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Actions with local $U(N)$ or $SU(N)$ invariance

Gauge fields: $B_\mu^a \rightarrow$ runs over generators T^a of $SU(N)$ or $U(N)$
(vector field - denoted by index μ)

$$\underline{B}_\mu = B_\mu^a T^a$$

$\rightarrow N \times N$ Hermitian matrices
traceless for $SU(N)$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - i [B_\mu, B_\nu]$$

$$S_{\text{gauge}} = -\frac{1}{2g^2} \int d^4x \text{Tr} (G_{\mu\nu} G^{\mu\nu})$$

invariant under $B_\mu \rightarrow U B_\mu U^{-1} - i (\partial_\mu U) U^{-1}$

In pert. theory, this doesn't contribute - it can't contribute to Feyn. rules

[if we consider $\int d^4x \text{Tr} (G_{\mu\nu} G^{\mu\nu})$ we can check that this is total derivative]

this is also gauge inv. - why not add this term? - though it has non-pert. effects, it doesn't contribute to pert.

In component form,

$$B_\mu^a \rightarrow R_{ab} B_\mu^b - \alpha_\mu^a$$

$$U T^a U^{-1} = R_{ba} T^b \rightarrow \text{defines } R_{ab}$$

$$i (\partial_\mu U) U^{-1} = \alpha_\mu^a T^a \rightarrow \text{defines } \alpha_\mu^a$$

For the boundary term from the total deriv. to vanish, you require fields to vanish - but do Feyn. rules, it can't contribute

$$S_{\text{fermion}} = \int d^4x \bar{\Psi} (i\gamma^\mu \underset{m}{D}_\mu - m) \Psi$$

where $\underset{m}{D}_\mu = \partial_\mu - i \underset{m}{B}_\mu$

Invariant under

$\Psi \rightarrow U\Psi$ together with
gauge trs. of $\underset{m}{B}_\mu^a$.

Suppose $R_U(U)$ represent U
& $R_A(T^a)$ represent T^a .

Here we have considered Ψ to be N -comp. - but it need not be necessarily N -comp.

Then $R_U(\mathbb{1} + i\epsilon^a T^a) = \mathbb{1} + i\epsilon^a R_A(T^a)$

Suppose $\Psi \rightarrow R(U)\Psi$

↳ defines

$R_A(T^a)$

$$\underset{m}{D}_\mu \Psi = \partial_\mu - i \underset{m}{B}_\mu^a R_A(T^a)$$

Define the repr. of the generators by this formula, given a repr. of the group

$$\int d^4x \bar{\Psi} (i\gamma^\mu \underset{m}{D}_\mu - m) \Psi$$

is gauge invariant.

Suppose a set of scalar fields

$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \end{pmatrix}$ transform as $\phi \rightarrow R_U(U)\phi$

Define: $\underset{m}{D}_\mu \phi = \left(\partial_\mu - i \underset{m}{B}_\mu^a R_A(T^a) \right) \phi$

Then, $(\underset{m}{D}_\mu \phi)^\dagger (\underset{m}{D}_\mu \phi)$ is gauge invariant.

NOTE $\underset{m}{D}_\mu \Psi \rightarrow R_U(U) \underset{m}{D}_\mu \Psi$ shows that the action is invariant.

