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$$\int \prod_A d\theta_A \prod_A \delta(f_A(\vec{\theta})) \det\left(\frac{\partial f_A}{\partial \theta_B}\right) = 1$$

if $f_A(\vec{\theta}) = 0$ has a unique solution

Proof :-

$$\text{Define } y^A = f_A(\vec{\theta})$$

$$\text{Then } \left(\prod_A d\theta_A \right) \det\left(\frac{\partial f_A}{\partial \theta_B}\right) = \prod_A dy^A$$

$$\text{Then, } \int \prod_A d\theta_A \prod_A \delta(f_A(\vec{\theta})) \det\left(\frac{\partial f_A}{\partial \theta_B}\right) = 1$$

$$\text{bec } \int \prod_A dy^A \prod_A \delta(y^A) = 1$$

$$\langle \mathcal{O} \rangle = \frac{\int \left(\prod_s d\psi_s \right) e^{iS(\vec{\psi})} \mathcal{O}(\vec{\psi})}{\int \left(\prod_s d\psi_s \right) e^{iS(\vec{\psi})}} \equiv \frac{N}{\mathcal{Z}}$$

$S(\vec{\psi})$ & $\mathcal{O}(\vec{\psi})$ are invariant under

$$\psi_s \rightarrow F_s(\vec{\psi}, \vec{\theta}) \quad \{ \theta_1, \theta_2, \dots \}$$

\rightarrow gauge trs. parameters

If G is the group of trs., and if $U(\vec{\theta})$ represent the group element, then:

$$U(\vec{\theta} + \delta\vec{\theta}) U(\vec{\theta})^{-1} = \mathbb{1} + i T^A \delta\omega^A \rightarrow \text{defines } \delta\omega^A$$

$$\delta\omega^A = S_{AB}(\vec{\theta}) \delta\theta^B + \dots \rightarrow \text{defines } S_{AB}(\vec{\theta})$$

$$F_S(\vec{u}, \vec{\theta} + \delta\vec{\theta}) = F_S(\vec{F}(\vec{u}, \vec{\theta}), \delta_w \vec{\theta})$$

Using this we showed

$$N = \int \left(\prod_A d\theta^A \det S \right) \int \left(\prod_S d\psi_S \right) e^{iS(\vec{u})} \mathcal{O}(\vec{\psi}) \left(\prod_A \delta(\mathcal{H}_A(\vec{\psi}) - B_A) \right) \det \left(\begin{array}{cc} \frac{\partial \mathcal{H}_A(\vec{\psi})}{\partial \psi_S} & \frac{\partial F_S(\vec{u}, \vec{\psi})}{\partial \psi_C} \end{array} \right)$$

← this matrix can be written as →

$$M_{AC}(\vec{u}) \equiv \frac{\partial \mathcal{H}_A(\vec{F}(\vec{u}, \vec{\psi}))}{\partial \psi^C} \Big|_{\vec{\psi} = 0}$$

because \vec{F} at $\vec{\psi} = 0$ is \vec{u}

$$\therefore N = \left(\int \left(\prod_A d\theta^A \right) \det S(\vec{\theta}) \right) N'$$

where $N' = \int \left(\prod_S d\psi_S \right) e^{iS(\vec{u})} \mathcal{O}(\vec{u})$

$$\left(\prod_A \delta(\mathcal{H}_A(\vec{u}) - B_A) \right) \det M(\vec{u})$$

changing the int. variable ψ_S to u_S

Now, $D = \left(\int \left(\prod_A d\theta^A \right) \det S(\vec{\theta}) \right) D'$

where $D' = \int \left(\prod_S d\psi_S \right) e^{iS(\vec{u})} \prod_A \delta(\mathcal{H}_A(\vec{u}) - B_A) (\det M(\vec{u}))$

$$\frac{N}{D} = \frac{N'}{D'}$$

(We don't know how to write feyn. rules in presence of $\prod_A \delta(\mathcal{H}_A(\vec{u}) - B_A)$ & $\det M(\vec{u})$ — so we need some more manipulation)

