

7/9/06

λ_{\max} of T \Downarrow $T'' = (U_2)^{1/2} U_1 (U_2)^{1/2}$
 λ_{\max} of T''

where $U_2 = \exp\left(\beta^* \sum_j \tau_j^z\right)$
 $U_1 = \exp\left(\beta \sum_i \tau_x^i \tau_x^{i+1}\right)$

$\tau_{\pm}^z = \frac{1}{2} (\tau_x^z \pm i \tau_y^z)$

~~$U_2 = \exp\left(\beta^* \sum_{j=1}^N (\tau_{+}^z + \tau_{-}^z - 1)\right)$~~

$U_2 = e^{\beta^* \sum_{j=1}^N (\tau_{+}^z + \tau_{-}^z - 1)}$

$U_1 = e^{\beta \sum_{j=1}^N (\tau_{+}^z + \tau_{-}^z) (\tau_{+}^{z+1} + \tau_{-}^{z+1})}$

Introduce new variable c_j^+, c_j^-

$c_j^+ = \exp\left(-i\pi \sum_{m=1}^{j-1} \tau_m^+ \tau_m^-\right) \tau_j^+$

$c_j^- = \tau_j^- \exp\left(+i\pi \sum_{m=1}^{j-1} \tau_m^+ \tau_m^-\right)$

$c_j^+ c_j^- = \tau_j^+ \tau_j^- \left. \begin{array}{l} c_j^+ = \exp\left(-i\pi \sum_{m=1}^{j-1} c_m^+ c_m^-\right) \tau_j^+ \\ c_j^- c_j^+ = \tau_j^- \tau_j^+ \end{array} \right\}$

$c_j^- c_j^+ = \tau_j^- \tau_j^+ \Rightarrow \tau_j^+ = c_j^+ \exp\left(i\pi \sum_{m=1}^{j-1} c_m^+ c_m^-\right)$

(Note that $\exp\left(-i\pi \sum_{m=1}^{j-1} \tau_m^+ \tau_m^-\right)$ commutes with τ_j^+ because $m \neq j$)

Also, $\tau_j^+ = \exp\left(i\pi \sum_{m=1}^{j-1} c_m^+ c_m^-\right) c_j^+$

Similarly, $\tau_j^- = c_j^- \exp\left(-i\pi \sum_{m=1}^{j-1} c_m^+ c_m^-\right)$

c_j^+ at a given site depends on all other sites

c_j^+ are non-local operators relative to τ_j^+

It is better to think as a non-local change of variables instead of thinking them as operators

You can think τ_j^+ to be a local operator

$$\tau_+ \tau_- = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$e^{i\pi \tau_+ \tau_-} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$e^{-i\pi \tau_+ \tau_-} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$e^{2i\pi \tau_+ \tau_-} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\therefore e^{2i\pi \tau_+^j \tau_-^j} = I$$

$$e^{2i\pi C_j^\dagger C_j} = I$$

This is possible only bcos $\tau_+ \tau_-$ has diagonal entries; otherwise we would have to diagonalise & then $\exp(\dots)$

Direct prodn with identity every where except at the j th place

$$\exp(i\pi C_j^\dagger C_j) = I \otimes \dots \otimes I \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes I \dots \otimes I$$

$$\therefore \frac{2^N}{2} = 2^{N-1} \text{ e. values are } +1$$

$$\& \frac{2^N}{2} = 2^{N-1} \text{ " are } -1$$

$$\begin{aligned} \{C_j, C_k\} &= C_j C_k + C_k C_j \\ &= \tau_j^{-1} \exp\left(i\pi \sum_{m=1}^{j-1} \tau_m^+ \tau_m^-\right) \\ &\quad \exp\left(i\pi \sum_{m=1}^{k-1} \tau_m^+ \tau_m^-\right) \tau_k^- \\ &\quad + \tau_k^{-1} \exp\left(i\pi \sum_{m=1}^{k-1} \tau_m^+ \tau_m^-\right) \\ &\quad \exp\left(i\pi \sum_{m=1}^{j-1} \tau_m^+ \tau_m^-\right) \tau_j^- \end{aligned}$$

these exponents commute (for diff. sites they obviously commute; at the same site they are the same op. & hence commute)

