

8/9/06

Non-abelian gauge theories

Draw analogy with QED.

Recall :- QED action is

$$S = \int d^4x \mathcal{L}$$

where $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi} \{ i\gamma^\mu (\partial_\mu - ieA_\mu) - m \} \Psi$

such that $\partial_\mu = \frac{\partial}{\partial x^\mu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Dirac indices aren't written explicitly

Symmetry of \mathcal{L}

① Global symmetry :-

$$\Psi(x) \rightarrow e^{ie\lambda} \Psi(x), \quad A_\mu(x) \rightarrow A_\mu(x)$$

↓
constant

This is valid even in the absence of gauge fields A_μ & their coupling to Ψ .

② Local symmetry / gauge symmetry :-

$$\Psi(x) \rightarrow e^{ie\lambda(x)} \Psi(x), \quad A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \lambda(x)$$

Here gauge field is essential to make \mathcal{L} invariant

→ gauge fields are essential.
Not a symmetry of free fermion action.
But it is a symmetry of the

Maxwell action without fermions.

We will be led to postulating the presence of gauge fields if we start with free fermionic action & then take $\lambda(x)$ & demand it to be a sym. of Z

Define Covariant derivative :-

$$D_\mu \Psi = \partial_\mu \Psi - ie A_\mu \Psi$$

(Reason for defining this \rightarrow)

Under local gauge transformation

$$D_\mu \Psi \rightarrow e^{ie\lambda(x)} D_\mu \Psi$$

Proof :-

$$\begin{aligned} D_\mu \Psi &\rightarrow \partial_\mu (e^{ie\lambda(x)} \Psi) - ie (A_\mu + \partial_\mu \lambda) (e^{ie\lambda(x)} \Psi) \\ &= e^{ie\lambda(x)} \partial_\mu \Psi + ie \partial_\mu \lambda(x) e^{ie\lambda(x)} \Psi - ie A_\mu e^{ie\lambda(x)} \Psi - ie \partial_\mu \lambda(x) e^{ie\lambda(x)} \Psi \\ &= e^{ie\lambda(x)} (\partial_\mu \Psi - ie A_\mu \Psi) \end{aligned}$$

Though $\partial_\mu \Psi$ transforms in a complicated way, $D_\mu \Psi$ transforms in a nice fashion - that's why it is called the covariant deriv.

Note \rightarrow For $e^{ie\lambda(x)}$ we have $(\partial_\mu - ie A_\mu) e^{ie\lambda(x)} = e^{ie\lambda(x)} (\partial_\mu \lambda - ie A_\mu)$
 $(ie) \partial_\mu \lambda = (\partial_\mu \lambda - ie A_\mu) + ie A_\mu$
 \downarrow sym. in μ, ν
 $\therefore F_{\mu\nu} = 0$

$$\begin{aligned} [D_\mu, D_\nu] \Psi &= D_\mu D_\nu \Psi - D_\nu D_\mu \Psi \\ &= (\partial_\mu - ie A_\mu(x)) (\partial_\nu - ie A_\nu(x)) \Psi(x) - \nu \leftrightarrow \mu \\ &= -ie F_{\mu\nu}(x) \Psi(x) \end{aligned}$$

