Consider a set of variables $u_1, u_2, \ldots, u_n$, which are the integration variables.

We want to calculate

$$\langle 0 \rangle = \frac{\int S''(\mathbf{u}) \, e^{i S'(\mathbf{u})} \, O(\mathbf{u})}{\int S''(\mathbf{u}) \, e^{i S'(\mathbf{u})}}$$

$n = 4 NK$ for pure gauge theory

$$\int \text{dim. of } G$$

$\# \text{ of lattice points}$

Symmetries:

$\begin{array}{c}
\text{Set of sym. tran.}
\{ \Phi \}
\text{labelled by } \theta_a's
\end{array}$

At each lattice pt., we will have $K$ gauge

$\text{trs. parameters}$

$- \text{So total } = NK$

$G \rightarrow \text{gauge group}$

$\mathcal{G} = G \times G \times \cdots \times G$ ($N$ times)

$S(\mathbf{u}) = S(F(\mathbf{u}, \mathbf{o}))$ action is invariant

$O(\mathbf{u}) = O(F(\mathbf{u}, \mathbf{o}))$ $O$ is a gauge invariant operator

$u_i = F_i(\mathbf{u}, \mathbf{o})$, then $\prod_{i=1}^{N} du_i = \prod_{i=1}^{N} du_i$

i.e.,

$$\det \left( \frac{\partial F_i(\mathbf{u}, \mathbf{o})}{\partial u_j} \right) = 1$$
If these are not satisfied, then there is no hope to get gauge invar. result because the integration measure in change of variables shouldn't change, in addition to $\mathcal{O}(\mu^1) = \mathcal{O}(F^2(\mu^3, \delta))$

Example: Check that $\prod_{\mu=0}^{\infty} \prod_{a=1}^{K} \mathcal{T}_0 A^a_{\mu}$ is gauge invariant.

(Recall that the integral measure doesn't change under gauge invariance in this case.)

We can separately take the upper limit $\mu = 0$ and integrate the above measure in $\delta$.

$\langle 0 \rangle = \frac{\mathcal{S}(\prod_{\mu=0}^{\infty} \mathcal{T}_0 \delta \mu) e^{i \mathcal{S}(\delta) \mu} \mathcal{O}(\delta)}{\mathcal{S}(\prod_{\mu=0}^{\infty} \mathcal{T}_0 \delta \mu) e^{i \mathcal{S}(\delta) \mu}}$

Suppose we have some representation of $G$.

Generators of $G$ are $T^a$ [$a = 1, 2, ...$].

Now, $U(0 + S\delta) U(\delta)^{-1} = 1 + i T^a \delta \mathcal{O}^a \rightarrow \delta \mathcal{O}^a$.

Our relation is independent of the rep. (or parameterised rep.) we choose.

The solution set, $\delta \mathcal{O}^a$ and $\delta \mathcal{O}$ will of course depend on the choice of rep.

A physical field can't be rep. by diff. reps. at diff. pts.

But mathematically, there is no reason why $\mu_1, \mu_2$ can't be in the same rep.
We don't make any attempt about the parametrisation of $U(1)$. We have$
abla^\alpha = \sum_{\beta} S_{\alpha \beta}(\varphi) \delta \theta^\beta \rightarrow$ general str. of $\delta \mathcal{O}$

Now,

$$N = \int \mathcal{D}u \cdot e^{iS(\varphi)} \mathcal{O}(\varphi)$$

Gauge transform of $\varphi$

This is $Nk$ dimensional (but it is parametrised by $\varphi^\alpha$):

The whole space is $4Nk$ dimensional.

For closed only, then integrate around it won't give $\times$

Total volume $= (N\kappa)^N$

$\rightarrow k$-dim. Group

[For each lattice site $\varphi_i$, we have a finite vol. $V_\alpha$ (for a finite group) - but there are $N$ such pts. - $\times$ in the $N \rightarrow \infty$ limit? we get $N$]

[In lattice gauge theory, we can't gauge fix because we take a finite num. of lattice pts. - $(N\kappa)^N$ cancels in $N \rightarrow \infty$ limit]
Gauge fixing condition:

\[ H_A(\vec{u}) = B_A \]

\[ \text{constants} \]

\[ F(\vec{u}, \vec{0}) = B_A \]

For any \( \vec{u} \), \( H_A(F(\vec{u}, \vec{0})) = B_A \) is true for a single \( \vec{0} \).

Naive guess: Insert

\[ \prod \delta(\vec{u} - B_A) \]

in the integrand.

But we can't recover the integral to begin with by this procedure, as we have seen already.

Think of \( \vec{u} \) as a fixed object here.

Begin with the identity:

\[ \prod \delta(x) \prod \delta(f_0(\vec{x})) = \frac{1}{\det \left( \frac{\partial f_0(\vec{x})}{\partial \vec{x}} \right)} \]

In other words,

\[ \prod \delta(x) \prod \delta(f_0(\vec{x})) \det \left( \frac{\partial f_0(\vec{x})}{\partial \vec{x}} \right) = 1 \]

Sum over all \( f_0(\vec{x}) = 0 \) for multiple zeros.

Dim. of the gauge slice

\[ = 4NK - NK = 3NK \]

We need to have NK eqn's to have 3NK-dim gauge slice in 4Kn-dim space.
\[ N = \int \mathcal{T} \mathcal{T} \, du \cdot e^{i \zeta(\overline{u})} \mathcal{O}(\overline{u}) \]
\[ \times \int \mathcal{T} \mathcal{T} \, d\overline{u} \cdot \mathcal{T} \mathcal{T} \delta \left( \mathcal{A}_A(\mathcal{F}(\overline{u}, \overline{\Theta})) - \mathcal{B}_A \right) \cdot \frac{\det \left( \partial \mathcal{A}_A(\mathcal{F}(\overline{u}, \overline{\Theta})) \right)}{\partial \overline{\Theta}} \]

(We will first do the \( \Theta \)-int. for a fixed \( \overline{\Theta} \) & then the \( \overline{\Theta} \)-int.)

Define \( \mathcal{V}_i = \mathcal{F}_i(\overline{u}, \overline{\Theta}) \)

and change variable from \( \overline{u} \) to \( \overline{\Theta} \).

Using:

\[ S(\overline{u}) = S(\overline{\Theta}) \]
\[ \mathcal{O}(\overline{u}) = \mathcal{O}(\overline{\Theta}) \]
\[ \mathcal{T} \mathcal{T} \, du = \mathcal{T} \mathcal{T} \, d\overline{u} \]
\[ \mathcal{V}_i \mathcal{V}_i = \mathcal{V}_i \mathcal{V}_i \]

Then:

\[ N = \int \mathcal{T} \mathcal{T} \, d\overline{u} \cdot e^{i \zeta(\overline{u})} \mathcal{O}(\overline{u}) \]
\[ \times \int \mathcal{T} \mathcal{T} \, d\overline{u} \cdot \mathcal{T} \mathcal{T} \delta \left( \mathcal{A}_A(\mathcal{F}(\overline{u}, \overline{\Theta})) - \mathcal{B}_A \right) \cdot \frac{\det \left( \partial \mathcal{A}_A(\mathcal{F}(\overline{u}, \overline{\Theta})) \right)}{\partial \overline{\Theta}} \]

(there is still some \( \Theta \)-dependence in this part of the integral so \( \Theta \)-dep. part is still not decoupled.)
\[ \frac{\partial H_A(F(\vec{w}, \vec{v}_B))}{\partial \vec{v}_B} = \frac{\partial H_A(\vec{v}_B)}{\partial \vec{v}_B} \cdot \frac{\partial F(\vec{w}, \vec{v}_B)}{\partial \vec{v}_B} \]

by Chain Rule

**Defn of** \( \frac{\partial F_i(\vec{w}, \vec{v})}{\partial \vec{v}_B} \)

\[ F_i(\vec{w}, \vec{v} + \delta \vec{v}) - F_i(\vec{w}, \vec{v}) = \frac{\partial F_i(\vec{w}, \vec{v})}{\partial \vec{v}_B} \cdot \delta \vec{v}_B \]

Use the group property

\[ U(\vec{v} + \delta \vec{v}) U(\vec{v})^{-1} = 1 + i \delta \vec{v} \cdot \vec{v}^A \cdot \vec{v}_A \]

\[ \Rightarrow U(\vec{v} + \delta \vec{v}) = (1 + i \delta \vec{v} \cdot \vec{v}^A \cdot \vec{v}_A) U(\vec{v}) = U(\delta \vec{v} \cdot \vec{v}^A) U(\vec{v}) \]

(This physically means) \[ \Rightarrow \] Trs. by \( \vec{v} + \delta \vec{v} = \text{trs. by } \vec{v} \] followed by a trs. by \( \delta \vec{v} \cdot \vec{v}^A \)

\[ F_i(\vec{w}, \vec{v} + \delta \vec{v}) = F_i(F(\vec{w}, \vec{v}^A), \delta \vec{v} \cdot \vec{v}^A) \]

\[ = F_i(\vec{v}, \delta \vec{v} \cdot \vec{v}^A) \]

\[ \delta \vec{v} \cdot \vec{v}^A \text{ is the change from } \delta \vec{v} \text{ to } \delta \vec{v} \]

The infinitesimal steps, that cancel, are those that do not mix \( \delta \vec{v} \) with \( \delta \vec{v}^A \)

- \( \delta \vec{v} \cdot \vec{v}^A \) is generic
- This is correct
The gauge transformation takes us from $0$ to $0 + 5\theta$.

For any Abelian group $G$,

$$S_{\omega} O = SO$$

of the group $G$ space has curvature.

Identity $I$.

What group element should you multiply with $A$ to give $U(\varphi, \omega) = \exp(-i\theta A)$?

These transformations follow group properties. So, the second element is the product of the first and third elements.

$\exp(-i\theta A)$ rotates the generators in the sense that it does not commute with $\theta A$.

Can't add the arguments of $\exp(-i(\theta + d\omega) T^A)$ and $\exp(-i\theta T^A)$.
\[ N = \int \left( \prod_A d\Omega^A \right) \int \prod_i d\phi_i \cdot e^{i S(\phi)} \delta(\phi) \]

This isn't a square matrix.

\[ \left\{ \frac{\partial H_A(\phi)}{\partial \phi_i} \bigg|_{\phi=0} \right\} \]

This isn't a square matrix.

\[ \det \left( \frac{\partial H_A(\phi)}{\partial \phi_i} \bigg|_{\phi=0} \right) \]

One square matrix.

\[ \det \left( \frac{\partial H_A(\phi)}{\partial \phi_i} \bigg|_{\phi=0} \right) \]

Another square matrix.

\[ \det \left( \frac{\partial f_i(\phi, \psi)}{\partial \phi} \bigg|_{\psi=0} \right) \]

\[ \left( \prod_A d\Omega^A \right) \det S(\phi) \int \prod_i d\phi_i \cdot e^{i S(\phi)} \delta(\phi) \]

\[ \prod_A \delta(H_A(\phi) - B_A) \]

Hence:

\[ \left( \prod_A d\Omega^A \right) \det S(\phi) \int \prod_i d\phi_i \cdot e^{i S(\phi)} \delta(\phi) \]

Factor out & cancels with the denominator.

\[ \det \left( \frac{\partial H_A(\phi)}{\partial \phi_i} \bigg|_{\phi=0} \right) \]

Haar measure (of the gp.)

\[ \tau = 0 \]

This is the standard measure of the gp. elements.

This measure is independent of the parametrisation.
Sampling is done in this region, the first case. Making a small slope, less grazing in the first case.

Slope set: how it samples.

So we are forced to include the det
\[
\frac{\partial \mathbf{R}(\mathbf{r})}{\partial \mathbf{r}} \cdot \frac{\partial \mathbf{F}(\mathbf{r}, \mathbf{\phi})}{\partial \mathbf{\phi}}
\]

To compensate for the fact that the sampling of pts. is deep on slope.

\[\text{Out: } \text{limit of summation - so this is necessary that sampling is uniform}\]

We have factored out the & cancelled out of the gauge group element. (But one factor for the config. space is still remaining)