

Any function = Even function + Odd function

Examples :-

1) θ_1

is an odd function

(one non-zero coeff. & is odd)

2) $\theta_1 \theta_2$

is an even function

(one non-zero coeff. & is even)

2) $\sin \theta_1 = \theta_1 - \frac{\theta_1^3}{3!} + \frac{\theta_1^5}{5!} - \dots$

$= \theta_1$

\rightarrow

odd fn

$\because \theta_1^2 = 0$

first pretend it is bosonic & then consider the fact that θ_1 is a grassmann variable

3) $e^{i\theta_1} = 1 + i\theta_1 - \frac{1}{2}\theta_1^2 + \dots$

$= 1 + i\theta_1$

first expand in usual Taylor series expr.

Differentiation :- (gt is just a formal operation)

① $\frac{\partial}{\partial \theta_i} \theta_j = \delta_{ij}$

② $\frac{\partial}{\partial \theta_i} (F G) = \frac{\partial F}{\partial \theta_i} G + (-1)^F F \frac{\partial G}{\partial \theta_i}$

\downarrow
 $= \begin{cases} 1 & \text{if } F \text{ is even} \\ -1 & \text{if } F \text{ is odd} \end{cases}$

Example :-

① $\frac{\partial}{\partial \theta_1} (\theta_1 \theta_2) = \theta_2 - (-1)\theta_2(1) = \theta_2$

this is not really $(-1)^F$ raised to the power F

② $\frac{\partial}{\partial \theta_1} (\theta_2 \theta_1) = (0)\theta_1 + (-1)\theta_2(1) = -\theta_2$

2 ways

$\frac{\partial}{\partial \theta_1} (\theta_2 \theta_1) = \frac{\partial \theta_2}{\partial \theta_1} \theta_1 - \theta_2 \frac{\partial \theta_1}{\partial \theta_1} = -\theta_2$
 $\frac{\partial}{\partial \theta_1} (-\theta_1 \theta_2) = -\theta_2$

Results

① $\frac{\partial}{\partial \theta_i} \frac{\partial}{\partial \theta_j} F = - \frac{\partial}{\partial \theta_j} \frac{\partial}{\partial \theta_i} F$ for any F .

Proof :-

Define $\vec{\theta}' = (\theta_1, \dots, \theta_i, \dots, \theta_j, \dots, \theta_n)$

$$F = A(\vec{\theta}') + \theta_i B(\vec{\theta}') + \theta_j C(\vec{\theta}') + \theta_i \theta_j D(\vec{\theta}')$$

$$\therefore \frac{\partial}{\partial \theta_i} \frac{\partial}{\partial \theta_j} F$$

$$= -D(\vec{\theta}')$$

$$\& \frac{\partial}{\partial \theta_j} \frac{\partial}{\partial \theta_i} F = D(\vec{\theta}')$$

Doing a Taylor series expr. only in θ_i & θ_j 's but of course the rest of the dependence will be there

→ This shows that

$$\frac{\partial}{\partial \theta_i} \frac{\partial}{\partial \theta_j} F = - \frac{\partial}{\partial \theta_j} \frac{\partial}{\partial \theta_i} F$$

$$\Rightarrow \frac{\partial^2}{\partial \theta_i^2} F = 0$$

Simply bcs Taylor series expr. can't contain more than one power of θ

② Consider two functions F & G .
Then $FG = GF$ if either F or G is even.

(If either of them is even, you can move through picking up a '-' sign) → every term of F through " " " a or vice versa

