How is ϕ^4 theory renormalizable?
According to the given criteria we must add ϕ^3 vertex.

$$\int g^{(3)} \phi^3 \, d^4x$$

$$[g^{(3)}] = 1$$

$$S = \int \left(-\tfrac{1}{2} \partial^\mu \phi \partial_\mu \phi - \tfrac{1}{2} m^2 \phi^2 - g/_{4!} \, \phi^4 \right) d^4x$$

has a symmetry under $\phi \to -\phi$

action doesn't change

$\Rightarrow$ All n-point fn vanishes identically for odd n. $\to$ there is no 3-pt. fn. $\downarrow$ no diag to contribute to it

3-pt. fn is identically zero

$\underset{\text{hence}}{\Rightarrow}$ we do not need a 3-pt. vertex to cancel divergence.

Modification $\to$

If there is an underlying symmetry, then it is enough if we add all terms in the action which are invariant under that symmetry and whose coefficients have dimension ≥ 0.

terms not inv. under this sym. aren't generated

For this to work, the regularized theory must also have that symmetry.

e.g. $\to$ $\phi \to -\phi$ symmetry holds in $(4-\epsilon)$ dimension also.

(If regularization breaks this sym, you need the counterterm)

(Most of the reqd. sym. are preserved by dim. reg.)

$$\left\langle \prod_{i=1}^{N} \phi(x_i) \right\rangle = \underbrace{(-1)^N}_{\text{must be } 1} \left\langle \prod_{i=1}^{N} \phi(x_i) \right\rangle$$

$$\int [\mathcal{D}\phi] e^{iS[\phi]} \prod_i \phi(x_i) \Big/ \int [\mathcal{D}\phi] e^{iS(\phi)}$$

Now $\chi = -\phi$

$$= \frac{\int [\mathcal{D}\chi] \overbrace{e^{iS[-\chi]}}^{= e^{iS[\chi]}} \prod_{i=1}^{N} \chi(x_i) (-1)^N}{\int [\mathcal{D}\chi] \underbrace{e^{iS[-\chi]}}_{= e^{iS[\chi]}}}$$

$$= (-1)^N \left\langle \prod_{i=1}^{N} \phi(x_i) \right\rangle$$

$$\frac{\partial}{\partial k^2} \int \frac{(d\,l)^k}{(l)^{\alpha-2}}$$

$$\underset{\text{div.}}{A} + \underset{\text{div.}}{Bk^2} + \text{finite}$$

we must have $(n+2)\left(\frac{2-D}{2}\right) + D - 2 \neq 0$ for coupling const.

in $D=2$ dim.

$$(n+2)\left(\frac{2-D}{2}\right) + D = 2$$

$(n+2)$ pt. fn. has max. deg. of divergence

$$= (n+2)\frac{2-D}{2} + D = 2$$

For $\phi^n \partial_\mu \phi \partial^\mu \phi$, to cancel the div. for