{\Gamma}{a_0} - \frac{a_1}{a_0}\lambda^2 - \frac{a_2}{a_0}\lambda^3$$

finite

Iteration

$$\frac{\Gamma}{a_0} = \lambda$$

$$\lambda = \frac{\Gamma}{a_0} - \frac{a_1}{a_0}\left(\frac{\Gamma}{a_0}\right)^2$$

infinite

Thus when we will invert it, we will get ~~the fun~~ coeff as ∞

$$\sigma = b_0 + b_1\lambda + b_2\lambda^2$$

infinities

$$\sigma = b_0 + b\left(\frac{\Gamma}{a_0} - \frac{a_1}{a_0}\left(\frac{\Gamma_1}{a_0}\right)\right)^2$$

$+ b_2\lambda^2$

$$\sigma = b_0 + b\left(\frac{T}{a_0} - \frac{a_1}{a_0}\left(\frac{r_1}{a_0}\right)\right)^2 + b_2\left(\frac{T}{a_0} - \frac{a_1}{a_0}\left(\frac{r}{a_0}\right)\right)^2$$

$b_0 \longrightarrow$ 1st term $\longrightarrow$ finite.

$$\sigma = b_0 + \frac{b_1}{a_0} r + r^2 \left[\frac{2b_1 a_1}{a_0^3} - \frac{b_2 a_1}{(a_0)^3}\right]$$

we have to make sure, whatever ~~theory~~ predicts give us a finite ~~result~~.

A & B - input parameter.

C, D - predict.

eventually ~~the~~ ∞ goes.

Infinities are coming from very short distance. But it happens bcoz short dist. effect goes may be modified

~~modi~~ necessarily involving, short dist cutoff & div., we can ~~since~~ ~~cutoff~~ still make predictions

$$C = F_C(A, B)$$

$\longrightarrow$ Theory having property is Renormalizable.

when divergence gets cancelled.

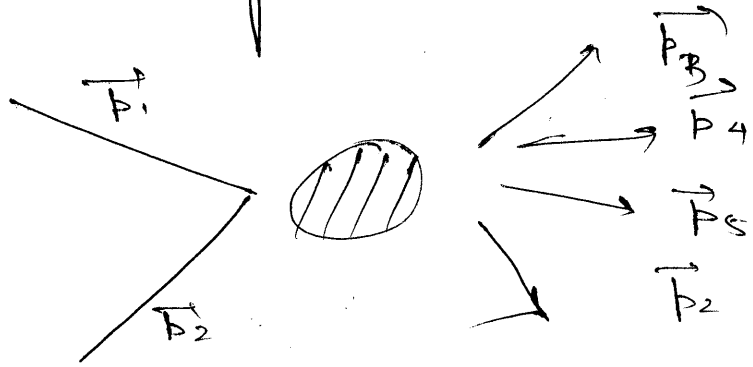
We have to check this for every theory. λ - higher order, we encounter infinities.

We are renormalizing the parameters & are writing in terms of physical quantities.

$m_p \longrightarrow$ m is getting renormalized into m_p.

Other Physical quantities :-

Scattering amplitude :-



We are throwing two scalers with $\vec{p_1}$ & $\vec{p_2}$ and the observing 4 particles.

What is the prob. that $\vec{p_1}, p_2 \longrightarrow p_3, p_4, p_5$ & p_2.

Go back to free K.G. theory.

$$\phi(\vec{x},0) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[e^{i\vec{k}\cdot\vec{x}} \left(a(\vec{k},0) + a^{+}(-\vec{k},0) \right) \right.$$

$$\phi(\vec{x},0) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{e^{i\vec{k}\cdot\vec{x}}}{\sqrt{2\omega_k}} \left[(a(\vec{k},0) + a^+(-\vec{k},0)) \right)$$

$$\phi(\vec{x},0)\,|0\rangle$$

$$= \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \; a^+(-\vec{k},0)\,|0\rangle$$

$\vec{k} \longrightarrow -\vec{k}$,

$$= \int \frac{d^3k}{(2\pi)^{3/2}} e^{-i\vec{k}\cdot\vec{x}} \; a^+(\vec{k},0)\,|0\rangle.$$

$a^+(\vec{k},0)\,|0\rangle = \cancel{a^+(\vec{k})}\,|\vec{k}\rangle$.

One particle state with mom. $\vec{k}$.

$f(\vec{x})$: some fuc. of $\vec{x}$. (localized in some regions) $\rightarrow$ picked at certain region & falls outside

$$|f\rangle \stackrel{\text{def}}{=} \int d^3x \; f(\vec{x}) \; \phi(\vec{x},0)|0\rangle$$

Ex :-

$$|f\rangle = \int \frac{d^3k}{\sqrt{2\omega_k}} \cdot \hat{f}(\vec{k}) \; |\vec{k}\rangle$$

$$\hat{f}(\vec{k}) = \int \frac{d^3\cancel{k}}{(2\pi)^{3/2}} f(\vec{x}) \; e^{-i\vec{k}\cdot\vec{x}}$$

Fourier transform of $f(\vec{x})$.

We are describing one particle state with weight factor.

$f(x')$

In NR limit $\omega_k = m$.

$f(x) \equiv$ position space wavefunc.

$f(\vec{x}') =$ probability amplitude for being at x.

if the mom. space is small,

Then $f(\vec{x}', 0) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{-i\vec{k}\cdot\vec{x}'} \frac{1}{\sqrt{2\omega_{k=0}}} |\vec{k}\rangle$

How the state evolved?

$$-i\frac{\partial}{\partial t}|\vec{k}\rangle = H|\vec{k}\rangle$$

$$= \sqrt{k^2+m^2}\,|\vec{k}\rangle.$$

$$|\vec{k}, t\rangle = e^{i\sqrt{k^2+m^2}\,t}|\vec{k}, 0\rangle$$

All the states evolve independently

$$|f, t\rangle = \int \frac{d^3k}{\sqrt{2\omega_k}} \underbrace{f(\vec{x}')\, e^{-i\sqrt{k^2+m^2}\,t}}_{\hat{f}(\vec{k}, t)} |\vec{k}\rangle.$$

Define

$$f(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int e^{i\vec{k}\cdot\vec{x}}\, \hat{f}(\vec{x}, t)\, d^3k$$

momentum eigenstate evolve simply under time evolution.
Thus we can write

$$|f,t\rangle = \int d^3x \cdot f(\vec{x},t)\, \phi(\vec{x},0)|0\rangle$$

$$= \int d^3x\, f(\vec{x},t)\, \phi(\vec{x},0)|0\rangle.$$

If we want to calculate $|f\rangle$ what happens to ~~the~~ ~~state~~ after time t, ~~then~~ it is simply $f(\vec{x},0) \longrightarrow f(\vec{x},t)$.

$|f\rangle$ - initial state

become into $|f,t\rangle$.

we are not changing the ~~states~~ Basis states $\phi(\vec{x},0)|0\rangle$ remains fixed.

weight factor changes $f(\vec{x},t)$

Consider:-

$$|f_1, \ldots f_n\rangle = \int d^3x_1\, f_1(x_1) \ldots \int d^3x_n\, f_n(\vec{x}_n)\, \phi(\vec{x}_1,0) \ldots \phi(\vec{x}_n,0)|0\rangle$$

$$= \int d^3x_1\, f_1(x_1) \ldots \int d^3x_n\, f_n(\vec{x}_n)\, \phi(\vec{x}_1,0) \ldots \phi(\vec{x}_n,0)|0\rangle$$

$\longrightarrow$ n particle states.

$$= \int \frac{d^3 k_1}{\sqrt{2\omega_{\vec{k}}}} \hat{f}(k_1) \cdots \frac{d^3 k_n}{\sqrt{\omega_{\vec{k}_n}}} \hat{f}(k_n)$$

$$|k_1, \ldots \vec{k}_n\rangle.$$

$|\vec{k}_1 \ldots \vec{k}_n\rangle$ - evolves as

$$= e^{-iEt}$$

$$E = \sqrt{k_1^2 + m_1^2} + \sqrt{k_2^2 + m_2^2} + \cdots + \sqrt{k_n^2 + m_n^2}$$

Thus

$|f_1, \ldots f_n, t\rangle$

$$= \int \frac{d^3 k_1}{\sqrt{2\omega_{\vec{k}_1}}} \hat{f}(\vec{k}_1) e^{-i\sqrt{k_1^2+m_1^2}\,t} \cdots \int \frac{d^3 k_n}{\sqrt{2\omega_{k_n}}} \hat{f}(\vec{k}_n) e^{-i\sqrt{k_n^2+m^2}\,t}$$

$$|\vec{k}_1, \ldots \vec{k}_n\rangle$$

~~then~~ we have to assume that there are localized into diff region.

$|f_1, \ldots f_n, t\rangle$

$$= \int d^3x_1\, f_1(\vec{x}_1, t)\, \vec{\Phi}(x_1, 0) \int d^3x_2\, f_2(\vec{x}_2, t)\, \phi(x_2) \cdots |0\rangle$$

where $f_i(\vec{x}, t)$ is Fourier transf. of $\hat{f}_i(\vec{k})\, e^{-i\sqrt{k^2+m^2}\,t}$.

we factored out the integral into n separation factors,

As if the wavefunc. of individual particles are evolving separately, i-th particle only knows about fi. True because particles are non interacting.

For interacting theory, picture is diff. even if we create state f1 ... fn are localized diff.

Initially they will start evolving differently. Eventually at some pt. they will come together, will interact & then will go away. There will be spreading.

We will use the notion of multiparticle state.

There should be some state in the theory, which have the prop. that they represent particles that are localized in different position which will initially independent separately & then it will interact & evolve gradually.

Lecture 21 18/03/2011

We are dealing with Free field theory

if we have state

$$\int \prod_{i=1}^{n} \frac{d^3 k_i}{\sqrt{2\omega_{\vec{k}_i}}} \hat{f}(\vec{k}_i) |\vec{k}_1, \ldots \vec{k}_n\rangle .$$

under time evolution it becomes,

$$\int \prod_{i=1}^{n} \frac{d^3 k_i}{\sqrt{2\omega_{\vec{k}_i}}} \hat{f}(\vec{k}_i)\, e^{-i\sqrt{k_i^2+m^2}\,t} |\vec{k}_1, \ldots \vec{k}_n\rangle ,$$

we defined,

$$f(x) = \int \frac{d^3 k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot x} \hat{f}_i(\vec{k})$$

$$\hat{f}_i(\vec{k}) \longrightarrow f_i(\vec{x}, t).$$

$$f_i(\vec{x}, t) = \int \frac{d^3 k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \hat{f}_i(\vec{k})\, e^{-i\sqrt{k_i^2+m^2}\,t},$$

each $f_i(x)$ evolves independent.

free particle,

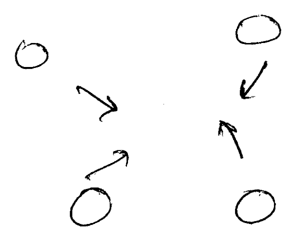
Normally we are using Heisenberg but when we talk about evolution of states → Schrödinger ~~eqn~~ picture.

Interacting theory

In the interacting we know that each molecule will interact with the other particles.

if there are one particle states nothing will happen but for 2 particle states - interaction will take place.

we are at time t = 0



Suppose we evolve for ~~back~~ in the time.

In the future ~~different~~ particles will be moving away from each other. ~~But~~ So this state we will expect that in the future it can be represented by l.c of states that are moving from each other.

If 2 particles form a ~~B~~ bound state, then we will consider the bound state as one particle.

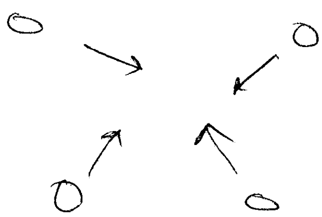
Final states $\equiv$ it should contain linear comb. of all ~~state~~,

In final state - we have to define what kind of particle carries what momentum

In Heisenberg picture -

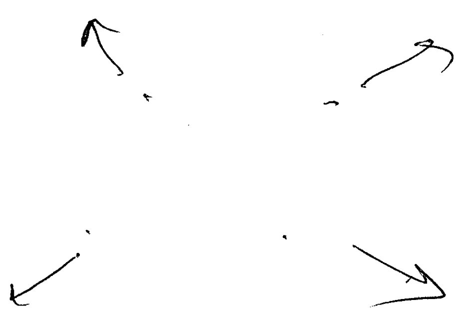
state at $t = 0$,

if we have evolved it by Schrödinger picture



$H \to$ time independent Hamiltonian

$$|\psi\rangle = \sum_i a_i |\phi_i\rangle$$



$$e^{-iHt}|\psi\rangle = \sum_i a_i e^{-iHt} |\phi_i\rangle$$

It is possible to construct basis of states $\{|k_1, \ldots k_n\rangle_{out}\}$

which in the far future evolve like $|k_1, \ldots k_n\rangle_{out}$.

what happens when we go backward in time.

As we go far back in time

superposition of states where diff. particle are separated by moving towards each other.

$|\vec{k}_1, \ldots \vec{k}_n\rangle$ in

$|\vec{k}_1, \ldots \vec{k}_n\rangle$ in $\neq$ $|\vec{k}_1, \ldots \vec{k}_n\rangle$ out ... (i)

if $|\vec{k}_1, \ldots \vec{k}_n\rangle$ in $=$ $|\vec{k}_1, \ldots \vec{k}_n\rangle_{out}$

then no interaction.

Even though eqn (i) holds we can say, the eigen value of energy :-

$H |\vec{k}_1, \ldots \vec{k}_n\rangle$ in

= Sum of the energies of individual particles $|\vec{k}_1, \ldots \vec{k}_n\rangle$.

No freedom on chasing what the energy is.

$$H |\vec{k}_1, \ldots \vec{k}_n\rangle = \sum_i \sqrt{k_i^2 + m_p^2}\, |k_1, \ldots \vec{k}_n\rangle_{in}$$

Because of there sepn. there is no interaction.

Energy eigenstate always remain ~~the~~ same.
fixing the energy doesn't fix all the quantum nos.

$$P^l_{\otimes} \left| k_1, \ldots k_n \right\rangle$$
$$= \left(\sum_i k_i^l \right) \left| \vec{k}_1, \ldots \vec{k}_n \right\rangle_{in}$$

we can use the same logic for out state.

particles are separated

Thus
$$H \left| \vec{k}_1, \ldots \vec{k}_n \right\rangle_{out} = \left(\sum_i \sqrt{k_i^2 + m_p^2} \right) \left| \vec{k}_1, \ldots \vec{k}_n \right\rangle_{out}$$

$$p^l \left| \vec{k}_1, \ldots \vec{k}_n \right\rangle_{out}$$
$$= \left(\sum_i k_i^l \right) \left| \vec{k}_1, \ldots \vec{k}_n \right\rangle_{out}.$$

~~So~~ if they are truely mom. eigenstates, they can not be localized into space.

~~As it~~ In the past we are considering mom. precisely, particle wavefunc. is spread out to good extend but ~~as~~

they are ~~not~~ separated by large distance. So there wouldn't be any overlapping.

The information about the interaction can be calculated from the matrix elements.

Precise postulate :- There exist ~~state~~

complete set of states ~~with~~

$|\vec{k}_1, \ldots \vec{k}_n\rangle$ in such

$$\hat{H}|\vec{k}_1, \ldots \vec{k}_n\rangle_{in} = \left(\sum_i \sqrt{\vec{k}_i^{\,2} + m_p^2}\right)|\vec{k}_1, \ldots \vec{k}_n\rangle_{in}$$

$$\vec{P}\,|\vec{k}_1, \ldots \vec{k}_n\rangle_{in} = \left(\sum_i \vec{k}_i\right)|\vec{k}_1, \ldots \vec{k}_n\rangle_{in}$$

such that

$$e^{-i\hat{H}t}\int \frac{d^3k_1}{\sqrt{2\omega_{\vec{k}_1}}} \cdots \frac{d^3k_n}{\sqrt{2\omega_{\vec{k}_n}}} \prod_{i=1}^{n} \hat{f}_i(\vec{k}_i)\,|\vec{k}_1 \ldots \vec{k}_n\rangle$$

$$\omega_{\vec{k}} = \sqrt{m_p^2 + |\vec{k}|^2},$$

$$= \int \frac{d^3k_1}{\sqrt{2\omega_{\vec{k}_1}}} \cdots \frac{d^3k_n}{\sqrt{2\omega_{\vec{k}_n}}} \prod_{i=1}^{n} \hat{f}_i(\vec{k}_i)\, e^{-i\omega_{\vec{k}_i}t}\,|\vec{k}_1 \ldots \vec{k}_n\rangle_{in}$$

represents n incoming particles

neither wave-func. $f_1(\vec{x}, t) = \int d^3k\, e^{-i\omega_k t}\, \hat{f}_1(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$

small spread

$$\hat{f}(\vec{k}) = \frac{1}{(2\pi)^{3/2}} \int d^3x \, e^{-ikx} \, e^{-\frac{(x-a)^2}{2\sigma^2}}$$

$$= \frac{e^{-a^2}}{(2\pi)^{3/2}} \int d^3x \, e^{-ikx} \, e^{-\frac{1}{2\sigma^2}(x^2 + a^2 -}$$

$$\simeq e^{-i\vec{k}\cdot\vec{a}} \, e^{-\frac{1}{2}\sigma^2 \vec{k}^2} \longrightarrow \text{spread about } 0,$$

if ~~we~~

$$f(\vec{x}) = e^{-\frac{(\vec{x}-\vec{a})^2}{2\sigma^2}} \, e^{+i\vec{l}\cdot\vec{x}}.$$

$$\hat{f}(\vec{k}) = e^{-\frac{\sigma^2}{2}(\vec{k}-l)^2} \text{ const.}$$

σ should be large.

so ~~spreading~~ is less in mom. ~~space~~ ~~spreading~~

but σ should be ~~small~~ ~~large~~ (but finite) in x-space.

But they won't interact.

For outgoing ~~states~~

$$\int \frac{d^3k_1}{\sqrt{2\omega_{\vec{k}_1}}} \cdots \frac{d^3k_n}{\sqrt{2\omega_{\vec{k}_n}}} \prod_{i=1}^{n} \hat{f}_i(\vec{k}_i) e^{-i\omega_{\vec{k}_i} t} |\vec{k}_1, \cdots \vec{k}_n\rangle_{in}$$

~~provided~~

~~$f_i(\vec{x}_1, t)$~~ are

~~they~~ represent n outgoing partic

provided $\vec{f}_i(\vec{x},t)$ are non overlapping for large ~~negative~~ positive t.

At the pt. of interaction, they won't represent large far away particle.

Now we will define S matrix,

$$S(p_1, \ldots, p_n | k_1, \ldots k_n)$$

$$= {}_{out}\langle \vec{p}_1 \ldots \vec{p}_n | \vec{k}_1, \ldots \vec{k}_m \rangle_{in}$$

non zero overlapp. with m ~~out~~ incoming particles & n outgoing particle.

Two questions :-
In a given field theory how do we calculate the S. matrix.
Even if we calculate S matrix how we can relate it to something that is measurable in the experiments ?

We will begin with (2).

In a free field theory :-

$$S(\vec{p}_1, \dots \vec{p}_n | \vec{k}_1, \dots \vec{k}_m)$$

$$= S_{mn} [\delta(\vec{k}_1 - \vec{p}_1) \dots \delta(\vec{k}_n - \vec{p}_n)$$

$$+ \text{all permutations of } \vec{k}_1 \text{ to } \vec{k}_n]$$

→ we can calculate it to any p.

In an interacting theory we define $T(\vec{p}_1, \dots \vec{p}_n | \vec{k}_1, \dots \vec{k}_m)$ via

$$S(\vec{p}_1, \dots \vec{p}_n | \vec{k}_1, \dots \vec{k}_n)$$

$$= S_{mn}\, \delta(\vec{p}_1 - \vec{k}_1) \dots \delta(\vec{p}_n - \vec{k}_n)$$ ↳ forward scattering,

$$+ \text{perm of } \vec{k}_1 \dots \vec{k}_n$$

$$+ i\, T(p_1 \dots \vec{p}_n | \vec{k}_1, \dots \vec{k}_m)$$

we are defining at t=0 in state (~~that~~ in past evolves from n incoming particles) & similarly out state (t=0).

Both in states and out states are eigenstates of energy mom.

⇒ In state should be linear comb. of out state, T should have a specific structure

$$T(\vec{p}_1, \ldots \vec{p}_n \,|\, \vec{k}_1, \ldots \vec{k}_m)$$

$$= (2\pi)^4 \, \delta^{(3)}\left(\sum_{i=1}^{m} \vec{k}_i - \sum_{i=1}^{n} \vec{p}_i \right) \delta\left(\sum_{i=1}^{m} \omega_{\vec{k}_i} - \sum_{i=1}^{n} \omega_{\vec{p}_i} \right).$$

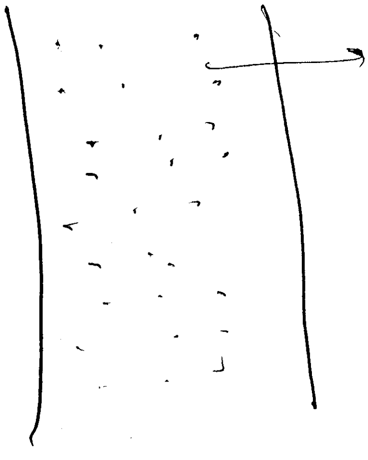
$$\prod_{i=1}^{n} \left\{ \frac{1}{\sqrt{2\omega_{\vec{p}_i}}\,(2\pi)^{3/2}} \right\} \prod_{i=1}^{m} \left\{ \frac{1}{\sqrt{2\omega_{\vec{k}_i}}\,(2\pi)^{3/2}} \right\}$$

$$M(\vec{p}_1, \ldots \vec{p}_n \,|\, \vec{k}_1, \ldots \vec{k}_m)$$

various other prop. are also encoded in S

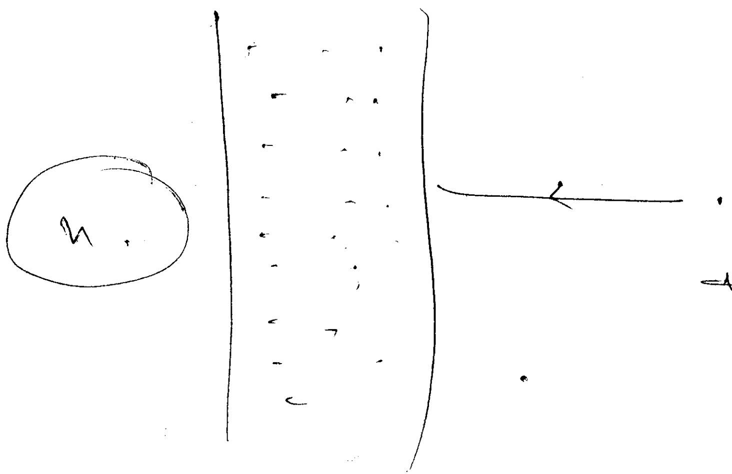
In cond mat. we relate conductivity to S matrix.

Let us imagine that we have some kind of ~~for~~ target that contains single particle (like state)



lots of e^- sitting inside

electronic wavefunc. are non overlapping.



take another particle of this type & throw it to the target.

Final state produced. with what prop particles are produced repto mom. p_1, p_2, p_n.

Probability of scattering into n particles of momentum in range $d^3p_1 \ldots d^3p_n$
(mom. in certain ranges)

$$d^3p_1 \ldots d^3p_n \left| {}_{out}\langle \bar{p}_1, \ldots \bar{p}_n | f_1, f_2 \rangle_{in} \right|^2$$

f_1 is the wavefunc. of the incoming particle.

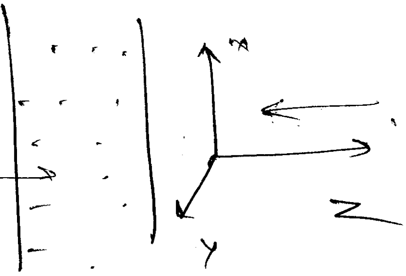
f_2 - wavefunc. of the target partcl.

f_2 can be any of this (1 to n)

thus we have sum over all contribution.

f_1 in position ~~space~~ is localized around $(x,y)=(0,0)$

There is some real func centred around $\overset{(x,y)}{(0,0)}$ origin



f in momentum ~~space~~ it is localized around

$k_1^x=0,\ k_1^y=0.\ k_1^z=k_{1z}$

we can not localize ~~Data~~ the mom. & coordinate simultaneously

incoming particle is coming along z-axis.

$\rightarrow$ $\vec{k}$ it is localized around $(0,0,k_{1z})$

$\vec{f}_2$:- In momentum space for static target it should be localized around 0.

for moving f_2 should be localized around $(0,0,k_{2z})$ in momentum space & $\vec{b}$ in position space

$\vec{b}$ is arbitrary. This reflects the fact ~~that~~ ~~there~~ are many of ~~this~~ particles we have to integrate over all $\vec{b}$.

$$\int d^3b \; \rho \; d^3p_1 \ldots d^3p_n \left| {}_{out}\langle \vec{p}_1, \ldots \vec{p}_n | f_1 \, f_2^{\vec{b}} \rangle_{in} \right|^2$$

$$\frac{1}{n!} \int d^3b \; \rho \; d^3p_1 \ldots d^3p_n \left| \langle \vec{p}_1 \ldots \vec{p}_n | f_1, f_2^{\vec{b}} \rangle_{in} \right|^2.$$

ρ = target particles per unit vol.

for finding total prob. we should integrate over all p.

exchange of $\vec{p}_1, \vec{p}_2$ is $\Rightarrow$ same state. (as all the factors are identical)

for electron-positron scattering there won't be $n!$.

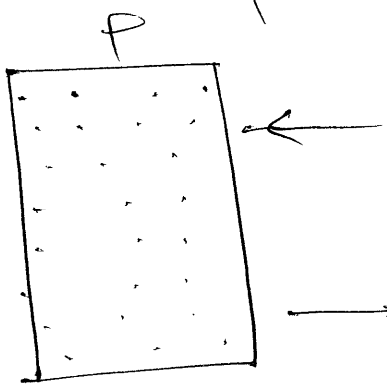
one of final states.

If the T matrix is known or M is known, we can calculate, probability, final form should be independent of f_1 & f_2.

Lecture - 22   22/03/2011

Whenever there is a non linear term in the Lagrangian → we call it interaction term.

In state & out state denote particle with definite momentum.



→ target can move.

Outstate - producing certain set of outgoing particles within certain range

Collision takes place one particle at a time.

Collision is always between two particle.

In state ≡ 2 particle state
One target particle!

target particles are incoherent.

In state

$$|f_1, f_2\rangle_{in} = \int \tilde{f}_1(\vec{k}_1)\,\tilde{f}_2(\vec{k}_2)\,\frac{d^3k_1\, d^3k_2}{\sqrt{2\omega_{\vec{k}_1}}\sqrt{2\omega_{\vec{k}_2}}}\,|\vec{k}_1, \vec{k}_2\rangle_{in}.$$

We will chose

$\tilde{f}_1$ & $\tilde{f}_2$ so that it localized in momentum space & position space we will assume that there is no overlap between $\hat{f}_1$ & $\hat{f}_2$. We will ~~suppo~~ consider classical ~~mon~~ particle.

Overlap :- $\langle \hat{f}_2(-\vec{k}) | \hat{f}_1(\vec{k}') \rangle$

no overlap means the inner prod. vanish.

Normalization :-

Initial state should be normalized

$$\int d^3k \frac{|\hat{f}_i(\vec{k})|^2}{2\omega_{\vec{k}}} = 1 \quad \text{for } i = 1, 2$$

$$\langle k_1' \, k_2' | k_1 k_2 \rangle = \delta^{(3)}(k_1' - k_1)\, \delta^{(3)}(k_2' - k_2) + \delta^{(3)}(k_1' - k_2)\, \delta(k_2' - k_1)$$

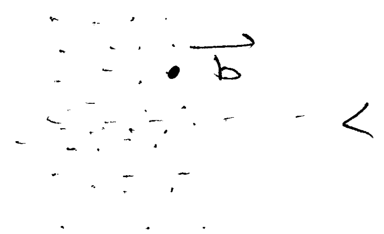
Probability of producing a final n-particle state in certain momentum range R.

$$\int_R d^3p_1 \ldots d^3p_n \left| {}_{out}\langle \vec{p}_1, \ldots \vec{p}_n | f_1 f_2 \rangle_{in} \right|^2$$

$\longrightarrow$ result for scattering of two particles

If we consider only the scattering, then $n = 2$.

Now let us consider. We have a target.



Target particle at the pt. $\vec{b}$ position space localized around $\vec{b}$.

~~Shape of all the w.func. is same.~~

$$f_2^{\vec{b}}(\vec{x}) = f_2(\vec{x}-\vec{b}) \longrightarrow \text{same func. for all target particles}$$

We have sum / integrate over all possible b, to get total ~~propo~~ probability of scattering from all the target particles, ρ = const

$$= \rho \int d^3b \int_R d^3p_1 \ldots d^3p_n \left| {}_{out}\langle p_1 \ldots p_n | f_1 f_2^b \rangle_{in} \right|^2$$

$$= \rho \int d^3b \int_R d^3p_1 \ldots d^3p_n \left| {}_{out}\langle p_1 \ldots p_n | f_1 f_2^b \rangle_{in} \right|^2 .$$

for large b, $|f_1 f_2^b\rangle$ should be small

$$f_2^{\vec{b}}(\vec{x}) = f_2(\vec{x}-\vec{b})$$

$$\hat{f}_2^{\vec{b}}(\vec{k}) = \int \frac{d^3x}{(2\pi)^{3/2}} e^{-i\vec{k}\cdot\vec{x}} f_2^{\vec{b}}(x)$$

$$= \int \frac{d^3x}{(2\pi)^{3/2}} \, e^{-ik\cdot x} \, f_2(\vec{x} - b)$$

$$\vec{x} = b + y$$

$$= \int \frac{d^3y}{(2\pi)^{3/2}} \, e^{-ik\cdot b} \, e^{-ik\cdot y} \, f_2(\vec{y}).$$

$$= e^{-ik\cdot b} \, \hat{f}_2(k).$$

Thus $\hat{f}_2^{\,b}(\vec{k}) = e^{-i\vec{k}\cdot\vec{b}} \, f_2(\vec{k})$

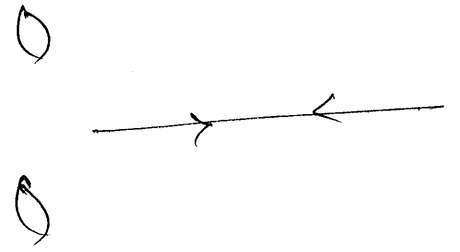
in position space if we shift the argument then in the mom. space it corr. to multiplication by a phase $e^{-i\vec{k}\cdot\vec{b}}$.

S matrix $\longrightarrow |\langle \vec{p}_1 \ldots \vec{p}_n | f_1 f_2^{\,b} \rangle_{in}|^2$.

gives us δ function (no scattering) + T,

we are interested in the part where outgoing mom. is diff. from mom.

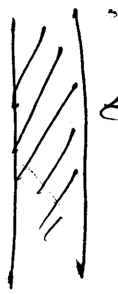
δ func. is common to all ~~theories~~



first term only contribute when the mom. is along the line.

$P \int_R \prod_{i=1}^{n} d^3p_i \int d^2 b_\perp \, db_3$

transverse $b = b_\perp$



$b_\perp = (b_1, b_2)$

$$\int \frac{d^3k_1}{\sqrt{2\omega_k}} \frac{d^3k_2}{\sqrt{2\omega_{\vec{k}_2}}} \frac{d^3k_1'}{\sqrt{2\omega_{\vec{k}_1'}}} \frac{d^3k_2'}{\sqrt{2\omega_{\vec{k}_2'}}}$$
$$\hat{f}_1(k_1)\, \hat{f}_2^{\,b}(k_2)$$

$$P \int_R \prod_{i=1}^{n} d^3p_i \int d^2b_\perp \, db_3$$
$$\int \frac{d^3k_1}{\sqrt{2\omega_{\vec{k}_1}}} \frac{d^3k_2}{\sqrt{2\omega_{\vec{k}_2}}} \frac{d^3k_1'}{\sqrt{2\omega_{\vec{k}_1'}}} \frac{d^3k_2'}{\sqrt{2\omega_{\vec{k}_2'}}} f_1(k_1') f_2^{\,b}(k_2)$$
$$\hat{f}_1(\vec{k}_1)^* \hat{f}_2^{\,b *}(k_2') \; T(\vec{p}_1, \vec{p}_2 \, \vec{p}_n \cdot \vec{k}_1 \, \vec{k}_2)$$
$$T(\vec{p}_1 \, \vec{p}_2 \ldots \vec{p}_n, \vec{k}_1', \vec{k}_2')$$

$$= \int e^{-i\vec{k}_2 \cdot \vec{b}} \, e^{i\vec{k}_2' \cdot \vec{b}} \, d^2 b_\perp \, db_3$$

Target in the transverse dime. ~~in the~~ is in infinite extent.
As in the periphery, far away particles don't contribute.

Assume that $\vec{b}_\perp$ integral extends over the full x-y plane.

$$= \int db_3 \, e^{-i(k_{2z} - k_{2z}')b_3} (2\pi)^2 \delta^{(2)}(\vec{k}_{2\perp} - \vec{k}_{2\perp}')$$

$$= \int db_3 \, e^{-i(k_{2z} - k_{2z}')b_3} (2\pi)^2 \delta^{(2)}(\vec{k}_{2\perp} - \vec{k}_{2\perp}')$$

There is a delta func. in T also

$$T(\vec{p}_1, \vec{p}_2, \ldots \vec{p}_n; \vec{k}_1, \vec{k}_2) = \delta^{(4)}\left(\sum_{i=1}^{2} p_i - \sum_{i=1}^{2} k_i\right)$$

$$\mathcal{M}(\vec{p}_1, \ldots, \vec{p}_n, \vec{k}_1, \vec{k}_2) \prod_{i=1}^{n} \frac{1}{\sqrt{2\omega_{\vec{p}_i}}} \frac{1}{\sqrt{2\omega_{\vec{k}_1}}} \frac{1}{\sqrt{2\omega_{\vec{k}_2}}}$$

Let us first collect the delta func:

$$\delta^{(2)}(\vec{k}_{2\perp} - k_{2\perp}') \, \delta^{(4)}(\Sigma p_i - k_1 - k_2)$$
$$\delta^{(4)}(\Sigma p_i - k_1' - k_2')$$

~~if we have $\delta(x)$~~ if we have $\delta(x)$, then to any other term we have add x.

As $\delta^{(4)}\left(\Sigma p_i - k_1 - k_2\right)$

Contribute ~~at $\delta^{(4)}$~~ only at $\Sigma p_i = k_1 + k_2$

then

$$\delta^{(4)}\left(k_1 + k_2 - k_1' - k_2'\right)$$

$$e^{-ib_3\left(k_{2z} - k_{2}'z\right)} \quad (2\pi)^2 \to \text{coeff. of } T.$$

---

thus we have

$$\delta^{(2)}\left(\vec{k}_{2\perp} - k_{2\perp}'\right)$$

$$\delta^{(4)}\left(k_1 + k_2 - k_1' - k_2'\right) \; \delta^{(4)}\left(\Sigma p_i - k_1 - k_2\right)$$

$$\delta^{(2)}\left(\vec{k}_{1\perp} + \cancel{\vec{k}_{2\perp}} - k_{1\perp}' - \cancel{k_{2\perp}'}\right) \; \delta\left(k_{1z} + k_{2z} - k_{1z}' - k_{2z}'\right)$$

$$\delta\left(\omega_{\vec{k}_1} + \omega_{\vec{k}_2} - \omega_{\vec{k}_1'} - \omega_{\vec{k}_2'}\right)$$

$k_{2\perp}'$. from this we can say

$$\vec{k}_{1\perp} = k_{1\perp}' \quad \& \quad \vec{k}_{2\perp} = k_{2\perp}'$$

$$\delta\left(\sqrt{k_{1z}^2 + k_{1\perp}^2 + m_p^2} + \sqrt{k_{2z}^2 + \vec{k}_{2\perp}^2 + m_p^2} - \sqrt{(k_{1z}')^2 + k_{1\perp}^2 + m_p^2} - \sqrt{(k_{2z}')^2 + k_{2\perp}^2 + m_p^2}\right)$$

$$\to k_{1z} = k_{1z}'$$

$$k_{2z} = k_{2z}'$$

both the eqn. gets satisfied

$K_1'z = K_1z \quad , \quad K_2'z = K_2z$

$\delta(K_1'z - K_1z)\, \delta(K_2z' - K_2z)$

~~proper~~

$\cancel{\delta\left(\sqrt{K_1z^2 + K_1\perp^2 + mp^2}\right.}$

$\cancel{K_1z = K_2z' + K_1z' - K_2}$

$K_2z' = K_1z + K_2z - K_1z'$.

$$\delta\left(\sqrt{K_1z^2 + K_1\perp^2 + mp^2} + \sqrt{K_2z^2 + K_2\perp^2 + mp^2} - \sqrt{K_1z'^2 + K_1\perp^2 + mp^2} - \sqrt{(K_1z + K_2z - K_1z')^2 + K_2\perp^2 + mp^2}\right)$$

$$= \delta\left(\sqrt{K_1z^2 + K_1\perp^2 + mp^2} + \sqrt{K_2z^2 + K_2\perp^2 + mp^2} - \sqrt{K_1z'^2 + K_1\perp^2 + mp^2} - \sqrt{(K_1z + K_2z - K_1z')^2 + K_2\perp^2 + mp^2}\right).$$

$\delta(K_1z + K_2z - K_1'z - K_2z')$.

$$\delta\left(f(K_1z')\right) = \delta(K_1z' - K_1z)\,\frac{1}{|f'(K_1z)|}$$

$$\left|\frac{-K_1z'}{\omega_{\vec{K}_1'}} + \frac{K_1z + K_2z - K_1z'}{\omega_{\vec{K}_1 + \vec{K}_2 - \vec{K}_1'}}\right|^{-1}$$

$K_1'z = K_1z$

$K_2z' = K_2z$

$$\left|\cancel{\frac{k_{12}}{\omega_{k_1}}}\right| = \left|\frac{k_{12}}{\omega_{k_1}} - \frac{k_{22}}{\omega_{k_2}}\right|^{-1}$$

$\frac{k_{12}}{\omega_{k_1}}$ = vel. of the first particle

~~b_3~~

$$e^{-ib_3(k_{22}-k_2'{}_z)}$$

$(2\pi)^2$

$\| \; 1.$

Thus

$\int d b_3$ gives me ~~the thickness~~

~~of the target~~

~~Probability = PL $\int_R \prod_{i=1}^{n} d^3 p_i$~~ $\cancel{v_{12} - v_{22}}$ $v_{12} - v_{2z}$

Probability

$$= PL \int_R \prod_{i=1}^{n} d^3 p_i \int \frac{d^3 k_1}{2\omega_{\vec{k}_1}} \frac{d^3 k_2}{2\omega_{\vec{k}_2}} \hat{f}_1(\vec{k}_1)\, \hat{f}_2(\vec{k}_2)\, \hat{f}_1(\vec{k}_1)^* \hat{f}_2(\vec{k}_2)^*$$

$$\cancel{T(\vec{p}_1, \vec{p}_2, \ldots} \; M(\vec{p}_1, \vec{p}_2, \ldots \vec{p}_n; \vec{k}_1, \vec{k}_2)^*.$$

$$\delta^{(4)}\left(\sum p_i - k_1 - k_2\right)(2\pi)^{10} \left|\frac{-k_{12}}{\omega_{\vec{k}_1}} + \frac{k_{22}}{\omega_{\vec{k}_2}}\right|^{-1}$$

$$\times \frac{1}{(2\pi)^{3(n+2)}} \prod_{i=1}^{n} \frac{1}{2\omega_{\vec{p}_i}} \frac{1}{2\omega_{k_1}} \frac{1}{2\omega_{k_2}}$$

Probability :-

$$= PL \int_R \prod_{i=1}^{n} d^3p_i \int \frac{d^3k_1}{2\omega_{\vec{k}_1}} \frac{d^3k_2}{2\omega_{\vec{k}_2}} \hat{f}_1(\vec{k}_1)\, \hat{f}_2(\vec{k}_2)$$

$$\hat{f}_1^{*}(\vec{k}_1)\, \hat{f}_2^{*}(\vec{k}_2)$$

$$M(\vec{p}_1, \vec{p}_2, \ldots \vec{p}_n; \vec{k}_1, \vec{k}_2)$$

$$M(\vec{p}_1, \vec{p}_2, \ldots \vec{p}_n; \vec{k}_1, \vec{k}_2)^{*}$$

$$\delta^{(4)}\left(\Sigma p_i - k_1 - k_2\right) \frac{(2\pi)^{10}}{(2\pi)^{3(n+2)}} \left| -\frac{k_{12}}{\omega_{\vec{k}_1}} + \frac{k_{22}}{\omega_{\vec{k}_2}} \right|$$

$$\left(\prod_{i=1}^{n} \frac{1}{2\omega_{\vec{p}_i}}\right) \frac{1}{2\omega_{\vec{k}_1}} \frac{1}{2\omega_{\vec{k}_2}},$$

~~Assume~~ It should be independent of the initial wavefunc.

Assume $\hat{f}_1(\vec{k}_1)$ is peaked around ~~some~~ value $\vec{k}_1 = \vec{l}_1 = (0,0,l_{12})$ & $\hat{f}_2(k_2)$ is peaked around some value $\vec{k}_2 = \vec{l}_2$ $(0,0,l_{22})$

if $\hat{f}_1(\vec{k}_1)$, $\hat{f}_2(k_2)$ is sharply ~~peaked~~ we will replace k_1 by l_1 & k_2 by l_2

→ replace k_1 by l_1 & k_2 by l_2.

$$\int \frac{d^3k_1}{2\omega_{\vec{k}_1}} \hat{f}_1(\vec{k}_1)\, \hat{f}_1(\vec{k}_1)^{*} = 1.$$

$$\& \int \frac{d^3k_2}{2\omega_{\vec{k}_2}} \hat{f}_2(k_2)\, \hat{f}_2(k_2)^{*} = 1$$

$$= FL \int_R \prod_{i=1}^{n} d^3 p_i \, |M(\vec{p}_1, \ldots \vec{p}_n ; \vec{l}_1, \vec{l}_2)|^2$$

$$\delta^{(4)}\left(\Sigma p_i - l_1 - l_2 \right) |v_{1z} - v_{2z}|^{-1} (2\pi)^4$$

$$\frac{1}{(2\pi)^n} \prod_{i=1}^{n} \frac{1}{2\omega_{\vec{p}_1}} \prod_{i=1}^{2} \frac{1}{2\omega_{\vec{l}_1}} \qquad v_{1z} = \frac{l_{1z}}{\omega_{\vec{l}_1}}, \quad v_{2z} = \frac{l_{2z}}{\omega_{\vec{l}_2}}.$$

if the particles are identical then there would be symmetry factor of $\frac{1}{n!}$

In the c.m frame, it is more simplified.

In Quantum Mechanics $2 \to 2$ can be defined similarly.

we can not simultaneously localize position & momentum space. restriction in position comes from apparatus. So ~~there~~ we can ~~not~~ localize ~~momentum~~ momentum.

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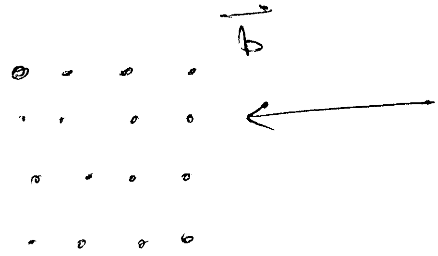


using this wavepackets allowed us to seem over various target particles over various distribution.

we chose $|f_i|^2$ to be delta func.

probability of scattering :-

$$PL \int_R \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 (2\omega_{\vec{p}_i})} \left| M(\vec{p}_1, \dots \vec{p}_n | \vec{l}_1, \vec{l}_2) \right|^2$$

$$(2\pi)^4 \delta^{(4)}\left(\sum p_i - l_1 - l_2\right) |v_1 - v_2|^{-1} \frac{1}{2\omega_{\vec{k}_1}}$$

$$\frac{1}{2\omega_{\vec{k}_2}}$$

for identical particles :- $1/n!$

But for now we won't consider the $n!$

measure of how many particles colliding

For two particle final states

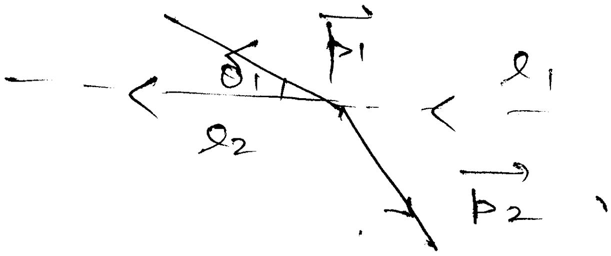
$$\delta^4(p_1 + p_2 - l_1 - l_2)$$

For 2 particles - 6 mom. is fixed.

$$\delta^{(3)}\left(\vec{p}_1+\vec{p}_2-\vec{\ell}_1-\vec{\ell}_2\right)\,\delta\left(\omega_{\vec{p}_1}+\omega_{\vec{p}_2}-\omega_{\vec{\ell}_1}-\omega_{\vec{\ell}_2}\right)$$

$$\vec{p}_2=\vec{\ell}_1+\vec{\ell}_2-\vec{p}_1 \Rightarrow \text{fixed } \vec{p}_2$$

$$\delta\left(\sqrt{p_1^2+m_p^2}+\sqrt{(\vec{\ell}_1+\vec{\ell}_2-\vec{p}_2)^2+m_p^2}-\omega_{\ell_1}-\omega_{\vec{\ell}_2}\right)$$



$$\delta\left(\sqrt{p_1^2+m_p^2}+\sqrt{(\ell_{1z}+\ell_{2z}-p_1\cos\theta)^2+p_1^2\sin^2\theta+m_p^2}-\omega_{\ell_1}-\omega_{\vec{\ell}_2}\right)$$

$$=\delta\left(p_1-\bar{p}_1\right)\Bigg/\left|\frac{\bar{p}_1}{\sqrt{\bar{p}_1^2+m^2}}-\frac{(\ell_{1z}+\ell_{2z})\cos\theta_1-\bar{p}_1}{\sqrt{(\ell_{1z}+\ell_{2z}-\bar{p}_1\cos\theta)^2+\bar{p}_1^2\sin^2\theta+m_p^2}}\right|$$

$\bar{p}_1$ soln. to

$$\sqrt{\bar{p}_1^2+m_p^2}+\sqrt{(\ell_{1z}+\ell_{2z}-\bar{p}_1\cos\theta)^2+\bar{p}_1^2\sin^2\theta_1+m_p^2}.$$

probability of ~~scattering~~.

$$= FL \Big\{ \frac{1}{(2\pi)^6} \int p_1^2 \sin\theta_1 \, d\theta_1 \, d\phi_1 \, \Big| M(p_1, p_2, l_1, l_2) \Big|^2_{\substack{p_2 = l_1 + l_2 - p_1 \\ p_1 = \bar{p}_1}}$$

$$(2\pi)^4 \, |v_1 - v_2|^{-1} \frac{1}{2\omega_{\vec{l_1}}} \frac{1}{2\omega_{\vec{l_2}}} \, .$$

$$\left| \frac{\bar{p}_1}{\sqrt{\bar{p}_1^2 + m_1^2}} - \frac{(l_{12} + l_{22}) \cos\theta_1 \bar{p}_1}{\sqrt{(l_{12} + l_{22} - \bar{p}_1 \cos\theta)^2 + \bar{p}_1^2 \sin^2\theta + m_p^2}} \right|$$

$$\frac{1}{2\omega_{\bar{p}_1}} \frac{1}{2\omega_{\vec{p_2}}} \, \Big\} .$$

we did the $\vec{p}_2$ integral in the cartesian coordinates.

~~⊕~~ If we want the probability to be independent of target. then we should divide it by FL.

$$[FL] = \frac{1}{L^2} \quad L^{-2} .$$

$$[\text{probability}] = M^0 L^0 T^0 .$$

Thus

$$[\{ \;\; \}] = L^2 .$$

$\longmapsto$ crosssection.

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2} \, p_1^2 \, |M(\vec{p_1}, \vec{p_2}, \vec{l_1}, \vec{l_2})|^2$$

$$\frac{1}{(2\omega_{\vec{p_1}})(2\omega_{\vec{p_2}})(2\omega_{\vec{l_1}})(2\omega_{\vec{l_2}})} \qquad |v_1 - v_2|^{-1}$$

$$\left[\frac{p_1}{\sqrt{p_1^2 + m_p^2}} - \frac{(l_{1z} + l_{2z})\cos\theta - p_1}{\sqrt{(l_{1z} + l_{2z} - p_1 \cos\theta)^2 + p_1^2 \sin^2\theta_1 + m_p^2}} \right]^{-1}$$

$\vec{p_2} = \vec{l_1} + \vec{l_2} - \vec{p_1}$

$\vec{p_1} = \vec{p_1}$

$d\Omega = \sin\theta_1 \, d\theta_1 \, d\phi_1$

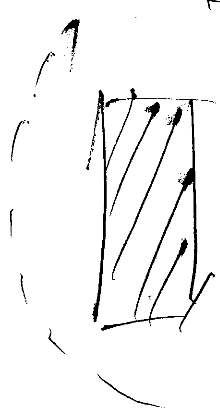
→ dimensionless

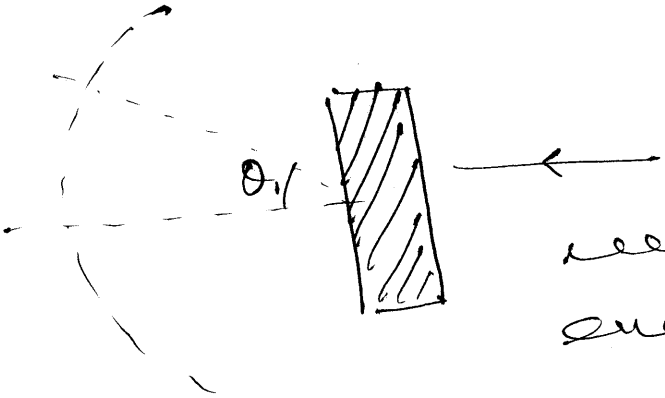
→ Dimension of Area = differential crosssection

measures

→ no. of scattered particle in the certain solid angles.

→ Repeat it many times





experimentally the no. of particle goes'

we have to isolate every event.

Even if we are ~~throwing~~ such beam of particles but we are considering the particles one by one

One incident particle = Pl x
if ~~there~~ are N particles ~~then~~ we should multiply it by N.

If we have a collider, then the calculation is best done in the CM frame. Two momentum are equal & opp. in the frame we are working.

Size of ~~the~~ appratus can be taken ~~to~~ ∞.

For indistinguishable particle $\frac{1}{2}!$ but we don't know which particle we are measuring. But ~~there are 2 contribution~~

let us see what happens when we use ~~the~~ centre of mass frame

For calculating total crosssection,
$\frac{d\sigma}{d\Omega}$ gives us prob. of finding
it along any Ω.

Total crosssection $\int \frac{d\sigma}{d\Omega} d\Omega$.

if we have ~~distinguishables~~
indistinguishable particle then
$\frac{1}{2} \times 4\pi$ or $\frac{1}{2} \times 4\pi$ $1 \times 2\pi$

In the C M frame :-

$\vec{q_1} + \vec{q_2} = 0$.

$q_{1z} + q_{2z} = 0$. & $\omega_{q_1} = \omega_{q_2}$

$v_2 = -v_1$

$\omega_{p_1} = \omega_{p_2}$

Thus from conservation

$\omega_{p_1} + \omega_{p_2} = \omega_{q_1} + \omega_{q_2}$

$\Rightarrow 2\omega_{\vec{q_1}} = 2\omega_{\vec{q_2}} = E_{cm}$

$\Rightarrow \omega_{\vec{q_1}} = \omega_{\vec{q_2}} = E_{cm}/2$.

$p_1 = |\vec{p_1}| = |\vec{q_1}|$

→ final vel. same as initial vel.
means they go out they go out

with certain angle with the magnitude of mel.

$$\frac{1}{4\pi^2} e_1^2 \; |M(\vec{p_1}, \vec{p_2}; \vec{e_1}, \vec{e_2})|^2 \frac{1}{E_{cm}^4}$$

$$\frac{1}{2v_1} \frac{1}{2v_1}$$

$$v_1 = \frac{e_1}{\omega_{\vec{e_1}}} \quad , \quad \frac{e_1}{v_1} = \omega_{\vec{e_1}} = \frac{E_{cm}}{2}$$

$$\frac{1}{16\pi^2} \frac{1}{4} \frac{E_{cm}^2}{4} \times |M(\vec{p_1}, \vec{p_2}, \vec{e_1}, \vec{e_2})|^2 \frac{1}{E_{cm}^4}$$

$$= \frac{1}{\pi^2 64 E_{cm}^2} |M|^2$$

$$= \frac{1}{64\pi^2 E_{cm}^2} |M|^2 .$$

first calculate it in the C.M frame.

Quantum F.T comes in calculating $|M|$.

Goal :- To calculate $S(\vec{p_1} .. \vec{p_n}, \vec{k_1} .. \vec{k_m})$
$= {}_{out}\langle \vec{p_1}, .. \vec{p_n} | \vec{k_1} .. \vec{k_m} \rangle_{in}$.

go back to free field theory & recall some results from free field theory

$$\phi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}.\vec{x}} \frac{1}{\sqrt{2\omega_{\vec{k}}}} \left(a(\vec{k},t) + a^{+}(\vec{k},t) \right)$$

$$= \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}.\vec{x}} \frac{1}{\sqrt{2\omega_{\vec{k}}}} \left\{ a(\vec{k}) e^{-i\omega_{\vec{k}} t} + a^{+}(-\vec{k}) e^{i\omega_{\vec{k}} t} \right\}.$$

$$a(\vec{k}) = a(\vec{k},t=0)$$
$$a^{+}(\vec{k}) = a^{+}(\vec{k},t=0)$$

$a(\vec{k}) = a(\vec{k},t=0)$, $a^{+}(\vec{k}) = a^{+}(\vec{k},t=0)$

try to express a & a^{+} in terms of ϕ.

$$\partial_0 \phi(\vec{x},t) = \pi(\vec{x},t) \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}.\vec{x}} (-i) \sqrt{\frac{\omega_{\vec{k}}}{2}} \left\{ a(\vec{k}) e^{-i\omega_{\vec{k}} t} - a^{+}(-\vec{k}) e^{i\omega_{\vec{k}} t} \right\}$$

Define

$$f_k(\vec{x},t) = e^{i(\vec{k}.\vec{x} - \omega_{\vec{k}} t)} \frac{1}{\sqrt{(2\pi)^3 2\omega_{\vec{k}}}}$$

Ex :-

$$i a^{+}(\vec{k}) = \int d^3x \, f_{\vec{k}}(\vec{x},t) \overleftrightarrow{\partial_0} \phi(\vec{x},t)$$

$$-i a(\vec{k}) = \int d^3x \, f_{k}(\vec{x},t)^{*} \overleftrightarrow{\partial_0} \phi(\vec{x},t)$$

Defn.

$$A \overleftrightarrow{\partial_0} B = A \partial_0 B - (\partial_0 A) B .$$

R.H.S $\longrightarrow$ time dependent

but $a^{+}(\vec{k})$ - doesn't depend upon time - check.

Free field theory $\rightarrow$ a & a^{+} are creation annihilation operator.

In the interacting theory we define

$$a_t(\vec{k}) = i \int d^3x \, f_{k}^{*}(\vec{x},t) \overleftrightarrow{\partial_0} \phi(\vec{x},t)$$

$$a^{+}_{t}(\vec{k}) = -i \int d^3x \, f_{k}(\vec{x},t) \overleftrightarrow{\partial_0} \phi(\vec{x},t)$$

$f_k(\vec{x},t)$ defined in the same way ~~ext~~ except that $\omega_{\vec{k}} = \sqrt{k^2 + m_p^2}$.

$\longrightarrow$ no longer time independent $\rightarrow$ check.

because ϕ satisfies Interacting theory

$a, a^\dagger$ by themselves cannot be thought as creation & annihilation operator.

we will use this operator to create in & out state from the vacuum

4:00 - 8:00

~~chg~~

$\langle \Omega | O_1 O_2 | \Omega \rangle$

$\sum_n \langle \Omega | O_1 | n \rangle \langle n | O_2 | \Omega \rangle$

if we create O_1 (2) in such a way that

$\langle \Omega | O_1 | n \rangle = 0$

then the single particle state will contribute.

we have the adjust O_1 in such a way that the in state & ~~out state~~ will contribute

we have to ~~define~~ now ~~design~~ devise the matrix element such that n particle state contribute.

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$$A \overleftrightarrow{\partial_0} B = A \partial_t B - (\partial_t A) B$$

$$f_k(\vec{x}, t) = \frac{1}{\sqrt{(2\pi)^3 2\omega_{\vec{k}}}} e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

$$\omega_{\vec{k}} = \sqrt{\vec{k}^2 + m_p^2}$$

$$a_t^+(\vec{k}) = -i \int d^3x \, f_{\vec{k}}(\vec{x}, t) \overleftrightarrow{\partial_0} \phi(\vec{x}, t)$$

$$a_t(\vec{k}) = i \int d^3x \, f_k^*(\vec{x}, t) \overleftrightarrow{\partial_0} \phi(\vec{x}, t)$$

└→ time ~~dependent~~

They play special role in computing

$$\partial_t a_t^+(\vec{k}) = -i \int d^3x \, \partial_t \Big[f_{\vec{k}}(\vec{x}, t) \partial_t \phi(\vec{x}, t) - \partial_t f_{\vec{k}}(\vec{x}, t) \phi(\vec{x}, t) \Big]$$

$$\cancel{-i \int d^3x \, \big[\partial_t f_k(\vec{x}, t) \partial_t \phi(\vec{x}, t)\big)}$$

$$= -i \int d^3x \Big[f_k(\vec{x}, t) \partial_t^2 \phi(\vec{x}, t) - \partial_t^2 f_{\vec{k}}(\vec{x}, t) \phi(\vec{x}, t) \Big]$$

$$\partial_t f_k = -\omega_k^2 f_k = -(k^2 + m_p^2) f_{\vec{k}}$$

$$= (\nabla^2 - m_p^2) f_{\vec{k}}.$$

$$\partial_t a_t^+(\vec{k}) = -i\int d^3x \left(f_k \partial_t^2 \phi + (-\nabla^2 + m_p^2) f_k \phi \right)$$

$$= \cancel{-i\int d^3x\, f_k \partial_t^2 \phi}$$

$$= -i\int d^3x\, f_k \left(\partial_t^2 - \nabla^2 + m_p^2\right) \phi$$

$$= -i\int d^3x\, f_k \left(-\Box + m_p^2\right) \phi$$

if ϕ had been a free field with mass m_p

then

$$\partial_t a_t^+(\vec{k}) = -i\int d^3x\, f_k \left(-\Box + m_p^2\right)\phi$$

$$\partial_t a_t(\vec{k}) = i\int d^3x\, f_k^* \left(-\Box + m_p^2\right)\phi$$

~~there is a time~~

In the defn. of a_t^ϕ, there is ~~the~~ ~~the~~ a time dependence inside $f_{\vec{k}}$.

Define :-

$$a_{in}^+(\vec{k}) = \frac{1}{\sqrt{2}} \lim_{T\to\infty} a^+_{-T(1-i\epsilon)}(\vec{k})$$

$$\langle \Omega | \phi(0) | \vec{k} \rangle = \frac{Z}{2\omega_{\vec{k}}(2\pi)^3}$$

$$\langle \Omega | T(\phi(x)\,\phi(y)) | \Omega \rangle = i \int \frac{d^4x}{(2\pi)^4} \frac{e^{ik\cdot x}}{-k^2 - m_p^2 + i\epsilon} Z$$

Z fixes the pole of ~~two~~ ~~pt~~ func.

$$\hat{a}_{in}(\vec{k}) = \frac{1}{\sqrt{Z}} \lim_{T\to\infty} a_{-T(1-i\epsilon)}(\vec{k})$$

$$a^{+}_{out}(\vec{k}) = \frac{1}{\sqrt{Z}} \lim_{T\to\infty} a^{+}_{T(1-i\epsilon)}(\vec{k})$$

$$\hat{a}_{out}(\vec{k}) = \frac{1}{\sqrt{Z}} \lim_{T\to\infty} a_{T(1-i\epsilon)}(\vec{k})$$

we'll prove that:

$$a^{+}_{in}(\vec{k}_1)\,|\vec{k}_2 \ldots \vec{k}_n\rangle_{in} = |\vec{k}_1 \ldots \vec{k}_n\rangle_{in}$$

annihilation operator

we will prove that

$$\hat{a}^{+}_{out}(\vec{p}_1)\,|\vec{p}_2, \ldots \vec{p}_n\rangle_{out} = |\vec{p}_1, \ldots \vec{p}_n\rangle_{out}$$

$${}_{out}\langle \vec{p}_2, \ldots \vec{p}_n | \,\hat{a}_{out}(\vec{p}_1) = {}_{out}\langle \vec{p}_1, \ldots \vec{p}_n |$$

we always calculate it as real

$$\hat{a}_{in}(\vec{k}) \,|\vec{k}_1, \ldots \vec{k}_n\rangle_{in}$$

$$= \sum_{i=1}^{n} \delta^{(3)}(\vec{k}-\vec{k}_i)\,|\vec{k}_1 \ldots \vec{k}_{i-1}\, \vec{k}_{i+1}, \vec{k}_n\rangle_{in}$$

$$_{out}\langle \vec{p}_1, \ldots \vec{p}_n|\, a^{+}_{out}(\vec{p})$$

$$= \sum_{i=1}^{n} \delta^{(3)}(\vec{p}-\vec{p}_i)\,\langle \vec{p}_1 \ldots \vec{p}_{i-1}\, \vec{p}_{i+1} \ldots \vec{p}_n|$$

Assume these seen. & compute

$$S(p_1, \ldots p_n; k_1 \ldots k_m) = {}_{out}\langle \vec{p}_1, \ldots \vec{p}_n|\vec{k}_1 \ldots \vec{k}_m\rangle_{in}$$

Assume that none of the $\vec{p}_i$'s are equal to any of the $\vec{k}_i$'s.

out mom. is different from in, see put the detector away from the line of the initial mom. particle.

$$\cancel{S(\vec{p}_1)}\; S(\vec{p}_1 \ldots \vec{p}_n; \vec{k}_1 \ldots \vec{k}_m)$$

$$= {}_{out}\langle \vec{p}_1 \ldots \vec{p}_n|\, a^{+}_{in}(\vec{k}_1)\,|\vec{k}_2 \ldots \vec{k}_m\rangle$$

$$\cancel{= {}_{out}\langle \vec{p}_1, \ldots \vec{p}_n|\, a_{out}(\vec{k}_1) - a^{+}_{in}(\vec{k}_1)}$$

$$= - {}_{out}\langle \vec{p}_1, \dots \vec{p}_n | a_{out}^{+}(\vec{k}_1) - a_{in}^{+}(\vec{k}_1) | \vec{k}_2, \dots \vec{k}_m \rangle$$

$$= \frac{-1}{\sqrt{Z}} \lim_{T \to \infty} {}_{out}\langle \vec{p}_1, \dots \vec{p}_n | \left(a^{+}_{T(1-i\epsilon)}(\vec{k}_1) - a^{+}_{-T(1-i\epsilon)}(\vec{k}_1) \right) | \vec{k}_2, \dots \vec{k}_m \rangle$$

$$= \frac{-i}{\sqrt{Z}} \lim_{T \to \infty} \int_{-T(1-i\epsilon)}^{T(1-i\epsilon)} dt \, \langle \vec{p}_1 \dots \vec{p}_n | \partial_t a_t^{+}(\vec{k}_1) | \vec{k}_2, \dots k_m \rangle_{in}$$

$$= \frac{+i}{\sqrt{Z}} \lim_{T \to \infty} \int_{-T(1-i\epsilon)}^{T(1-i\epsilon)} dt \int d^3x \; {}_{out}\langle \vec{p}_1, \dots \vec{p}_n | f_{\vec{k}_1}(\vec{x}, t) \left(-\Box + m^2 \right) \phi(\vec{x}, t) | \vec{k}_2, \dots \vec{k}_m \rangle_{in}$$

$$\cancel{= \frac{i}{\sqrt{Z}} \int d^4x_1 \; {}_{out}\langle \vec{p}_1}$$

$$\cancel{= \frac{i}{\sqrt{Z}} \lim_{T \to \infty} \frac{i}{\sqrt{Z}} \int d^4x_1 \, f_{\vec{k}_1}(\vec{x}, t) \left(-\Box_{x_1} + m_p^2 \right) \phi(\vec{x}_1,}$$

$$= \frac{i}{\sqrt{Z}} \lim_{T \to \infty} \int d^4x_1 \cdot f_{\vec{k}_1}(\vec{x}_1, t) \left(-\Box_{x_1} + m_p^2 \right) {}_{out}\langle \vec{p}_1, \dots, \vec{p}_n | \phi(x_1) | \vec{k}_2, \dots \vec{k}_m \rangle_{in}$$

$${}_{out}\langle \vec{p}_1, \dots \vec{p}_n | \phi(x_1) | \vec{k}_2, \dots \vec{k}_m \rangle_{in}$$

$$= {}_{out}\langle \vec{p}_1, \dots \vec{p}_n | \phi(x_1) \, a^{+}_{in}(\vec{k}_2) | k_3 \dots \vec{k}_m \rangle_{in}$$

$$= -\,{}_{out}\langle \vec{p}_1 \ldots \vec{p}_n | \{ a_{out}^{+}(\vec{k}_2)\, \phi(x_1)$$
$$- \phi(x_1)\, a_{in}^{+}(\vec{k}_2) \} | \vec{k}_3, \ldots \vec{k}_m \rangle_{in}$$

$$= \frac{-1}{\sqrt{Z}} \lim_{T\to\infty} {}_{out}\langle \vec{p}_1 \ldots \vec{p}_n | \{ a^{+}_{T(1-i\epsilon)}\, \phi(x_1)$$
$$- \phi(x_1)\, a^{+}_{-T(1-i\epsilon)} \} | \vec{k}_3, \ldots \vec{k}_m \rangle_{in}$$

$$= \frac{-1}{\sqrt{Z}} \lim_{T\to\infty} \int_{-T(1-i\epsilon)}^{T(1-i\epsilon)} dt\, \partial_t\, {}_{out}\langle \vec{p}_1, \ldots \vec{p}_n | T(\phi(x_1)\, a_t^{+})$$
$$| \vec{k}_3 \ldots \vec{k}_m \rangle_{in}.$$

$$= \frac{1}{\sqrt{Z}} \lim_{T\to\infty} \int_{-T(1-i\epsilon)}^{T(1-i\epsilon)} \partial_t \langle \vec{p}_1, \ldots \vec{p}_n | \Big(i\, T \big(f_{\vec{k}_2}(x,t)$$
$$\overleftrightarrow{\partial}_t\, \phi(\vec{x},t)\, \phi(x_1) \big)$$
$$| \vec{k}_3, \ldots \vec{k}_m \rangle_{in}$$

So:

$$T\left(a_t^{+}(\vec{k}_2)\, \phi(x_1) \right)$$
$$= -i \int d^3x\; T\left(f_{\vec{k}_2}(\vec{x},t)\, \overleftrightarrow{\partial}_0\, \phi(\vec{x},t)\, \phi(x_1) \right)$$
$$= -i \int d^3x\; f_{\vec{k}_2}(\vec{x},t)\, T\left(\partial_t \phi(\vec{x},t) \right) \phi(x_1,t)$$
$$+ i \int d^3x\; \partial_t f_{\vec{k}_2}(\vec{x},t)\; T\left(\phi(\vec{x},t)\, \phi(x_1) \right)$$

$$\partial_t \{ \theta(t-t_1)\, \phi(\vec{x},t)\, \phi(\vec{x}_1,t_1) + \theta(t_1-t)\, \phi(\vec{x}_1,t_1)\, \phi(\vec{x},t) \}$$

$$= \delta(t-t_1) \left[\phi(\vec{x},t)\, \phi(\vec{x}_1,t_1) - \phi(\vec{x}_1,t_1)\, \phi(\vec{x},t) \right]$$

$$= \delta(t-t_1)\, \cancel{\phi(\vec{x},\vec{x}_1)} \left[\phi(\vec{x},t), \phi(\vec{x}_1,t_1) \right]$$

$$= 0 .$$

$$= -i \int d^3x\, f_{\vec{k}_2}(\vec{x},t)\, \partial_t T\left(\phi(\vec{x},t)\, \phi(x_1) \right)$$
$$+ i \int d^3x\, \partial_t f_{\vec{k}_2}(\vec{x},t)\, T\left(\phi(\vec{x},t)\, \phi(x_1) \right)$$

if it is $\dot{\phi}$ → then we can not proceed in ~~this~~ way.
$f_{\vec{k}}$ is a function.

$$\partial_t T\left(a_t^{+}(\vec{k}_2)\, \phi(x_1) \right)$$

$$= -i \int d^3x\, f_{\vec{k}_2}(\vec{x},t)\, \partial_t^2\, T\left(\phi(\vec{x},t)\, \phi(x_1) \right)$$
$$+ i \int d^3x\, \partial_t^2 f_{\vec{k}_2}(\vec{x},t)\, T\left(\phi(\vec{x},t)\, \phi(x_1) \right)$$

$$(\nabla^2 - m_\phi^2) f_{\vec{k}_2}$$

$$= i \int d^3x\, f_{\vec{k}_2}(\vec{x},t)\, \underset{(\Box - m_\phi^2)}{\cancel{\Box}}\, T\left(\phi(\vec{x},t), \phi(\vec{x}_1) \right)$$

we can also

$$= i \int d^3x\, f_{\vec{k}_2}(\vec{x},t) \left(\Box - m_p^2\right) T\left(\phi(\vec{x},t)\, \phi(x_1)\right)$$

$$= \frac{i}{\sqrt{Z}} \int_{-T(1-i\epsilon)}^{+T(1-i\epsilon)} dt \int d^3x\, f_{\vec{k}_2}(\vec{x},t) \left(-\Box_{\vec{x},t} + m_p^2\right) {}_{out}\langle \vec{p}_1 \ldots \vec{p}_n | T\left(\phi(x_1)\phi(x)\right) | \vec{k}_3, \ldots \vec{k}_m\rangle_{in}$$

$\phi \to$ falls of at large x.

$$= \left(\frac{i}{\sqrt{Z}}\right) \int d^4x_1\, f_{\vec{k}_1}(\vec{x}_1,t) \left(-\Box_{x_1} + m_p^2\right) {}_{out}\langle \vec{p}_1 \ldots \vec{p}_n | \phi(x_1) | \vec{k}_2 \ldots \vec{k}_m\rangle_{in}$$

$$= \left(\frac{i}{\sqrt{Z}}\right)^2 \int d^4x_1 \int d^4x_2\, f_{\vec{k}_1}(\vec{x}_1,t)\, f_{\vec{k}_2}(\vec{x}_2,t_2) \left(-\Box_{x_1} + m_p^2\right)\left(\Box_{\vec{x}_2} + m_p^2\right) \langle \vec{p}_1 \ldots \vec{p}_n | T\left(\phi(x_1)\phi(x_2)\right) | \vec{k}_3, \vec{k}_m\rangle$$

$\longrightarrow$ Do the same for

$\langle \vec{p}_1 \ldots \vec{p}_n \rangle$.

Final expression:-

$$|k_m\rangle_{in} = a_{in}^{\dagger}(\vec{k}_{in})\,|\Omega\rangle$$

$\hookrightarrow$ vacuum of the interacting theory.

$$S(\vec{p}_1 \ldots \vec{p}_n, \vec{k}_1 \ldots \vec{k}_m)$$
$$= \left(\frac{i}{\sqrt{Z}}\right)^{m+n} \int d^4x_1 \ldots \int d^4x_m \int d^4y_1 \ldots \int d^4y_n \; f_{\vec{k}_1}(x_1) \ldots f_{\vec{k}_m}(x_m)$$
$$f^*_{p_1}(y_1) \ldots f^*_{p_n}(y_n)\,(-\Box_{x_1} + m_p^2) \ldots (-\Box_{x_n} + m_p^2)\,(-\Box_{y_1} + m_p^2) \ldots (-\Box_{y_n} + m_p^2)$$
$$\langle \Omega | T\,(\phi(x_1) \ldots \phi(x_m)\phi(y_1) \ldots \phi(y_n))|\Omega\rangle$$

for in states product with f_k

for outstate f_k^*

Z can be calculated

Perturbative expansion - 2 pt. fun.

We will calculate the Feynman ... mom. space.

$$\boxed{i\frac{\partial}{\partial t} - \frac{\nabla^2}{2m} + V(r)} = 0$$

$$\psi\, i\frac{\partial \psi^*}{\partial t} - \frac{1}{2m}\nabla\psi^* \cdot \nabla\psi + V(r)\psi^*\psi$$

$$\Rightarrow \frac{1}{2m}\left[\psi\nabla^2\psi - \nabla\psi^*\nabla\psi\right]$$

$$S t - S\psi$$

$$\mathcal{L}(\psi, \partial_\mu \psi, \partial_\nu \partial_\mu \psi)$$

$$\delta\mathcal{L} = \frac{\partial \mathcal{L}}{\partial \psi}\delta\psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\delta(\partial_\mu\psi) + \frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\delta(\partial_\nu\partial_\mu\psi)$$

$$= \frac{\partial \mathcal{L}}{\partial \psi}\delta\psi + \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\delta\psi\right)$$

$$- \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\right)\delta\psi$$

$$\cancel{+ \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\partial_\nu\psi\right)}$$

$$\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\partial_\nu\delta\psi\right)$$

$$- \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\right)\partial_\nu\delta\psi$$

$$\to \partial_\nu\left(\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\right)\delta\psi\right)$$

$$- \partial_\nu\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\right)\delta\psi$$

$$\left[\frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\right) - \partial_\mu \partial_\nu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\right)\right]\delta\psi$$

$$+ \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\delta\psi\right) + \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\partial_\nu \delta\psi\right)$$

$$- \partial_\nu\left(\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\right)\delta\psi\right) = \delta\mathcal{L}$$

~~thus the eqn. of~~ motion

$$\left[\frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\right) - \partial_\mu \partial_\nu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\right)\right]\delta\psi$$

$$\cancel{+ \partial_\mu\left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\delta\psi + \frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \psi)}\partial_\nu \delta\psi\right.}$$

$$- \eta^\mu$$

$$+ \partial_\alpha\left(\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \psi)}\delta\psi\right) + \partial_\alpha\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\alpha \psi)}\partial_\nu \delta\psi\right)$$

$$- \partial_\nu\left(\partial_\alpha\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\alpha \psi)}\right)\delta\psi\right) = \delta\mathcal{L}.$$

$$\cancel{\partial_\alpha \left[\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \psi} \right.}$$

$$\partial_\alpha \left(\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \psi)} \delta\psi \right) + \partial_\alpha \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\alpha \psi)} \partial_\nu \delta\psi \right)$$

$$- \partial_\nu \left(\partial_\gamma \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\gamma \psi)} \right) \delta\psi \right)$$

$$\cancel{\partial_\alpha \left[\frac{\partial \mathcal{L}}{\partial(\partial} \right.} = \partial_\alpha \left(\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \psi)} \delta\psi \right)$$

$$+ \partial_\alpha \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\alpha \psi)} \partial_\nu \delta\psi \right)$$

$$- \eta^{\alpha\nu} \partial_\alpha \Big($$

$$\delta \mathcal{L} \doteq \partial_\alpha \left[\left(\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \psi)} \delta\psi \right) + \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\alpha \psi)} \partial_\nu \delta\psi \right) - \delta^\alpha_\nu \left(\partial_\gamma \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\gamma \psi)} \right) \delta\psi \right) \right]$$

$$\mathcal{L}\left(\phi, \partial_\mu \phi, \partial_\mu \partial_\nu \phi\right)$$

$$\delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \phi} \delta \phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta(\partial_\mu \phi) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \delta(\partial_\mu \partial_\nu \phi).$$

$$= \frac{\partial \mathcal{L}}{\partial \phi} \delta \phi + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta \phi \right) - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) \delta \phi$$

$$+ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\mu \partial_\nu \delta \phi$$

$$\longrightarrow \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\nu \delta \phi \right)$$

$$- \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \right) \partial_\nu \delta \phi$$

$$= \frac{\partial \mathcal{L}}{\partial \phi} \delta \phi + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta \phi \right) - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) \delta \phi$$

$$+ \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\nu \delta \phi \right) - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \right) \partial_\nu \delta \phi$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \right) \partial_\nu \delta\phi \qquad \begin{array}{l} \mathcal{L}(x) \leftrightarrow \partial\mathcal{L} \\ \phi(x+a) = \delta\phi \end{array}$$

$$= \cancel{\partial_\nu \left(\partial_\mu \right.} \; \partial_\nu \left(\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \right) \delta\phi \right)$$

$$- \partial_\nu \left(\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \right)\right) \delta\phi .$$

Thus

$$\delta \mathcal{L} = \left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) + \partial_\nu \left(\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \right)\right) \right] \delta\phi$$

$$+ \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi \right) + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\nu \delta\phi \right)$$

$$- \partial_\nu \left(\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \right) \delta\phi \right) - \cancel{\delta^\mu_\nu \mathcal{L}}$$

$$\cancel{\partial_\alpha} \left[\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi)} \delta\phi + \left(\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \partial_\nu \phi)} \partial_\nu \delta\phi \right) \right.$$

$$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\nu \delta\phi - \left(\partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \phi)} \right) \delta\phi \right) \right]$$

$$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\nu \delta\phi - \left\{ \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \phi)} \right) \delta\phi \right\} \right]$$

$$\phi \to \phi$$

$$x \to x$$

$$\cancel{\delta S = \int d^4x \, \mathcal{L}}$$

$$\delta S = \int d^4x \; \cancel{\partial_\alpha \mathcal{L}} \; (\delta \mathcal{L}) \qquad \frac{\partial \mathcal{L}}{\partial x^\mu} \delta x^\mu$$

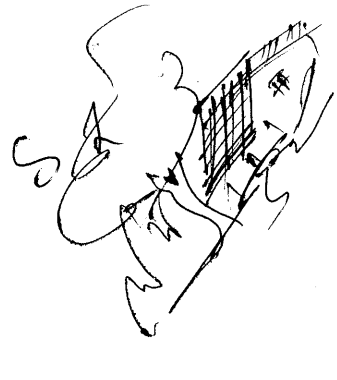
$$\to \partial_\mu \delta x^\mu \mathcal{L}$$

$$\to \cancel{\partial_\mu} ($$

$$\partial \left[\frac{\partial \mathcal{L}}{\partial x^\mu} \delta x^\mu \right]$$

$$= \mathcal{L}$$

$$\frac{\partial \mathcal{L}}{\partial x^\mu} \delta x^\mu = \frac{\partial \mathcal{L}}{\partial}$$



$$\delta^\mu{}_\nu \mathcal{L}$$

$$\mathcal{L}(x)$$

$\cancel{\partial \mathcal{L} =}$

$$\mathcal{L} = (\partial^\alpha \partial_\alpha \phi)(\partial^\beta \partial_\beta \phi)$$

$\cancel{j^\mu}$

$$\cancel{= \frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\nu \delta\phi}$$

$$\phi = \phi(x + a^\mu)$$

$$\delta\phi = a^\mu \frac{\partial \phi}{\partial x^\mu}$$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\nu \delta \phi$$

$$\cancel{= \eta^{\alpha\mu} \eta \, \delta^\alpha}$$

$$= \eta^\alpha{}_\mu \eta_{\alpha\nu} \partial^\beta \partial_\beta \phi \, \partial_\nu \left[a^\mu \frac{\partial \phi}{\partial x^\mu} \right]$$

$$- \cancel{\partial_\nu} \left[\eta \right. + \eta^\beta{}_\mu \eta_{\beta\nu} \partial^\alpha \partial_\alpha \phi \, \partial_\nu \left[a^\mu \frac{\partial \phi}{\partial x^\mu} \right]$$

$$- \partial_\nu \left[\eta^\alpha{}_\nu \eta_{\alpha\mu} (\partial^\beta \partial_\beta \phi) \, a^\mu \frac{\partial \phi}{\partial x^\mu} + \eta^\beta{}_\nu \eta_{\beta\mu} (\partial^\alpha \partial_\alpha \phi) \, a^\mu \frac{\partial \phi}{\partial x^\mu} \right],$$

$$= \eta^{\alpha}{}_{\mu}\eta_{\alpha\nu}\,\partial^{\beta}\partial_{\beta}\phi\;\partial_{\nu}\cancel{a^{\mu}}\;\partial_{\mu}\phi$$

$$+\ \eta^{\alpha}{}_{\mu}\eta_{\alpha\nu}\,\partial^{\beta}\partial_{\beta}\phi\; a^{\mu}\;\partial_{\nu}\partial_{\mu}\phi$$

$$+\ \eta^{\beta}{}_{\mu}\eta_{\beta\nu}\,\partial^{\alpha}\partial_{\alpha}\phi\left(\partial_{\nu}a^{\mu}\right)\frac{\partial\phi}{\partial x^{\mu}}$$

$$+\ \eta^{\beta}{}_{\mu}\eta_{\beta\nu}\,\partial^{\alpha}\partial_{\alpha}\phi\; a^{\mu}\,\partial_{\nu}\left(\frac{\partial\phi}{\partial x^{\mu}}\right)$$

$$-\ \eta^{\alpha}{}_{\nu}\eta_{\alpha\mu}\,\partial_{\nu}\left(\partial^{\beta}\partial_{b}\phi\right)a^{\mu}\frac{\partial\phi}{\partial x^{\mu}}$$

$$-\ \eta^{\alpha}{}_{\nu}\eta_{\alpha\mu}\left(\partial^{\beta}\partial_{\beta}\phi\right)\partial_{\nu}\left(a^{\mu}\frac{\partial\phi}{\partial x^{\mu}}\right)$$

$$-\ \eta^{\beta}{}_{\nu}\eta_{\beta\mu}\,\partial_{\nu}\left(\partial_{\alpha}\partial_{\alpha}\phi\right)a^{\mu}\frac{\partial\phi}{\partial x^{\mu}}$$

$$-\ \eta^{\beta}{}_{\nu}\eta_{\beta\mu}\left(\partial^{\alpha}\partial_{\alpha}\phi\right)\partial_{\nu}\left(a^{\mu}\frac{\partial\phi}{\partial x^{\mu}}\right)$$

$$= \partial^{\beta}\partial_{\beta}\phi\;\partial_{\alpha}a^{\alpha}$$

$$\int \mathcal{D}\phi \, \Box \phi \; dt \, d^3x$$

$$\phi(x+a) = \phi(x) + a^\mu \partial_\mu \phi$$

$$\Box \phi = \partial^\alpha \partial_\alpha \tilde{\phi}$$

$$= \partial^\alpha \partial_\alpha (\phi +$$

$$x + a = y$$

$$\int \phi \Box^2 \phi$$

$$\phi(x+a) = \phi(x) + a^\mu \partial_\mu \phi$$

$$\cancel{\partial^\alpha \partial_\alpha [\phi] =}$$

$$\partial^\alpha \partial_\alpha \left(\phi(x) + a^\mu \partial_\mu \phi \right)$$

$$= \partial^\alpha \left[\partial_\alpha \phi + a^\mu \partial_\alpha \partial_\mu \phi \right]$$

$$= \partial^\alpha \partial_\alpha \phi + a^\mu \partial^\alpha \partial_\alpha \partial_\mu \phi$$

$$= \Box^2 \phi + a^\mu \Box^2 \partial_\mu \phi .$$

$$\cancel{\partial^\beta \partial^\beta} =$$

$$\left(\phi + a^\mu \partial_\mu \phi \right)\left(\Box^2 \phi + a^\mu \Box^2 \partial_\mu \phi \right)$$

$$\cancel{\phi \, \Box^2 \phi + \phi a^\mu \Box^2}$$

$$\phi \Box^2 \phi + \phi a^\mu \Box^2 \partial_\mu \phi$$

$$+ a^\mu \boxed{\partial_\mu \phi} \Box^2 \phi$$

$$\longrightarrow a^\mu \phi \Box^2 \phi$$

$$\delta S = \qquad - \phi \partial_\mu \left[a^\mu \Box^2 \phi \right]$$

$$\longrightarrow \phi \partial_\mu a^\mu \Box^2 \phi - a^\mu \partial_\mu \Box^2 \phi$$

$$S = \int dt\, d^3x \;\; \phi \, \Box^2 \phi$$

$$\cancel{\phi\,(x^\mu + a^\mu}$$

$$x \longrightarrow x^\mu + a^\mu$$

$$\phi(x) = \phi(x + a^\mu)$$

$$= \phi(x) + \cancel{\partial^\mu}\, a^\mu \partial_\mu \phi$$

$$\cancel{\partial^\alpha \partial_\alpha}$$

$$\partial^\beta \partial_\beta \; \partial^\alpha \partial_\alpha \left[\phi(x) + a^\mu \partial_\mu \phi \right]$$

$$\cancel{= \partial^\beta \partial_\beta \partial^\alpha \partial_\alpha \phi(x)}$$

$$\partial^\alpha \Big[\partial_\alpha \phi(x) + (\partial_\alpha a^\mu) \partial_\mu \phi$$

$$+ a^\mu \partial_\alpha \partial_\mu \phi \Big]$$

$$\partial^\alpha \partial_\alpha \phi(x) + (\partial^\alpha \partial_\alpha a^\mu) \partial_\mu \phi$$

$$+2 (\partial_\alpha a^\mu)(\partial^\alpha \partial_\mu \phi)$$

$$+ a^\mu \partial^\alpha \partial_\alpha \partial_\mu \phi .$$

$$\partial_\beta \Big[\partial^a \partial_a \phi + (\partial^\alpha \partial_a a^\mu)\, \partial_\mu \phi$$
$$+ 2(\partial_\alpha a^\mu)(\partial^\alpha \partial_\mu \phi)$$
$$+ a^\mu \partial^a \partial_a \partial_\mu \phi \Big]$$

$$= \partial_\beta \partial^a \partial_\alpha \phi + \partial_\beta (\partial^a \partial_\alpha a^\mu)\, \partial_\mu \phi$$
$$+ (\partial^\alpha \partial_a a^\mu)\, \partial_\beta \partial_\mu \phi$$
$$+ 2(\partial_\beta \partial_\alpha a^\mu)(\partial^\alpha \partial_\mu \phi) \qquad \Box\phi\, \Box\phi$$
$$+ 2(\partial_\alpha a^\mu)(\partial_\beta \partial^\alpha \partial_\mu \phi) \qquad \partial^a \partial_\alpha \phi\; \partial^\beta \partial_\beta \phi$$
$$+ (\partial_\beta a^\mu)(\partial^\alpha \partial_\alpha \partial_\mu \phi) \qquad \int d^4x\, \partial^a \partial_\alpha \phi\, \partial^\beta \partial_\beta \phi$$
$$\cancel{\int d^4x\, \partial^a} \Big[$$
$$+ a^\mu (\partial_\beta \partial^\alpha \partial_\alpha \partial_\mu \phi)$$

$$= \partial_\beta \partial^\alpha \partial_a \phi + \partial_\beta (\partial^a \partial_\alpha a^\mu)\, \partial_\mu \phi$$
$$+ (\partial^\alpha \partial_a a^\mu)\, \partial_\beta \partial_\mu \phi$$
$$+ 2(\partial_\beta \partial_a a^\mu)(\partial^a \partial_\mu \phi)$$
$$+ 2(\partial_\alpha a^\mu)(\partial_\beta \partial^\alpha \partial_\mu \phi)$$
$$+ (\partial_\beta a^\mu)(\partial^a \partial_\alpha \partial_\mu \phi)$$
$$+ a^\mu (\partial_\beta \partial^\alpha \partial_\alpha \partial_\mu \phi)$$

$$\partial^\beta \Big[\partial_\beta \partial^\alpha \partial_\alpha \phi + \partial_\beta (\partial^\alpha \partial_\alpha a^\mu) \partial_\mu \phi$$
$$+ (\partial^\alpha \partial_\alpha a^\mu) \partial_\beta \partial_\mu \phi$$
$$+ 2 (\partial_\beta \partial_\alpha a^\mu)(\partial^\alpha \partial_\mu \phi)$$
$$+ 2 (\partial_\alpha a^\mu)(\partial_\beta \partial^\alpha \partial_\mu \phi)$$
$$+ (\partial_\beta a^\mu)(\partial^\alpha \partial_\alpha \partial_\mu \phi)$$
$$+ a^\mu (\partial_\beta \partial^\alpha \partial_\alpha \partial_\mu \phi) \Big]$$

$$= \partial^\beta \partial_\beta \partial^\alpha \partial_\alpha \phi$$
$$+ \partial^\beta \partial_\beta (\partial^\alpha \partial_\alpha a^\mu) \partial_\mu \phi$$
$$+ \partial_\beta (\partial^\alpha \partial_\alpha a^\mu)(\partial^\beta \partial_\mu \phi)$$
$$+ \partial^\beta (\partial^\alpha \partial_\alpha a^\mu)(\partial_\beta \partial_\mu \phi)$$
$$+ (\partial^\alpha \partial_\alpha a^\mu)(\partial^\beta \partial_\beta \partial_\mu \phi)$$
$$+ 2 (\partial^\beta \partial_\beta \partial_\alpha a^\mu)(\partial^\alpha \partial_\mu \phi)$$
$$+ 2 (\partial_\beta \partial_\alpha a^\mu)(\partial^\beta \partial^\alpha \partial_\mu \phi)$$
$$+ 2 (\partial^\beta \partial_\alpha a^\mu)(\partial_\beta \partial^\alpha \partial_\mu \phi)$$
$$+ 2 (\partial_\alpha a^\mu)(\partial^\beta \partial_\beta \partial^\alpha \partial_\mu \phi)$$

$$+\left(\partial^{\beta}\partial_{\beta}a^{\mu}\right)\left(\partial^{\alpha}\partial_{\alpha}\partial_{\mu}\phi\right)$$

$$+\left(\partial_{\beta}a^{\mu}\right)\left(\partial^{\beta}\partial^{\alpha}\partial_{\alpha}\partial_{\mu}\phi\right)$$

$$+\left(\partial^{\beta}a^{\mu}\right)\left(\partial_{\beta}\partial^{\alpha}\partial_{\alpha}\partial_{\mu}\phi\right)$$

$$+a^{\mu}\left(\partial^{\beta}\partial_{\beta}\partial^{\alpha}\partial_{\alpha}\partial_{\mu}\phi\right)$$

$$\cancel{=\Box^{2}\phi+\Box^{2}a^{\mu}\partial_{\mu}\phi}$$

$$=\Box^{2}\phi+\Box^{2}a^{\mu}\partial_{\mu}\phi$$

$$+2\partial_{\beta}\Box a^{\mu}\partial_{\beta}\partial_{\mu}\phi$$

$$+2\Box a^{\mu}\Box\partial_{\mu}\phi$$

$$+2\left(\Box\partial_{\alpha}a^{\mu}\right)\left(\partial^{\alpha}\partial_{\mu}\phi\right)$$

$$+4\left(\partial_{\beta}\partial_{\alpha}a^{\mu}\right)\left(\partial^{\beta}\partial^{\alpha}\partial_{\mu}\phi\right)$$

$$+4\left(\partial_{\alpha}a^{\mu}\right)\left(\partial^{\beta}\partial_{\beta}\partial^{\alpha}\partial_{\mu}\phi\right)$$

$$+a^{\mu}\Box^{2}\partial_{\mu}\phi$$

$$\Box^{2}\phi$$

$$\cancel{\delta S}=+\left(\Box^{2}a^{\mu}\right)\partial_{\mu}\phi+2\left(\partial_{\beta}\Box a^{\mu}\right)\left(\partial_{\beta}\partial_{\mu}\phi\right)$$

$$+2\Box a^{\mu}\Box\partial_{\mu}\phi+2\left(\Box\partial_{\alpha}a^{\mu}\right)\left(\partial^{\alpha}\partial_{\mu}\phi\right)$$

$$+4\left(\partial_{\beta}\partial_{\alpha}\right.$$

$$\phi \longrightarrow \phi\left(x + a^\mu \partial_\mu x\right)$$

$$= \phi(x) + a^\mu \partial_\mu \phi$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\nu \delta\phi - \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \phi)} \right) \delta\phi \right]$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \partial_\nu \delta\phi - \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \partial_\mu \phi)} \right) \delta\phi$$

$$\delta\phi = \frac{\partial \phi}{\partial x^\mu} \qquad \partial_\mu \phi$$

$$\mathcal{L} = (\partial^\alpha \partial_\alpha \phi)(\partial^\beta \partial_\beta \phi)$$

$$= \eta^\alpha{}_\mu \eta_{\alpha\nu} (\partial^\beta \partial_\beta \phi) \partial_\nu \partial_\mu \phi$$

$$+ \eta^\beta{}_\mu \eta_{\beta\nu} (\partial^\alpha \partial_\alpha \phi) \partial_\nu \partial_\mu \phi$$

$$- \partial_\nu \left(\eta^\alpha{}_\mu \eta_{\alpha\nu} (\partial^\beta \partial_\beta \phi) + \eta^\beta{}_\mu \eta_{\beta\nu} (\partial^\alpha \partial_\alpha \phi) \right) \partial_\mu \phi$$

$$= \eta^\alpha{}_\mu \eta^\alpha{}_\nu (\partial^\beta \partial_\beta \phi)(\partial_\nu \partial_\mu \phi) + \eta^\beta{}_\mu \eta_{\beta\nu} (\partial^\alpha \partial_\alpha \phi) \partial_\nu \partial_\mu \phi$$

$$- \eta^\alpha{}_\mu \eta_{\alpha\nu} \partial_\nu (\partial^\beta \partial_\beta \phi) \partial_\mu \phi - \eta^\beta{}_\mu \eta_{\beta\nu} \partial_\nu (\partial^\alpha \partial_\alpha \phi) \partial_\mu \phi$$

$a_{in}^{+}(\vec{k})\,|\vec{k}_1,\ldots\vec{k}_m\rangle_{in}$

$= |\vec{k},\vec{k}_1,\ldots\vec{k}_m\rangle_{in}$

Total momentum always commutes with the total energy.

we have ~~defined~~ in state such ~~that in the~~ past (asymptotic) ~~for~~ we have localized mom. space as well as position space such that ~~convolution is~~ overlap is 0.

~~classical~~ $T \longrightarrow -\infty$, $|\vec{k},\vec{k}_1,\ldots\vec{k}_m\rangle_{in}$ ~~has the~~ interpretation of $m+1$ ~~position~~ particles, with ~~these~~ momentum.

$$a_{in}^{+}(\vec{k}) = \frac{1}{\sqrt{Z}}\lim_{T\to\infty} a^{+}_{-T(1-i\epsilon)}(\vec{k})$$

$$a^{+}_{t}(\vec{k}) = -i\int d^3x\, f_{\vec{k}}(\vec{x},t)\overleftrightarrow{\partial_0}\phi(\vec{x},t)$$

$$f_{\vec{k}}(\vec{x},t) = \frac{1}{\sqrt{(2\pi)^3 2\omega_k}}\, e^{i(\vec{k}\cdot\vec{x}-\omega_{\vec{k}}t)}$$

Insert a complete set of states.

$a_{in}^{+}(\vec{k})\,|\vec{k}_1,\ldots\vec{k}_m\rangle_{in}$

$$= \underbrace{\sum_{\alpha}|\alpha\rangle\langle\alpha|}_{\text{complete basis}}\, a_{in}^{+}(\vec{k})\,|\vec{k}_1,\ldots\vec{k}_m\rangle_{in}$$

$$= -i\lim_{T\to\infty}\int d^3x\, f_{\vec{k}}(\vec{x},t)\overleftrightarrow{\partial_0}\sum_{a}|a\rangle\langle a|\,\phi(\vec{x},t)|\vec{k}_1,\ldots\vec{k}_m\rangle_{in}\big|_{t=-T(1-i\epsilon)}$$

$$\cancel{= e^{iH_1}} \; \phi(\vec{x},t) = e^{iHt}\,\phi(\vec{x},0)\,e^{-iHt}$$

$$= \frac{-i}{\sqrt{Z}} \lim_{T\to\infty} \int d^3x\, f_k(\vec{x},t) \overleftrightarrow{\partial_0} \sum_\alpha e^{i\left(E_\alpha - \sum_{i=1}^{m} \omega_{\vec{k}_i}\right)t}$$

$$|\alpha\rangle\langle\alpha|\,\phi(\vec{x},0)\,|\vec{k}_1 \ldots \vec{k}_m\rangle_{in}$$

explicite time dependence.

$$= \frac{i}{\sqrt{Z}} \sum_\alpha \lim_{T\to\infty} \int d^3x\, f_{\vec{k}}(\vec{x},t)\, i \left\{ E_\alpha - \sum_{i=1}^{m} \omega_{\vec{k}_i} + \omega_{\vec{k}} \right\}$$

$$e^{i\left(E_\alpha - \sum_{i=1}^{m} \omega_{\vec{k}_i}\right)t}\, |\alpha\rangle\langle\alpha|\,\phi(\vec{x},0)\,|\vec{k}_1 \ldots \vec{k}_m\rangle_{in} \Big|_{t=T(1-i\epsilon)}$$

$$\boxed{f_{\vec{k}}(\vec{x},t) = \frac{e^{i(\vec{k}\cdot\vec{x} - \omega_{\vec{k}} t)}}{\sqrt{(2\pi)^3\, 2\omega_{\vec{k}}}}}$$

we can also extract the $\vec{x}$ dependence.

$$e^{\left(-i\vec{p}(\alpha) + i \sum_i \vec{k}_i\right)\cdot\vec{x}} \langle\alpha|\,\phi(0,0)\,|\vec{k}_1 \ldots \vec{k}_m\rangle$$

$$\phi(\vec{x},0) = e^{-i\vec{p}\cdot\vec{x}}\,\phi(0,0)\,e^{i\vec{p}\cdot\vec{x}}$$

$\alpha \to$ is the momentum eigenstate as well as energy eigenstate

$$= \frac{-i}{\sqrt{Z}} \frac{1}{\sqrt{(2\pi)2\omega_{\vec{k}}}}$$

$$\sum |\alpha\rangle (2\pi)^3 \delta^{(3)}\left(\vec{k} + \sum_i \vec{k}_i - \vec{p}_\alpha\right)$$

$$e^{i\left(E_\alpha - \sum_{i=1}^{m} \omega_{\vec{k}_i} + \omega_{\vec{k}}\right)(-T)(1-i\epsilon)}$$

$$\langle \alpha | \phi(0\,0) | \vec{k}_1 \ldots \vec{k}_m \rangle$$

$$i\left\{E_\alpha - \sum_{i=1}^{m} \omega_{\vec{k}_i} + \omega_{\vec{k}}\right\}$$

$$\langle n | \phi(\vec{x},0) | \rangle$$

$$[P_i, \phi(\vec{x}_i)] = i\partial_i \phi(\vec{x})$$

$$\langle \alpha | [P_i, \phi(\vec{x},0)] | \vec{k}_1 \ldots \vec{k}_m \rangle$$

$$= i \frac{\partial}{\partial x_i} \langle \alpha | \phi(\vec{x},0) | \vec{k}_1 \ldots \vec{k}_m \rangle$$

$$\left(p_{\alpha(i)} - \sum_i k_i\right) \langle \alpha | \phi(\vec{x},0) | \vec{k}_1 \ldots \vec{k}_m \rangle$$

we get a D.E

$$i \frac{\partial}{\partial x_i} \langle \alpha | \phi(\vec{x},0) | \vec{k}_1 \ldots \vec{k}_m \rangle = \left(p_{(\alpha)i} - \sum_i k_i\right) \langle \alpha | \phi(\vec{x},0) | \vec{k}_1 \ldots \vec{k}_m \rangle$$

Only ~~those~~ mom. contributes is ~~the~~ ~~total~~ mom $= k + \sum k_i$

we are trying 2 establish if we take α to be basis of in state, then the state ~~that~~ contributes $(m+1)$ particle. ~~state~~

Exponential factor $e^{(-iT-\epsilon T)\left(E_\alpha - \sum_{i=1}^{m} \omega_{\vec{k}_i} - \omega_{\vec{k}}\right)}$ ~~$e^{(-iT-\epsilon T)(E}$~~

$\to$ contribution goes to 0 if $E_\alpha > \sum_{i=1}^{m} \omega_{\vec{k}_i} + \omega_{\vec{k}}$

$$E_\alpha \leq \sum_{i=1}^{m} \omega_{\vec{k}_i} + \omega_{\vec{k}}.$$

$\vec{P}_{(\alpha)} = \vec{k} + \sum_{i=1}^{m} \vec{k}'_i$ → momentum was fixed completely

Energy is not fixed completely but bounded by $E_\alpha \leq \sum_{i=1}^{m} \omega_{\vec{k}_i} + \omega_{\vec{k}}$.

we took $|\vec{k}_1 \ldots \vec{k}_m\rangle$

& acted on this operator

$$a^\dagger_{in}(\vec{k}) \underbrace{\left[|\vec{k}_1, \ldots, \vec{k}_m\rangle \right]}_{\to\ m \text{ particle states.}}$$

Let us suppose

1 ⊘

2 ⊘ . . . m ⊘ ⊘ $\vec{k}$ → particle is localized → its creating some particle states

~~Como~~ localization means

$$f_{\vec{k}}(\vec{x},t) = \frac{e^{i(\vec{k}\cdot\vec{x} - \omega_{\vec{k}}(t))}}{\sqrt{(2\pi)^3\, 2\omega_{\vec{k}}}} \; e^{-\sigma(\vec{x} - \vec{x}_c(t))^2}$$

we have created some localized disturbance somewhere else

~~linear comb. of in state~~

→ it will create some addn. state (it won't interact)

$= \sum_{s=0}^{\infty} \int d^3\vec{l}_1 \ldots d^3\vec{l}_s \, f(\vec{l}_1, \ldots \vec{l}_s, \vec{k}_1 \ldots \vec{k}_m\rangle_{in}$

→ some $|\vec{l}_1, \vec{l}_2, \ldots \vec{l}_s, \vec{k}_1, \ldots \vec{k}_m\rangle_{in}$

It creates a multi particle state

Convolution:- Instead of taking $|\vec{k}\rangle$ if we multiply it with

$$|\vec{k}_1\rangle = \int d^3k \; e^{-f(\vec{k}'-\vec{k}_1)^2} e^{i\vec{k}.\vec{a}} |\vec{k}'\rangle$$

$\rightarrow$ large sharply peaked around k_1

This has the advantage that it is localized around the position space.

$\rightarrow$ Any state is a linear comb. of in state

The only intermediate states will contribute for which

$$\vec{P}(\alpha) = \vec{K}' + \sum_{i=1}^{m} \vec{k}'_i$$

for $S=0$. $\vec{P}(\alpha) = \sum_{i=1}^{m} \vec{k}'_m$.

$\rightarrow$ since it will violate mom conservation

α to be basis of in state only ~~these~~ in state contributes have

$f_0 = 0$.

$$\vec{P}_\alpha = \vec{K}_0 + \sum_{i=1}^{m} \vec{k}'_i \qquad f_1(\vec{k}_1', \vec{k}_1' \cdots \vec{k}_m) \alpha\, \delta^{(3)}(\vec{k}_1 - \vec{k})$$

f_0

$$f_2(\vec{l_1}, \vec{l_2}, \ldots) \propto \delta^{(3)}(\vec{l_1} + \vec{l_2} - \vec{k})$$

$$\sqrt{l_1^2 + m_p^2} + \sqrt{l_2^2 + m_p^2} \leq \sqrt{(l_1+l_2)^2 + m_p^2}$$

$$\Rightarrow \cancel{l_1^2} + \cancel{m_p^2} + \cancel{l_2^2} + m_p^2$$

$$+ 2\sqrt{l_1^2 + m_p^2}\sqrt{l_2^2 + m_p^2}$$

$$\leq \cancel{l_1^2} + \cancel{l_2^2} + \cancel{m_p^2}$$

$$+ 2 l_1 l_2$$

$2\sqrt{}$ ~~$A + B \leq$~~

$$2 l_1 l_2 \left[1 + \frac{m_p^2}{l_1^2}\right]\left[1 + \frac{m_p^2}{l_2^2}\right] \leq 2 l_1 l_2$$

$$\left(1 + \frac{m_p^2}{l_1}\right)\left(1 + \frac{m_p^2}{l_2^2}\right) \leq 1$$

$$\frac{m_p^2}{l_1^2} + "$$

we can go to a frame where

$$\vec{k} = 0$$

$$2\sqrt{l_1^2 + m_p^2} \leq m_p$$

It can not create 2 particles of the same mass.

Thus impossible to satisfy the constraint for any $s > 1$.

$a_{in}^{+}(\vec{k}) \,|\vec{k}_1, \ldots \vec{k}_m\rangle_{in}$

$= N(\vec{k}, \vec{k}_1, \ldots \vec{k}_m) \,|\vec{k}, \vec{k}_1, \ldots \vec{k}_m\rangle_{in}$

~~$|\vec{k}, \vec{k}_1, \ldots \vec{k}_m\rangle_{in}$~~ . → Normalization factor

Assumption for the normalization factor :-

Assume :- $N(\vec{k}, \vec{k}_1, \ldots \vec{k}_m)$ is independent of $\vec{k}_1 \ldots \vec{k}_m$

but depends on $\vec{k}$.

whatever the does $\vec{k}$

should not affect $\vec{k}_1 \ldots \vec{k}_m$

Similarly whatever happens in $\vec{k}$ is should not be affected by $\vec{k}_1, \ldots \vec{k}_m$.

$a_{in}^{+}(\vec{k}) \,|\vec{k}_1, \ldots \vec{k}_m\rangle_{in}$

$= N(\vec{k}) \,|\vec{k}, \vec{k}_1, \ldots \vec{k}_m\rangle_{in}$,

So we can take $m = 0$ '

$a_{in}^{+}(\vec{k}')\,|\Omega\rangle = N\,|\vec{k}'\rangle_{in}$ → One particle state

for single particle state (non interacting) in state & out state are same.

$$\langle \vec{k}'|\, a_{in}^{+}(\vec{k}')\,|\Omega\rangle$$

$$= N(k)\,\langle \vec{k}'|\vec{k}'\rangle$$

$$= N(\vec{k}')\,\delta^{(3)}(\vec{k}-\vec{k}')$$

l.h.s :-

$$\cancel{a_{in}^{+}(\vec{k}) =} \langle \vec{k}'|\left(\frac{i}{\sqrt{Z}}\right)\int \cancel{d^3x\, f_k(\vec{x},t)\overleftrightarrow{\partial_0}\phi}$$

$$\langle \vec{k}'|\left(\frac{i}{\sqrt{Z}}\right)\int d^3x\, f_{\vec{k}}(\vec{x},t)\,\overleftrightarrow{\partial_0}\,\phi(\vec{x},t)\,|\Omega\rangle$$

$$= \frac{-i}{\sqrt{Z}}\int d^3x\, f_{\vec{k}}(\vec{x},t)\,\overleftrightarrow{\partial_0}\,\langle k'|\,\phi(\vec{x},t)\,|\Omega\rangle$$

$$\rightarrow e^{i(\omega_{\vec{k}'}t - \vec{k}'\cdot\vec{x})}\,\langle \vec{k}'|\,\phi(0,0)\,|\Omega\rangle$$

$$\rightarrow \frac{\sqrt{Z}}{\sqrt{2\omega_{\vec{k}'}}\,(2\pi)^{3/2}}$$

$$= -\frac{i}{\sqrt{(2\pi)^3 2\omega_{\vec{k}}}} \frac{1}{\sqrt{(2\pi)^3 2\omega_{\vec{k}'}}} e^{i(\omega_{\vec{k}} + \omega_{\vec{k}'})} e^{i(\omega_{\vec{k}} - \omega_{\vec{k}'})t} \int d^3x \, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}$$

$$\delta(\vec{k}-\vec{k}') = \delta(\vec{k}-\vec{k}')$$

$$\Rightarrow N(\vec{k}) = 1 = N(\vec{k}')$$

Thus a_{in}^{+} on applying over in state will add $\vec{k}$ ~~for~~ 1 ~~~~ to ~~the~~ n particles.

It ~~breaks down~~ ~~for~~ massless particle. ~~It accounts~~ If we carry it in ~~the~~ same manner than we will get ~~the~~ ~~it b~~ divergence (infrared).

Lecture 26 31/03/2011

$$S(\vec{p}_1, \vec{p}_2, \ldots \vec{p}_n, \vec{k}_1, \ldots \vec{k}_m)$$

$$= {}_{out}\langle \vec{p}_1, \ldots \vec{p}_n | \vec{k}_1, \ldots \vec{k}_m \rangle_{in}$$

$$= \left(\frac{i}{\sqrt{Z}}\right)^{m+n} \int d^4x_1 \ldots d^4x_m \, d^4y_1 \ldots d^4y_n$$

$$f_{\vec{k}_1}(x_1) \ldots f_{\vec{k}_m}(x_m) \, f^*_{\vec{p}_1}(y_1) \ldots f^*_{\vec{p}_n}(y_n)$$

$$(-\Box_{x_1} + m_p^2) \ldots (-\Box_{x_m} + m_p^2)(-\Box_{y_1} + m_p^2)$$

$$\ldots (-\Box_{y_n} + m_p^2)$$

$$G^{(m+n)}(x_1, \ldots x_m, y_1, \ldots y_n)$$

$$\langle \Omega | T \, \phi(x_1) \ldots \phi(x_m) \, \phi(y_1) \ldots \phi(y_n) | \Omega \rangle$$

Define

$$\tilde{G}^{(s)}(k_1, \ldots k_s) = \int d^4x_1 \ldots d^4x_s \prod_{l=1}^{s} e^{-ik_l \cdot x_l} \, G^{(s)}(x_1, \ldots x_s)$$

$$G^{(s)}(x_1, \ldots x_s) = \int \frac{d^4k_1}{(2\pi)^4} \ldots \frac{d^4k_s}{(2\pi)^4} \prod_{l=1}^{s} e^{ik_l \cdot x_l} \, \tilde{G}^{(s)}(k_1, \ldots k_s)$$

$$\left(\frac{i}{\sqrt{Z}}\right)^{m+n} \int d^4x_1 \ldots d^4x_m$$

$$f_{\vec{k}_1}(x_1) \ldots f_{\vec{k}_m}(x_m) \, f^*_{\vec{p}_1}(y_1) \ldots f^*_{\vec{p}_n}(y_n)$$

$$\int \frac{d^4 k_1'}{(2\pi)^4} \cdots \frac{d^4 k_m'}{(2\pi)^4} \frac{d^4 p_1'}{(2\pi)^4} \cdots \frac{d^4 p_n'}{(2\pi)^4}$$

$$\left[\overline{m(k') \, \& \, n(p')}\right] \prod_{i=1}^{m} \left(k_i'^2 + m_p^2\right) \prod_{i=1}^{n} \left(p_i'^2 + m_p^2\right)$$

$$\prod_{i=1}^{m} e^{i k_i' x_i} \prod_{i=1}^{n} e^{i p_i' y_i} \, \hat{G}^{(m+n)} \begin{pmatrix} k_1', \ldots k_m' \\ p_1' \ldots p_n' \end{pmatrix}$$

x_1 integral $\Rightarrow (2\pi)^4 \, \delta^4(k_1 + k_1')$

$$\boxed{f_{\vec{k}}(\vec{x}, t) = \frac{1}{\sqrt{(2\pi)^3 2\omega_{\vec{k}}}} e^{i k \cdot x}, \quad k^0 = \omega_{\vec{k}}}$$

~~wherever~~ k_1' ~~appears~~

~~we~~ ~~set~~ it $-k_1$,

$$S(\vec{p}_1, \ldots \vec{p}_n; \vec{k}_1, \ldots \vec{k}_m),$$

$$= \left(\frac{i}{\sqrt{Z}}\right)^{m+n} \hat{G}^{(m+n)}(-k_1, \ldots -k_m, p_1, \ldots p_n)\Big|_{\substack{k_i^0 = \omega_{\vec{k}_i} \\ p_i^0 = \omega_{\vec{p}_i}}}$$

$$\prod_{i=1}^{m} \frac{1}{\sqrt{(2\pi)^3 2\omega_{\vec{k}_i}}} \prod_{i=1}^{n} \frac{1}{\sqrt{(2\pi)^3 2\omega_{\vec{p}_i}}}$$

$$\prod_{i=1}^{m} \left(k_i^2 + m_p^2\right) \prod_{i=1}^{n} \left(p_i^2 + m_p^2\right).$$

$$\longrightarrow k_i^2 = -\omega_k^2 + k^2 + m_p^2 - m_p^2 = 0.$$

We will cancel the poles of $\hat{G}^{(m+n)}(-k_1, \ldots -k_m, p_1, \ldots p_n)$.

~~Thus~~ at this stage we won't simplify it.

There is no integral.

$$S\left(\vec{p}_1,\ldots\vec{p}_n;\vec{k}_1,\ldots\vec{k}_m\right).$$

$$= \left[\left(\frac{i}{\sqrt{Z}}\right)^{m+n}\tilde{G}^{(m+n)}\left(-k_1,\ldots -k_m, p_1,\ldots p_n\right)\right.$$

$$\prod_{i=1}^{m}\frac{1}{\sqrt{(2\pi)^3 2\omega_{k_i}}}\prod_{i=1}^{n}\frac{1}{\sqrt{(2\pi)^3 2\omega_{\vec{p}_i}}}$$

$$\prod_{i=1}^{m}\left(k_i^2+m_p^2\right)\prod_{i=1}^{n}\left(p_i^2+m_p^2\right)$$

General structure of Feynman rules for $G^{(n)}(x_1,\ldots x_n) \longrightarrow$ coordinates

Let us take ϕ^4.

$\sum_V$ (sum of vertices, how many time we have bought power of λ)

$$\sum_V \frac{1}{V!}\left(-\frac{i\lambda}{4!}\right)^V \prod_{\alpha=1}^{V} d^4y_\alpha \longrightarrow \text{vertex coordinates}$$

$$\prod_{\beta=1}^{P}\Delta_F\left(z^{(1)}_\beta, z^{(2)}_\beta\right) \longrightarrow \text{coordinates of the two ends of the propagator}$$

$\rightarrow$ no. of propagators

Each $Z^{(i)}_{\beta}$ is either = some y_a
or some x_i

$\Rightarrow$ either comes from vertex

$$\Delta_F\left(Z_\beta^{(1)}\, Z_\beta^{(2)}\right) = \int \frac{d^4 k'_{(\beta)}}{(2\pi)^4} \frac{e^{i k'_\beta \left(Z_\beta^{(1)} - Z_\beta^{(2)}\right)}}{-k'^2_{(\beta)} - m^2 + i\epsilon}$$

Let us try to look at y_a.

arrow will be

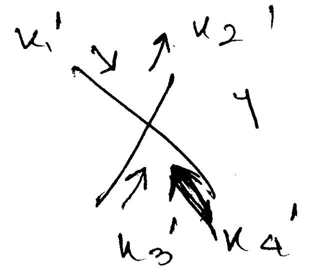
$$\overset{\xrightarrow{k'_{(\beta)}}}{Z^{(2)}_\beta \text{———} Z^{(1)}_\beta}$$

such that it goes from 2 to 1.

If $k'_{(\beta)}$ leaves a vertex Z_β it gives a factor of $e^{-ik'_{(\beta)}Z}$

if $k'_{(\beta)}$ enters a vertex Z it gives $e^{ik'_\beta Z}$.

For every propagator there came a momentum



$$e^{iy(k_1' + k_3' - k_2' + k_4')}$$

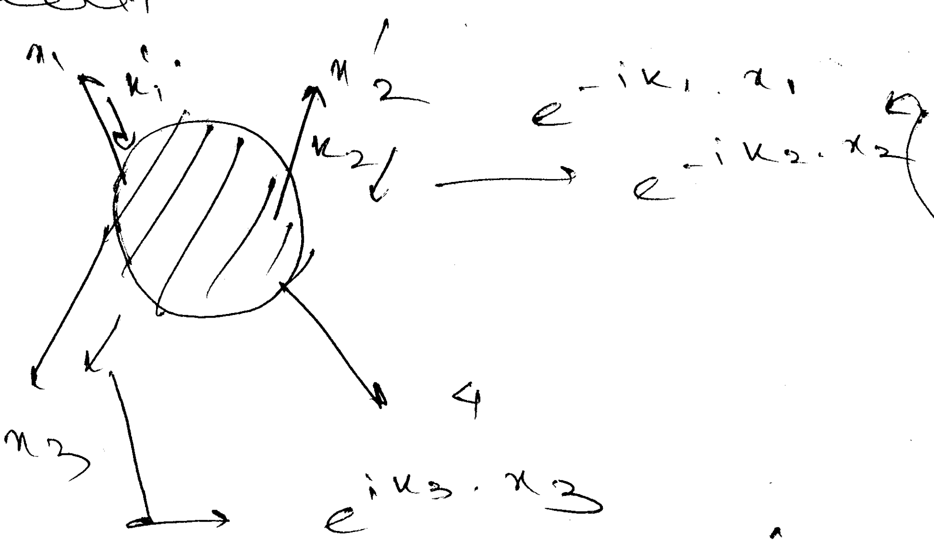
$\longrightarrow$ complete y dependence

$$\int d^4y \, e^{iy(k_1' + k_3' - k_2' + k_4')}$$

$$= (2\pi)^4 \, \delta^{(4)}(k_1' + k_3' - k_2' + k_4')$$

momentum conservation at every ~~the~~ vertex.

Now let us consider external vertices



$$\tilde{G}^{(n)}(l_1, l_2, \ldots l_n)$$

$$= \int d^4x_1 \ldots d^4x_n \, e^{-il_1 x_1} \, e^{-il_n x_n} \, G^{(n)}(x_1, \ldots x_n)$$

Let us ~~keep~~ substitute in

$$G^{(n)}(x_1, \ldots x_n)$$

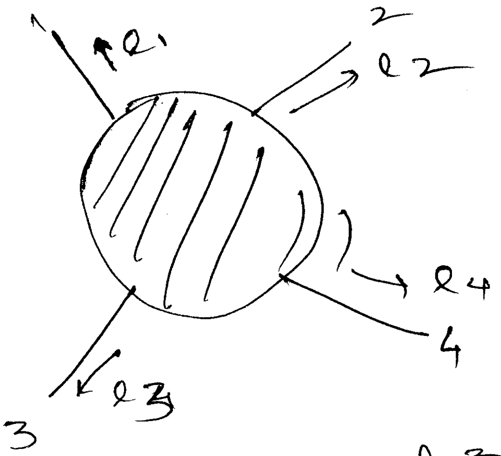
$$= (2\pi)^4 \, \delta^{(4)}(k_1 + l_1) \ldots$$

First we will do x_1 integral

Each propagator by fourier transform → replace

Let $k_1 = -l_1$

In terms of feynman diagram



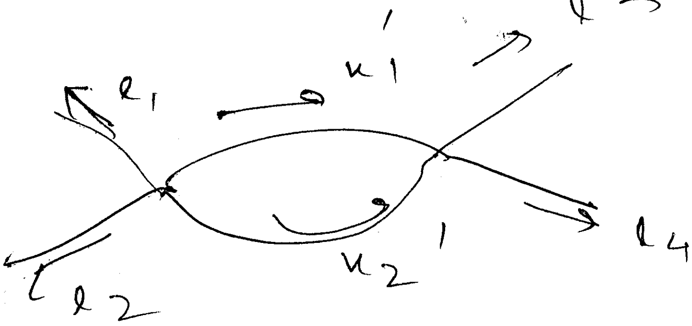
As l_1 is entering

$$\int \frac{d^4k_1'}{(2\pi)^4}\frac{d^4k_2'}{(2\pi)^4}(2\pi)^4(2\pi)^4\,\delta^4(-l_1-l_2-k_1'-k_2')\,\delta^4(k_1'+k_2'-l_3-l_4)$$

$$\downarrow$$

$$(2\pi)^4\delta^4(-l_1-l_2-l_3-l_4)$$

→ overall momentum conservation



At each vertex mom. is conserved, so total momentum adds up to 0

Because translation symmetry is associated with mom. conservation

Ex:- prove that

$$G^{(n)}(x_1, \ldots x_n) = G^{(n)}(x_1+a \ldots x_n+a)$$

$$\tilde{G}(k_1, \ldots k_n) \propto \delta^{(4)}(k_1+k_2 \ldots k_n)$$

If λ depends upon x, then integration won't give me δ func
- no conservation of mom.

(1)

Feynman rules for computing $\hat{G}^n(k_1, \ldots k_n)$.

i) Draw all Feynman diagrams with propagator labelled by momentum k_i'.

ii) On external vertex, choose $k_i' = k_i$ entering the vertex.

iii) For each vertex include $\left(\frac{-i\lambda}{4!}\right)(2\pi)^4$

$$= \left(\frac{-i\lambda}{4!}\right)(2\pi)^4 \; \delta^{(4)}\left(\sum \text{momentum entering it}\right)$$

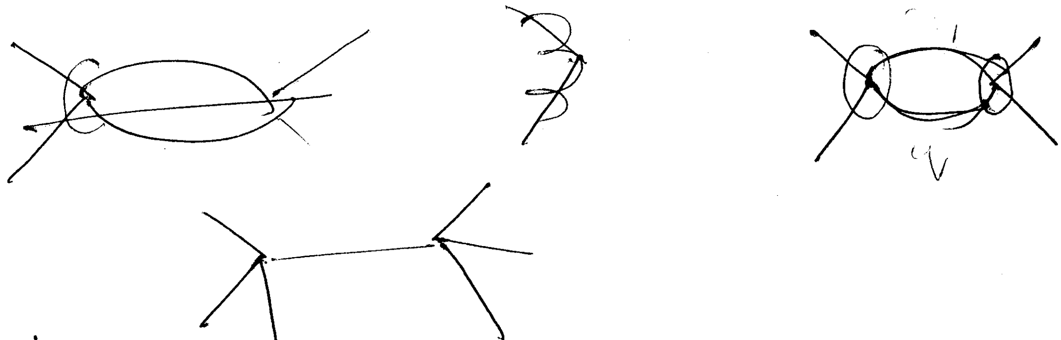
$\delta^{(4)}(\sum \text{mom})$

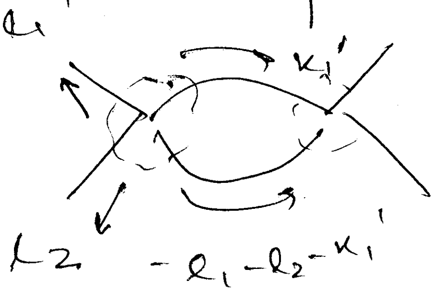
iv) For each propagator of momentum k_i' include a factor

$$\frac{i}{-k_i'^2 - m^2 + i\epsilon}$$

v) Integrate $\int \frac{d^4 k_i'}{(2\pi)^4}$ for each propagator except the ones attached to external vertices

6) multiply by combinatorial factors & $\frac{1}{V!}$ if the vertices.





Short distance divergence in the ~~position~~ space will appear large mom. distribution

Lecture 27. 01/04/2011

Momentum space Feynman rules

$$\hat{G}^{(n)}(k_1,\ldots,k_n) = \int \prod_{i=1}^{n}\left(d^4x_i\, e^{-ik_i\cdot x_i}\right) G^{(n)}(x_1,\ldots x_n)$$

1) For every external propagator connected to the i^{th} external vertex, set the momentum entering the external vertex to be k_i & include a factor of

$$\frac{i}{k_i^2 - m^2 + i\epsilon}$$

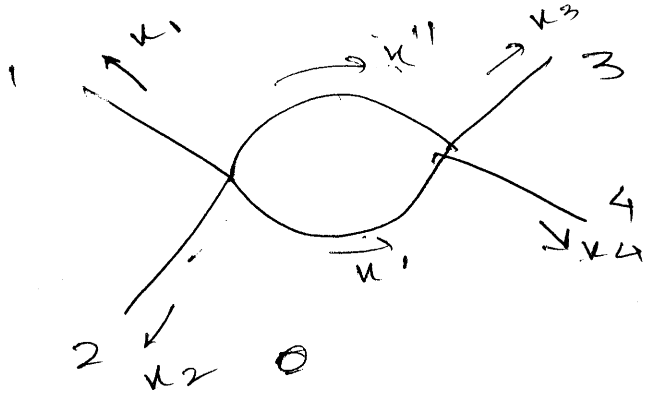
2) For every internal propagator ~~connected to the~~ carrying momentum k' include an integral $\int \frac{d^4k'}{(2\pi)^4}$ & a factor of $\frac{i}{k'^2 - m^2 + i\epsilon}$ in the integrand.

3) For every internal vertex, include a factor of $(2\pi)^4 \delta^{(4)}$ (Total momentum entering the vertex).

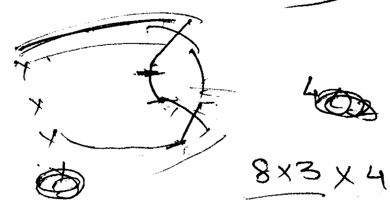
[Internal vertex comes from

expanding the interaction Hamiltonian)

4) Put in the combinatorial & vertex factor.



$\tilde{G}^{(4)}(k_1, k_2, k_3, k_4)$



$\frac{8\times3\times4\times3}{\times 2}$

$$\frac{i}{-k_1^2-m^2+i\epsilon}\,\frac{i}{-k_2^2-m^2+i\epsilon}\,\frac{i}{-k_3^2-m^2+i\epsilon}\,\frac{i}{-k_4^2-m^2+i\epsilon}$$

$$\int \frac{d^4k''}{(2\pi)^4}\,\frac{i}{-k''^2-m^2+i\epsilon}\,\frac{d^4k'}{(2\pi)^4}\,\frac{i}{-(k')^2-m^2+i\epsilon}$$

$$(2\pi)^4\delta(-k_1-k_2-k'-k'')\,.$$

$$(2\pi)^4(-k_3-k_4+k'+k'')\,.$$

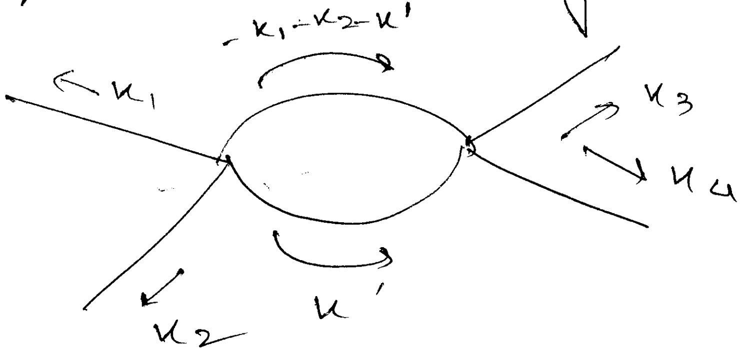
$$\frac{1}{2!}\left(\frac{-i\lambda}{4!}\right)^2\times 8\times3\times4\times3\times2\,.$$

$\to \delta^4(-k_1-k_2-k_3-k_4)$

overall mom. conservation.

$$\to \frac{i}{-(k_1+k_2+k')^2-m^2+i\epsilon}\,.$$

Another way



If

~~$-k_1-k_2$~~

we will write only those mom. which are not fixed.

$$\frac{i}{-k_1^2-m^2+i\epsilon}\,\frac{i}{-k_2^2-m^2+i\epsilon}\,\frac{i}{-k_3^2-m^2+i\epsilon}\,\frac{i}{-k_4^2-m^2+i\epsilon}$$

$$\int\frac{d^4k'}{(2\pi)^4}\,\frac{i}{-k'^2-m^2+i\epsilon}\,\frac{i}{-(-k_1-k_2-k')^2-m^2+i\epsilon}$$

$$\frac{1}{2!}\left(\frac{-i\lambda}{4!}\right)^2 8\times 3\times 4\times 3\times 2\,(2\pi)^4\delta^{(4)}(k_1+k_2+k_3+k_4)$$

when we have a loop.
One mom. is arbitrary
Divergence comes only from
loop integral.

If k is 3 mom., then in perturbation theory we have to sum over ~~mom~~ all ~~possible~~ states. ($\Rightarrow$ all possible.

energy)).

ϕ^3 theory

$$S = \int d^4x \left[-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{3!} :\phi^3: \right]$$

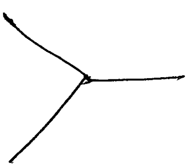
not a good Q. theory.

$$H = \int d^3x \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 + \frac{(\nabla\phi)^2}{2} + \frac{g}{3!}\phi^3 \right)$$

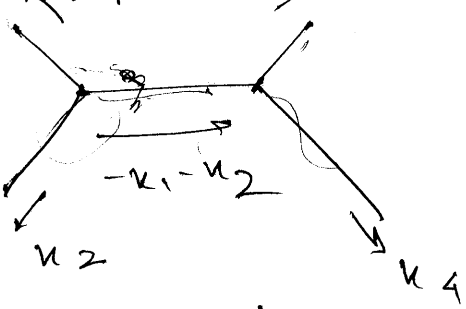
when ϕ goes to $-\infty$.

$g \longrightarrow \quad H \rightarrow -\infty$.

not bounded from below.



$\left(\frac{-ig}{3!} \right)$ contribution to $\tilde{G}^{(4)}(k_1, \dots k_4)$

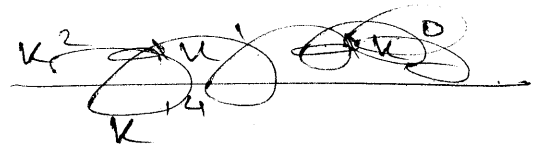


$$\frac{i}{-k_1^2 - m^2 + i\epsilon} \; \frac{i}{-k_2^2 - m^2 + i\epsilon} \; \frac{i}{-k_3^2 - m^2 + i\epsilon} \; \frac{i}{-k_4^2 - m^2 + i\epsilon}$$

$$\frac{1}{2!} \; \frac{i}{-(k_1+k_2)^2 - m^2 + i\epsilon} \; (2\pi)^4 (k_1+k_2+k_3+k_4)$$

$$\left(\frac{-ig}{3!} \right)^2 \times 72.$$

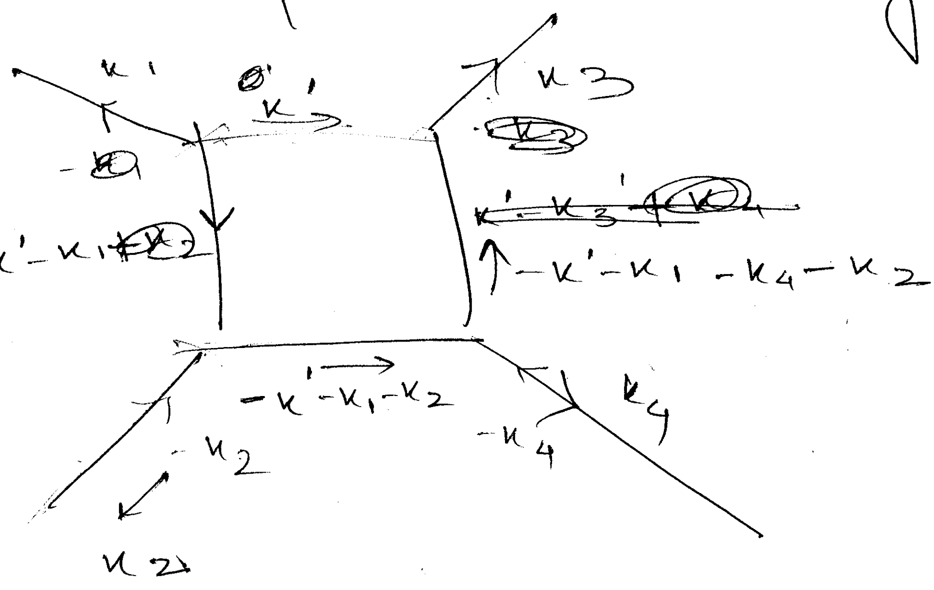
not divergent.



~~no~~ As long as no. of integral is more or equal

- ringed

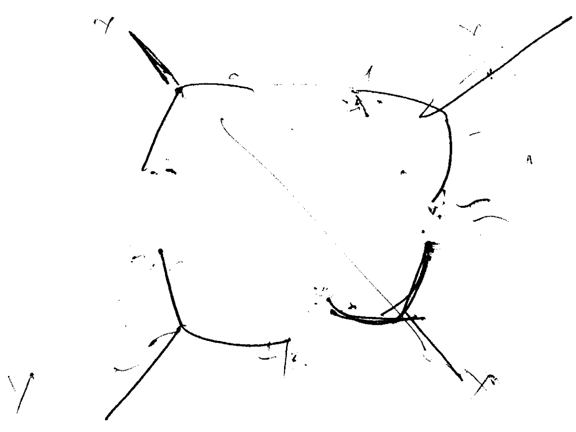
In ϕ^3 theory



$k'-k'-k_1-k_4-k_2$
$-k_3$
$=0$

$k'-k_3$

12



$12\times6\times3\times3$

$12\times9\times3\times3$

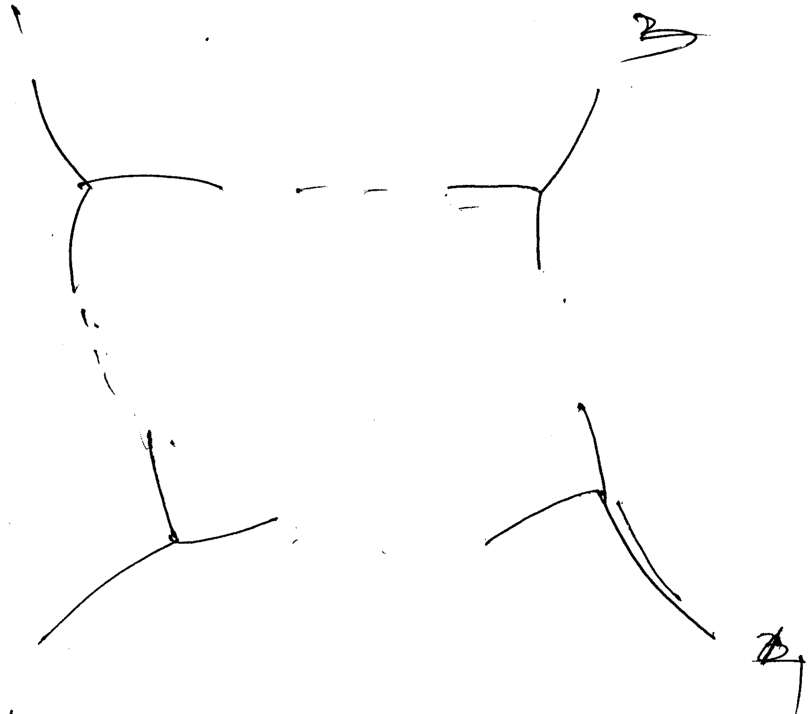
~~$12\times\ \times3\times3$~~

$6\times4\times2$

$(2\pi)^4 \, \delta^{(4)}(k_1 + k_2 + k_3 + k_4)$

$$\int \frac{d^4k'}{(2\pi)^4} \left(\prod_{j=1}^{4} \frac{i}{-k_j^2 - m^2 + i\epsilon} \right)$$

$$\frac{i}{-k'^2 - m^2 + i\epsilon} \; \frac{i}{-(k_1 + k')^2 - m^2 + i\epsilon} \; \frac{i}{-(k_1 + k_2 + k')^2 - m^2 + i\epsilon} \; \frac{i}{-(k_1 + k_2 + k_4 + k')^2 - m^2 + i\epsilon}$$

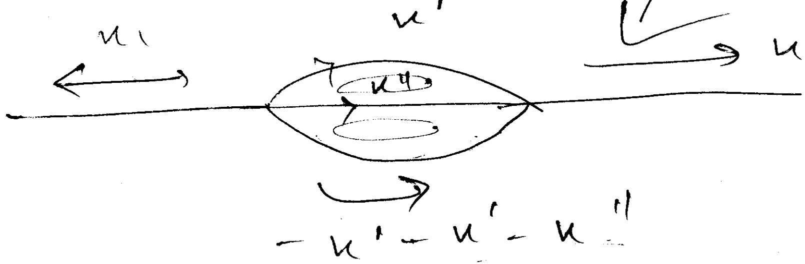


$$\frac{12 \times 9 \times 6 \times 4 \times 2}{\times 2}$$

$$\frac{i}{4!} \left(\frac{-ig}{3!} \right)^4$$

$8 \times 4 \times 3 \times 2.$

not divergent $\quad \frac{d^4k'}{k'^8}$



$$\to k\,2 \int \frac{1}{(2\pi)^4} \frac{d^4k' \, d^4k''}{-k'^2 - m^2 + i\epsilon} \; \frac{i}{(k'')^2 - m^2 + i\epsilon} \; \frac{i}{-(k_1 + k' + k'')^2 + i\epsilon - m^2}$$

$S = 1 + iT$

$$T(p_1, \ldots p_n; k_1, \ldots k_m) = (2\pi)^4 \delta^{(4)}\left(\sum p_i - \sum k_i\right) M\begin{pmatrix} p_1, \ldots p_n, \\ k_1, \ldots k_m \end{pmatrix}$$

comes from the Green's fn.

$$S(p_1, \ldots p_n; k_1, \ldots k_m) = \left(\frac{i}{\sqrt{2}}\right)^{m+n} \prod_{i=1}^{m} \frac{1}{\sqrt{(2\pi)^3 2\omega_{\vec{k}_i}}} \prod_{i=1}^{n} \frac{1}{\sqrt{(2\pi)^3 2\omega_{\vec{p}_i}}}$$

$$\prod_{i=1}^{n} (k_i^2 + m_p^2) \prod_{i=1}^{n} (p_i^2 + m_p^2) \; \hat{G}(p_1, \ldots p_n; -k_1, \ldots -k_m)$$

~~they~~ vanish when we evaluate $p_i^0 = \omega_{p_i}$ & $k_i^0 = \omega_{k_i}$

As long as $p_i \neq k_i$

S is same as iT,

~~To get~~ to calculate T

we have to multiply

S by $-i$

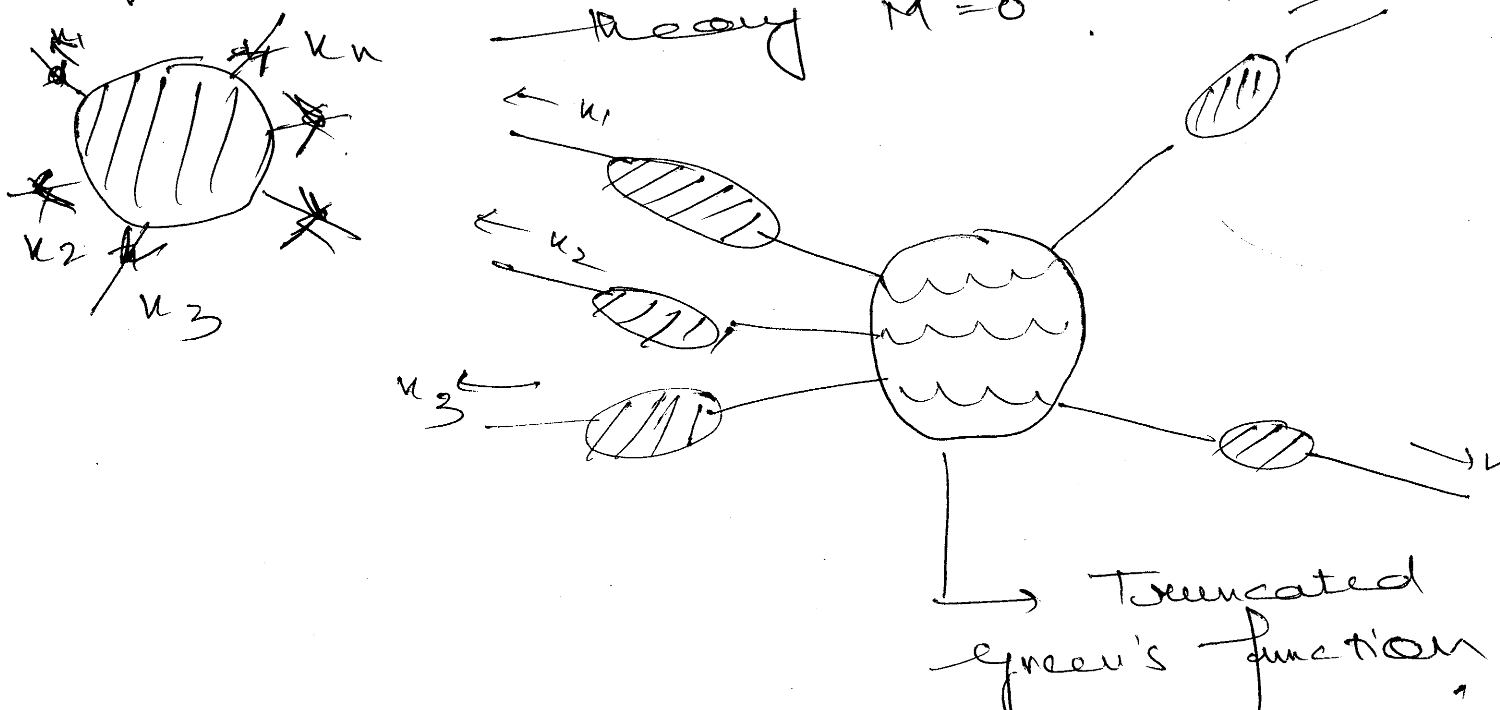
& To calculate M we

(1) Have to multiply the expression by $-i$

(2) Remove the $(2\pi)^4 \delta^{(4)}(\Sigma p_i - \Sigma k_i)$ factor from $\tilde{G}^{(n)}$.

These factors come out from the poles.

For free field theory $M = 0$.

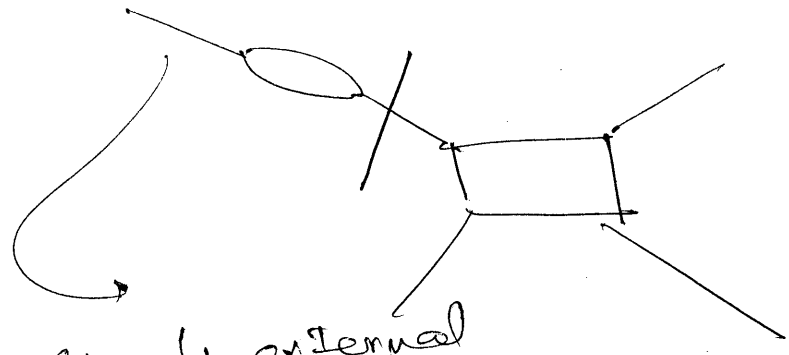


Truncated Greens function

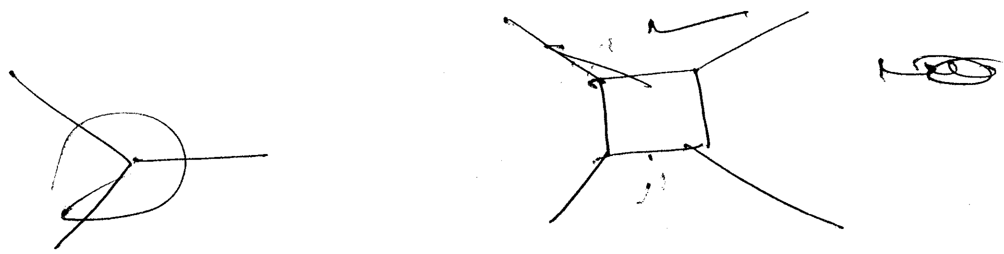
Sum of those graphs which cannot be divided into two parts, with one part containing a single external line, (by keeping a single internal line), / product of external propagators.

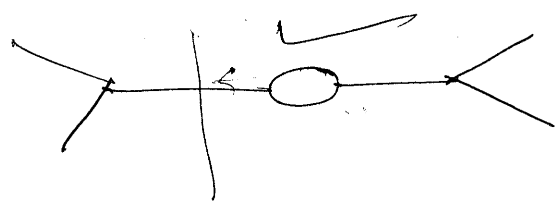
ϕ^3

× not included.

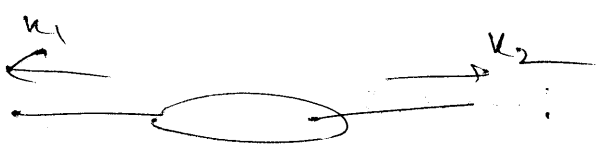


Single external lines





We have to include the graphs similar to above one & remove the external leg contribution.

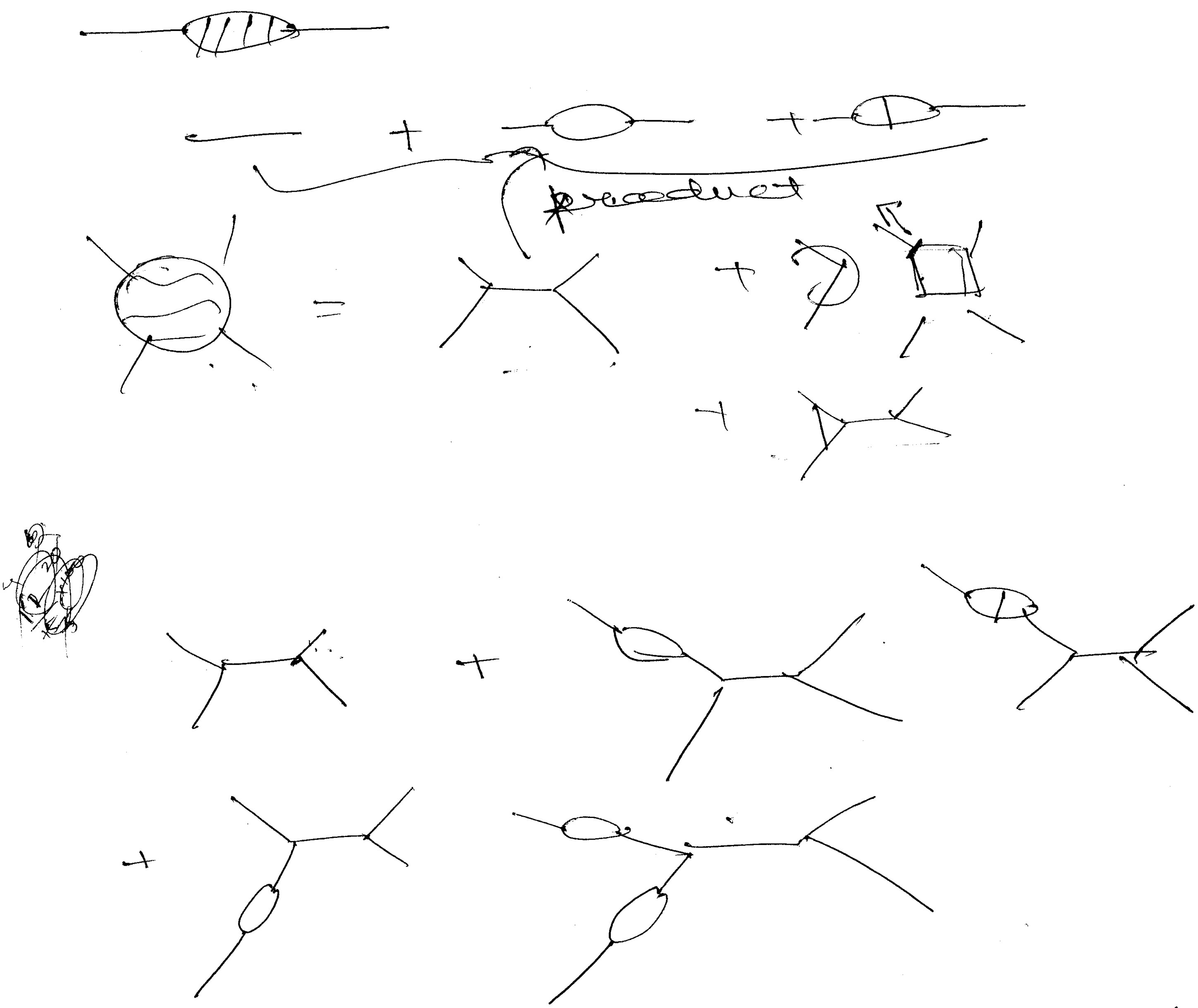


: Two point function,

$$\tilde{G}^{(2)}(k_1, k_2)$$

General graphs ≡ Truncated green's func × two pt. func.

In ϕ^3



We are getting product of all these ~~two~~ pt. func.

Full two pt. func. $k^2 = -m_p^2$.

$\prod (k_i^2 + m_p^2) \longrightarrow$ Two pt. func has a pole.

For S matrix calculation

we can replace two pt. func

by $\frac{2}{-k_i^2 - m_p^2 + i\epsilon}$

Combinatorials factor will match ?!.

Lecture 28 - 06/04/2011

Review

$$\mathcal{M}(\vec{p}_1, \ldots \vec{p}_n, \vec{k}_1, \ldots, \vec{k}_m)$$

• ~~moment~~

$$= \left(\frac{i}{\sqrt{Z}}\right)^{m+n} \left[\prod_{i=1}^{m} (k_i^2 + m_p^2) \prod_{i=1}^{n} (p_i^2 + m_p^2) \; \tilde{G}^{(m+n)}(-k_1, \ldots -k_m, p_1, \ldots p_n) \right]_{\substack{k_i^0 = \omega_{\vec{k}_i} \\ p_i^0 = \omega_{\vec{p}_i}}}$$

In the S-matrix: $(2\pi)^4 \delta^{(4)}(\Sigma p_i - \Sigma k_i)$

$$\prod_{i=1}^{n} \frac{1}{\sqrt{(2\pi)^3 \, 2\omega_{p_i}}} \prod_{i=1}^{m} \frac{1}{\sqrt{(2\pi)^3 \, 2\omega_{\vec{k}_i}}}$$

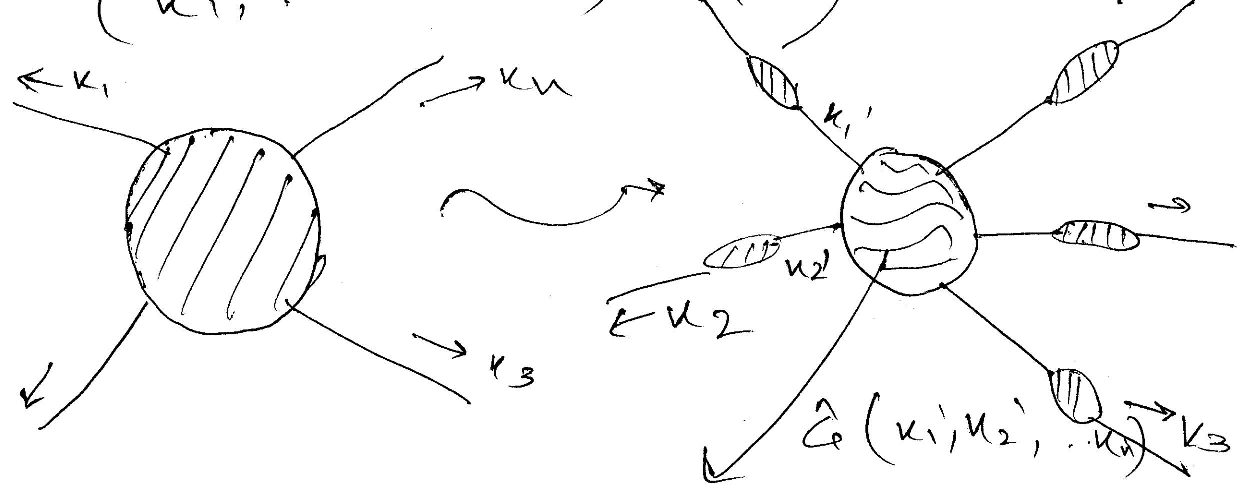
$$S = 1 + iT$$

T has precisely this factor.

$$T = \mathcal{M} \times (\ldots)$$

two pt. fn.

$$\tilde{G}^{(n)}(k_1, \ldots, k_n)$$



has the prop.

~~that the~~
~~we~~ include those graph which on cutting one internal ~~leg~~ won't give us an external line

$$\hat{G}(k_1, \ldots k_n)$$

$$= \int \frac{d^4 k_1'}{(2\pi)^4} \cdots \frac{d^4 k_n'}{(2\pi)^4} \hat{G}^{(2)}(k_1, -k_1')\, \hat{G}^{(2)}(k_2, -k_2') \cdots \hat{G}^{(2)}(k_n, -k_n')\, \hat{G}(k_1', \ldots k_n')$$

~~now the~~ func. of k near ~~the physical~~ mass $(k^2 = m_p^2)$

$$\hat{G}^{(2)}(k, k') = (2\pi)^4 \delta^{(4)}(k + k') \left[\frac{iZ}{-k'^2 - m_p^2} + \right].$$

this terms won't contribute as ~~$k^2 + m_p^2 - 0$~~ in ~~the~~ limit $(k^2 + m_p^2 - 0)$ ~~they~~ vanish.

$$= \prod_{i=1}^{n} \left\{ \frac{iZ}{-k_i^2 - m_p^2} + \ldots \right\} \hat{G}^{(n)}(k_1, \ldots k_n)$$

$iM(\vec{p}_1, \dots \vec{p}_n, \vec{k}_1, \dots \vec{k}_m)$

$= \left(\frac{i}{\sqrt{Z}}\right)^{m+n} \left\{ \prod_{i=1}^{m} (k_i^2 + m_p^2) \right\} \left\{ \prod_{i=1}^{n} (p_i^2 + m_p^2) \right\}$

$\prod_{i=1}^{m} \left\{ \frac{iZ}{-k_i^2 - m_p^2} + \dots \right\} \prod_{i=1}^{n} \left\{ \frac{iZ}{-p_i^2 - m_p^2} + \dots \right\}$

$\hat{G}^{(n)}(-k_1, \dots, -k_m, p_1, \dots p_n) \Big|_{k_i^0 = \omega_{\vec{k}_i}}$

$iM(\vec{p}_1, \dots \vec{p}_n, \vec{k}_1, \dots \vec{k}_m)$, $\quad p_i^0 = \omega_{\vec{p}_i}$

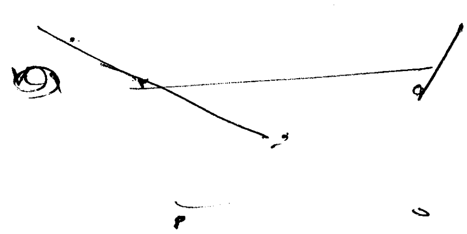
$= (\sqrt{Z})^{m+n} \, \hat{G}^{(n)}(-k_1, \dots -k_m, p_1, \dots k_n)$

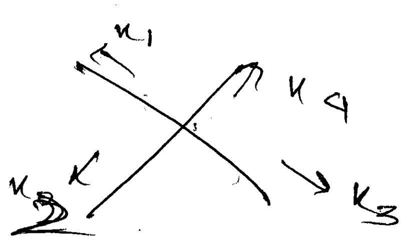
$k_i^0 = \omega_{\vec{k}_i}$,
$p_i^0 = \omega_{\vec{p}_i}$

Calculate

$\hat{G}^{(4)}$ $\qquad \hat{G}(k_1, k_2, k_3, k_4)$

in ϕ^4 theory





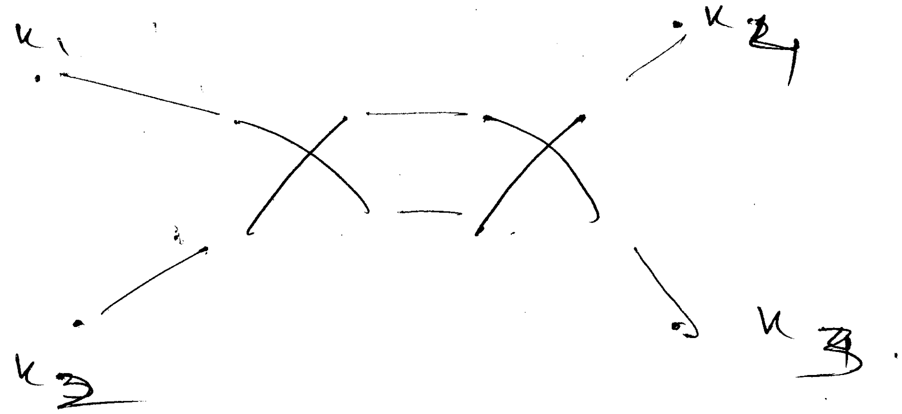
$\frac{-i\lambda}{4!} \times 4!$

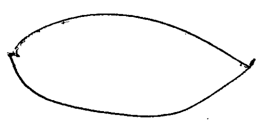
exclude the external

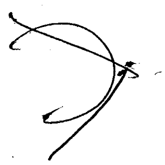
$= -i\lambda$

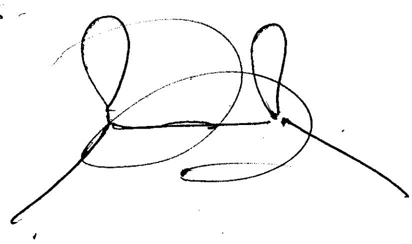
Calculate Diff c.s

$$\frac{d\sigma}{d\Omega} = (\cdot\,\cdot)\,|M|^2$$

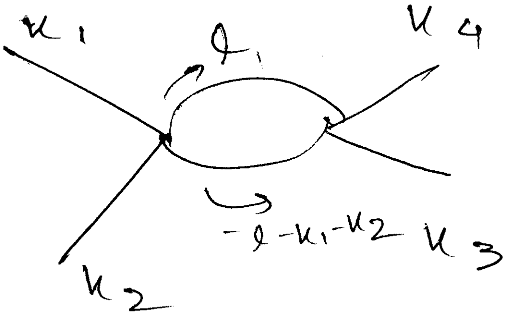


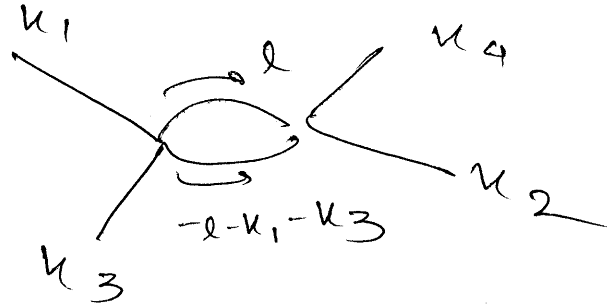


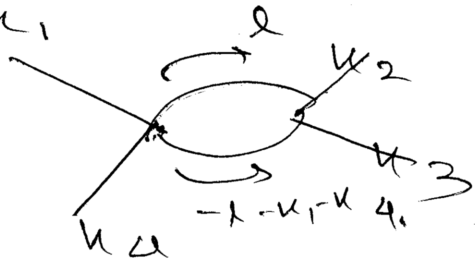




To order λ^2







8 $\qquad 8 \times 3 \times 4 \times 3 \times 2$

$$\int \frac{d^4 l}{(2\pi)^4} \times \frac{i}{l^2 - m^2 + i\epsilon} \cdot \frac{i}{(l+k_1+k_2)^2 - m^2 + i\epsilon}$$

$$\left(\frac{-i\lambda}{4!}\right)^2 \frac{1}{2!} \times 8 \times 3 \times 4 \times 3 \times 2$$

+ two more terms.

We didn't rewrite the external propagator factor.